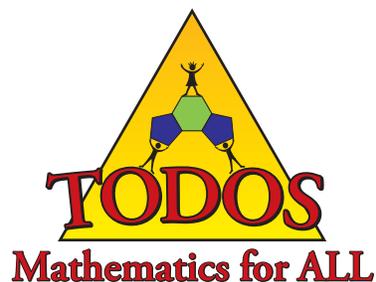


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# TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS



An Affiliate Organization of the National Council of Teachers of Mathematics



# ***TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS***

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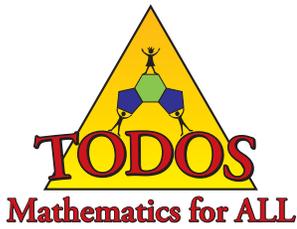
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## *TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS*

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## From the Editors

This issue reaches you a few months before the TODOS 2020 conference (<https://www.todos-math.org/todos-2020-conference->) where we hope to see many of you, authors, reviewers, and readers of *TEEM*. For the conference, we are preparing the second special issue of *TEEM* (the first one was in 2016, *TEEM* 7, “Mathematics Education: Through the Lens of Social Justice”). This upcoming special issue will be “Teaching Mathematics with Multilingual Learners: Actions and Innovations.” We received quite a large number of submissions and the editorial team for this special issue has been working hard on what we know will be an exciting and strong contribution to the field. If you have an idea for a special issue, feel free to email us ([teem@todos-math.org](mailto:teem@todos-math.org)). Keep in mind the guidelines and scope for *TEEM* (<https://www.todos-math.org/newteemwb>) and the TODOS mission (<https://www.todos-math.org/mission-goals>).

Over the last year we have noticed an increase in the number of submissions. This is great news! Keep them coming. This has led us to expand our editorial team. Starting with the next regular *TEEM*, Anthony Fernandes from the University of North Carolina at Charlotte will be joining us. Anthony has been a regular reviewer for *TEEM* for many years and brings a wealth of expertise to the journal. He is already busy handling some submissions. Welcome, Anthony!

This first issue of the eleventh volume of *TEEM* has three externally-reviewed articles covering topics that are particularly relevant to TODOS members, such as equity considerations in group work and topics related to dual language and multilingual settings, all of them in relation to mathematics education.

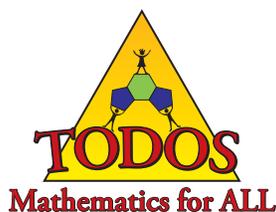
In “Teachers’ Grouping Strategies: Implications for Equity,” Cara Haines and Charles Munter invite readers to reflect on their strategies for equitably organizing small-group work. They do so by sharing the results of an interview-based study in which a group of secondary mathematics teachers described what they considered equitable approaches to arranging students for small-group work, and by considering the implications of those strategies for different dimensions of equity in the classroom.

The article by Rachel Reeder, “Twenty-One Days of First Grade Spanish Dual Language Immersion: A Nurturing Mathematics and Linguistics Incubation,” presents and illustrates four mathematics and linguistics practices as a framework in a first grade Dual Language Immersion classroom. The author offers details about these practices and examples of formal mathematics instruction.

In “Experiencing Multilingualism in the Math Classroom: A Conversation Starter with Teacher Candidates,” Laura Kennedy and Sandra Crespo share data and insights from a refined, experiential learning activity with prospective teachers. This activity was designed to help teachers consider how to support emerging multilingual students. The teachers’ reflections demonstrate the importance of building awareness and understanding of linguistic diversity in the mathematics classroom, but also suggest the need for more conversations and supports for teacher candidates to move beyond awareness.

As always, as editors, we are extremely grateful for the dedication and expertise of all our reviewers and authors. We are also very appreciative of the excellent editorial support provided by Associate Editor Lawrence M. Lesser and Layout Editor Susie W. Håkansson. *TEEM* gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* continues to serve the TODOS membership and that this issue serves as a resource for the community and a source of inspiration for future contributions to the journal.

**Marta Civil, Ksenija Simic-Muller, M. Alejandra Sorto, and Craig Willey**



## Teachers' Grouping Strategies: Implications for Equity

**Cara Haines**  
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### Abstract

In this article, we invite readers to reflect on their strategies for equitably organizing students for small-group work. We do so by sharing the results of an interview-based study of the ways in which a group of secondary mathematics teachers, working in an urban school district in which racial equity was an explicit focus, described equitable approaches to arranging students for small-group work. We share the grouping strategies that teachers described, and consider implications of those strategies for different dimensions of equity in the classroom.

### Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What are your strategies for organizing students for small-group work?
2. In what ways, if at all, do you consider each of those strategies as a means for working toward equity in your classroom?
3. Do you talk with colleagues and/or instructional leaders about strategies for grouping students? Explain.

**Cara Haines** ([carahaines@mail.missouri.edu](mailto:carahaines@mail.missouri.edu)) is a doctoral candidate in mathematics education at the University of Missouri. Her current research is focused on early-career teacher learning and the institutional settings in which such learning occurs.

**Charles Munter** ([munterc@missouri.edu](mailto:munterc@missouri.edu)) is an assistant professor of mathematics education at the University of Missouri. His research focuses primarily on inservice, secondary teacher learning and district-level policy contexts, including foci on characterizing high-quality instruction, the institutional setting of teaching, and designing for equity.

### Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 1321216. Thanks to the teachers for allowing us to interview them. Correspondence concerning this paper should be addressed to the first author.

## Teachers' Grouping Strategies: Implications for Equity

Cara Haines and Charles Munter

How can mathematics teachers work toward achieving equity in their classrooms? One of the more influential ways of thinking about the answer to that question in recent years has been in terms of Gutiérrez's (2007) four dimensions—*access*, *achievement*, *identity*, and *power*. The first two have been prominent in mathematics policy documents for many years (e.g., National Council of Teachers of Mathematics [NCTM], 2000). Students' *access* to and *achievement* in rigorous mathematics requires, for example, that students have opportunities to engage in and discuss tasks that promote reasoning and problem solving and develop conceptual understandings of the mathematics central to those tasks (Aguirre, Mayfield-Ingram, & Martin, 2013; Horn, 2012; NCTM, 2014).

Although less prominent historically, attention to Gutiérrez's (2007) other dimensions has been increasing in the field. *Identity* pertains to whether students feel like they can be themselves while they do mathematics (Aguirre et al., 2013; Gutiérrez, 2007). This means that as students learn mathematics, they must also experience affirmation of their racial, gender, and other social identities. Finally, *power*—at the classroom level—requires that students feel like their voice matters (Gutiérrez, 2009). This means that teachers must be aware of and work to negotiate relational dynamics in the classroom (Horn, 2012).

It is important to consider how such ways of *conceptualizing* equity can show up in the *practical* aspects of teaching. Here we consider how teachers' strategies for arranging students for small-group work – a common instructional practice often taken for granted – has implications for all of the dimensions of equity described above. Our discussion originates in what we have learned from listening to secondary mathematics teachers' perspectives on how strategies for arranging students in groups (of two or more) might act as a means for working toward equity in the classroom. The research question that guided this study was: How do teachers describe grouping strategies that are equitable,

and what are their rationales for employing those strategies?

### Method

This study was conducted within the context of a larger project that took a mathematics-specific approach to decreasing a racial opportunity gap (Flores, 2007) in a Northeastern U.S. urban school district serving a predominantly Black (55%) and white (33%) student population. The sample for this study included 37 secondary (Grades 6–12) mathematics teachers (8 of whom identified as people of color, and 29 as white) from two cohorts who were participants in the larger study. Teachers' participation in the project included two years of summer workshops that focused on confronting issues of racial inequity in mathematics. During these workshops, project leaders supported teachers in investigating and addressing inequities in school mathematics by: reconceiving what it means to know and do mathematics, discussing the historical marginalization of Black students, reflecting on their own (and students') racial and mathematical identities (Nasir, 2011), and developing and enacting ideas for more equitable practice (e.g., culturally relevant pedagogy, Ladson-Billings, 1995).

Our study's findings regarding teachers' perceptions of equitable grouping strategies are based on analyses of semi-structured interviews conducted with teachers during the first three years of the project. In those interviews, we prompted teachers to describe their perceptions of high-quality and equitable mathematics instruction. The project's research team—of which we were a part—attended explicitly to such perceptions in hopes that ongoing assessment of them would guide project leaders in supporting teachers' learning over time. For the analysis shared in this article, we focused primarily on teachers' responses to the following prompts:

- If you were to observe another teacher’s math classroom for one or more lessons, what would you look for to determine whether instruction was high-quality/equitable? Why?
- Would you expect to see students working in small- or whole-group settings? Why?
- If instruction were equitable, would you expect to see students grouped in a particular way? Why?
- Do you group your students? If so, how and why?

To analyze responses to those interview questions, we identified the grouping strategies that teachers described (*forms* of practice) and the corresponding rationales they provided, in terms of what they expected such strategies to achieve (intended *functions* of those practices) (Saxe, Gearhart, Franke, Howard, & Crockett, 1999). As alluded to earlier, in this article we use the findings of our analysis as an opportunity for reflection, as we focus primarily on how the grouping strategies and rationales that teachers described have implications for different dimensions of equity at the classroom level: access, achievement, identity, and power (Gutiérrez, 2007). In the sections that follow, we focus on two of the most common themes among teachers’ descriptions of grouping strategies (and likely common practices in many classrooms) and, for each, consider implications for equity and raise questions for further reflection.

### Grouping by “Ability”

Among the grouping strategies that teachers described as equitable, grouping students according to their perceived mathematics “ability” was one of the most popular. From about half of the teachers ( $n=19$ ), we heard arguments that both homogeneous ability grouping and mixed-ability grouping can be equitable strategies given their potential to afford opportunities for differentiation and student-to-student support. However, in their descriptions of how each of these grouping strategies might afford such opportunities, teachers framed student ability in two different ways: ability as a *static* trait of students, or ability as *fluid* and based on merely students’ current strengths and/or needs, with the former view much more common ( $n=17$ ) than the latter ( $n=2$ ). This is similar to the widely accepted (though also

critiqued—e.g., Kohn, 2015) notions of “growth mindset” versus “fixed mindset” (Dweck, 2006), particularly as it relates to *teachers’* mindsets about their students’ capabilities (Boaler, 2013). However, what we observed was less about teachers’ assumptions concerning general intelligence and more about whether they framed mathematical success as varying by topic.

As an example of framing student ability as static, consider the following teacher’s description of employing a mixed-ability grouping strategy:

You could kind of have like a mixed group where somebody who could be the top person in the class could be paired with somebody who’s a little bit less than that—and then another one who’s even lower than that. And have it in charge of the two kids to kind of help the smaller-achieving -- the lesser-achieving student.

In this response, the teacher suggested organizing small groups by “mixing” students according to the teacher’s own perceptions of students’ ability. Given that those perceptions are based on students’ achievement (i.e., “top person in the class”), the teacher’s framing of ability, here, implies a static view of what students are capable of doing during small-group work.

In contrast, other teachers framed student ability as subject to change, implying a more fluid view of students’ capabilities. Consider, for example, the way in which the following teacher described student ability in a rationale for employing a homogenous grouping strategy:

Sometimes [grouping is] based on ability. ‘Ok, you have this concept. You’re good. And this group over here doesn’t...’ So I can put that- you know, split them that way, and then I can work with the kids who don’t get it and give them some more individualized attention. The kids that have it can do some more on their own.

In this response, the teacher suggested homogeneously grouping students “based on ability,” but characterized ability not in terms of class ranking (as in the previous example), but in terms of proficiency with particular concepts. This implies a different approach to “ability grouping,” one based on students’ current strengths and needs, which may vary according to the mathematical concepts and skills students are learning.

## Implications for Equity

Across teachers' descriptions of both mixed-ability and homogeneous ability grouping, we found that either a "static" or "fluid" framing of ability could be invoked; there was not a one-to-one relation between type of grouping strategy and way of framing ability. Regardless of the grouping by "ability" strategy that teachers promote, differences in their framings of ability may lead to important differences in classroom equity. Here we consider the first two of Gutiérrez's (2007) four dimensions discussed earlier: students' *access* to learning opportunities; and students' *achievement*, or success.

Regarding implications for access, in cases in which teachers promoted grouping by static ability, only students whom teachers described as more capable than others were depicted as having consistent access to special, and perhaps richer, learning opportunities (e.g., being "in charge" of other students). While teachers who framed ability as fluid also described scenarios in which students with stronger understandings would have access to distinct opportunities (e.g., could "do some more on their own"), their views of ability may imply that the students with stronger understandings, and thus those with access to special opportunities, will consistently vary over time, depending on students' *current* strengths and needs.

Framing ability as a static trait may also perpetuate issues of academic status (Cohen, 1994; Horn, 2012). This can potentially lead to differences in the extent to which students experience *achievement*, including successful participation in small-group activity, and hinder the development of those students' mathematics identities. Research has shown, for example, that students with high status, or those consistently positioned as "in charge" of their peers, often issue directives to their classmates (Esmonde, 2009), isolate students with low status (Featherstone et al., 2011), and tend to dominate small-group discussions (Cohen & Lotan, 1997; Cohen, Lotan, Scarloss, & Arellano, 1999), ultimately diminishing some students' opportunities to meaningfully participate in small-group activity (Wood & Kalinec, 2012). Given the lack of participation—or *achievement*—that students with low status may experience, coupled with being consistently assigned to the "low-ability" group or grouped with "higher-

achieving" students, it may be especially difficult for those students to come to see themselves as smart, capable learners and doers of mathematics.

## Moving Beyond Grouping by (Static) Ability

Given the prevalence of accountability structures in our nation's education system, categorizing students according to their "abilities"—or, in many cases, achievement levels or test scores—is not uncommon. These broader narratives about mathematics ability are so pervasive that countering them is not an easy task, so teachers' framing of ability as a static trait is not necessarily surprising. However, given the potential impacts of such framings on students' access to learning opportunities and achievement, we argue that working toward classroom equity necessitates moving beyond conceptions of ability as static and focusing on the fluid ways that students develop and express forms of expertise and understandings.

We also recommend using grouping strategies that might minimize the risks posed by grouping according to ability—many of which are consistent with strategies promoted within the tradition of *Complex Instruction* (Cohen et al., 1999). For example, some research points to random grouping as a promising strategy (Horn, 2012). If coupled with continuous efforts to broaden notions of mathematical competence and public affirmation of different ways of participating (Cohen, 1994; Boaler & Staples, 2008; Horn, 2012), random grouping communicates that all students are capable of contributing to small-group work and, therefore, can alleviate status issues.

## "Diversity" Grouping

A second popular grouping strategy that teachers described as equitable was what we refer to as "diversity" grouping. Note that we write "diversity" in quotes: while we consider diversity to extend beyond social indicators such as race, gender, etc., because a majority of teachers focused explicitly on race and ethnicity, this is the type of "diversity" that we describe here.

Recall that at the time of our interviews, teachers were involved in professional development focused on issues of racial equity in secondary mathematics.

Therefore, that some teachers considered students’ race and/or ethnicity in relation to grouping is not surprising. Fifteen of the 37 teachers recommended arranging students in small groups in a way that mirrors the demographics of the whole class (e.g., if 25% of the students in a class were African American, then one would expect each small group of 4 to include one African American student). Most often, teachers did not provide a rationale for using this strategy, although they occasionally suggested that it could provide opportunities for students to get know and/or work with peers outside of their own social groups, or questioned the affordances of the strategy, implying that it is school leadership—and not necessarily teachers—who prefer “diversity” grouping. To help clarify, we provide examples of such descriptions in Table 1.

**Implications for Equity**

The responses about “diversity” grouping included in Table 1 suggest that teachers and, perhaps, school leaders consider the “mixing” of students with different racial identities as a means by which teachers can create a more equitable learning environment. While it may be possible that arranging racially-diverse groups of students could foster students’ appreciation of the variety of their peers’ ideas and, as suggested in the second response in Table 1, help students “get to know

each other a little bit more,” “diversity” grouping—or desegregation—is not inherently positive and may have negative implications for students’ *power* and *identity*, the other two of Gutiérrez’s (2007) four equity dimensions.

First, it is possible that “diversity” grouping may, over time, lead to *power* imbalances between the teacher and students. If, for example, teachers consistently decide with whom their students will work in small groups, or, on occasion, intentionally “break up” groups that students had chosen for themselves, then “diversity” grouping may diminish the power of students’ voice, decision-making, and sense of agency in the classroom.

We also wonder whether, in some cases, teachers’ suggestions to “mix” or “break up” racial groups of students may be rooted in ideologies of Black inferiority, which were invoked in the 1950s to grossly mischaracterize the real motivations for desegregation efforts: the unequal distribution of resources and unconstitutionality. If so, teachers’ decisions to group for “diversity”—or desegregate—may communicate negative views of Blackness (Martin, 2012) and, consequently, inhibit the development of students’ racial *identity*. That is, if teachers do in fact separate Black students because of race, then those students may be deprived of opportunities to feel that their mathematics classroom is a place where they can be themselves.

Table 1  
*Teachers’ Rationales for “Diversity” Grouping*

<b>Rationale</b>	<b>Representative Excerpts</b>
No rationale	<p>“I don’t think I’d like to see an all-girls group or an all-boys group. Would I have a problem with all African Americans, or all whites? Um, yeah, I don’t know- I just think that we need to disperse... You just need to mix it around.”</p> <p>“If I do see that one group is all African American males then yes, that is an issue. Um, that, that needs to be broken up.”</p>
Students can get to know/work with others outside of their own social groups	<p>“I have done, on purpose, groups where I’ve had one [student] of each [racial/ethnic group]. So like, they know each other, but here’s to get to know each other a little bit more...I feel that everybody should learn each other’s culture. So, I really do try to group them so that they can learn each other, like get to know each other better.”</p>
The strategy is preferred by district leadership	<p>“I have come to the realization that students are accomplishing more and having more conversations if I’m not forcing them to work with somebody specific...I know [my principal] doesn’t wanna see all African Americans in this section and all whites over here, but making sure that all students are mingled.”</p>

## Moving Beyond “Diversity” Grouping

We do not suggest that teachers should never make decisions about how or with whom students are grouped, and recognize that not all teachers who espouse “diversity” grouping have deficit views of students of color. However, we argue that there are more productive ways to respond to student diversity in the classroom, which likely require teachers to think about aspects of their instructional practice beyond grouping strategies (as many of the teachers we spoke to undoubtedly did). For example, teachers might positively influence students’ identity development and empower students by posing authentic tasks that are relevant to students’ lives and encouraging them to draw on their cultural and community “funds of knowledge” (Moll, Amanti, Neff, & González, 1992) as they problem solve (Aguirre et al., 2013).

Additionally, rather than intentionally breaking up groups of students, teachers may consider inviting students to choose their own groups more regularly, as suggested by the last teacher’s response included in Table 1. This is not to suggest that teachers should always allow students to choose their groups, because it is also important to foster positive relationships and collaboration among all students in the classroom. However, affording such opportunity for choice in the classroom can provide an important support for students’ developing sense of autonomy (Williams, Wallace, & Sung, 2016). Additionally, as psychologist and educator Beverly Tatum explained in her book, *“Why Are All The Black Kids Sitting Together in the Cafeteria?” and Other Conversations About Race*, for students of color, time interacting in school with peers in one’s own racial identity group is often important for adolescents’ racial identity development, and that “connecting with one’s Black peers in the process of identity development is important and should be encouraged” (1997, p. 69). Therefore, shifting decision-making power from teacher to students may not only provide opportunities for students’ voices to be heard and preferences to be honored, but, in some cases, may also contribute to creating a classroom environment in which students feel like they can be themselves.

## Pursuing Equity Potential of Small-Group Work

As illustrated by the examples in this article, our investigation revealed variation in the grouping strategies that teachers promoted and the ways in which they described student ability and diversity. Through our discussion of how those differences may have important implications for students’ *access, achievement, identity, and power*, we invited teachers to reflect on how their own grouping strategies and views of students may influence classroom equity. Additionally, as alternatives to ability-based and diversity-based grouping strategies, we offered practical ways in which teachers may work to avoid the potential pitfalls of and move beyond those strategies:

- Resist their own static views of student ability and focus on the fluid ways that students develop and express forms of expertise and understandings.
- Randomly group students and publicly affirm different ways of participating to communicate that all students are capable of contributing to small-group work and alleviate status issues (Horn, 2012).
- Question whether “desegregation” in the context of organizing students for small-group work is an inherently good thing.
- Leverage student diversity through other practices, such as posing authentic tasks that are relevant to students’ lives and encouraging students to draw on their cultural and community knowledges and experiences as they problem solve (Aguirre et al., 2013).
- Occasionally or regularly invite students to decide who they work with (Williams et al., 2016).

It is our hope that considerations such as these might support teachers in their instructional decision-making, and that such reflection might aid them in being even more purposeful in their use of common practices such as small-group work.

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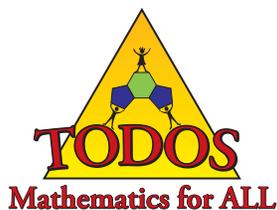
### **Discussion And Reflection Enhancement (DARE) Post-Reading Questions**

1. In your view, what reinforces “static” views of student ability (e.g., labels based on standardized testing, tracking, etc.)?
2. In what ways beyond what were discussed in this article might static views of ability be harmful for students?
3. What implications does adopting a “fluid” view of students’ mathematical capabilities have for the nature of small-group activity?
4. What concerns does the idea of allowing students to choose their own small groups raise for you (with respect to both social and mathematical aspects of the classroom)? Are there productive ways to address those concerns?
5. Have you felt inclined to “desegregate” your classroom? If so, what do you perceive to be underlying that inclination?

***The mission of TODOS: Mathematics for ALL is to advocate for equity and high quality mathematics education for all students—in particular, Latina/o students.***

Five goals define the activities and products of TODOS: Mathematics for ALL

1. To advance educators' knowledge and ability that lead to implementing an equitable, rigorous, and coherent mathematics program that incorporates the role language and culture play in teaching and learning mathematics.
2. To develop and support educational leaders who continue to carry out the mission of TODOS.
3. To generate and disseminate knowledge about equitable and high quality mathematics education.
4. To inform the public and influence educational policies in ways that enable students to become mathematically proficient in order to enhance college and career readiness.
5. To inform families about educational policies and learning strategies that will enable their children to become mathematically proficient.



## **Twenty-One Days of First Grade Spanish Dual Language Immersion: A Nurturing Mathematics and Linguistics Incubation**

**Rachel Reeder**  
Utah State University

### **Abstract**

Analogous to a 21-day incubation period of a chick, four mathematics and linguistics practices are employed as a framework in a first grade Dual Language Immersion classroom. These practices include: 1) Concrete-Representational-Abstract instructional sequences, 2) Receptive and Productive Language Modalities, 3) Gradual Release of Responsibility, and 4) Number Representations and Subitizing. The article illustrates how the framework can be implemented by offering details about mathematics and management routines and examples of formal mathematics instruction.

### **Discussion And Reflection Enhancement (DARE) Pre-Reading Questions**

1. How would you describe a modeling sequence employed during mathematics instruction?
2. Identify a couple of examples from your own mathematics instruction where students are given the opportunity to use productive modes of communication (i.e., speaking and writing).
3. Describe the role of regular routines in mathematics instruction and discuss the pros and cons of maintaining the routines without deviation.

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## Twenty-One Days of First Grade Spanish Dual Language Immersion: A Nurturing Mathematics and Linguistics Incubation

Rachel Reeder

Twenty-one days makes a notable difference in the linguistic and conceptual development of first graders in a Dual Language Immersion (DLI) math class, similar to that of a 21-day incubation period for chicks. Science affirms a strong environmental impact on embryonic development with repercussions that impact future growth (De Smit et al., 2006). A similar impact is observed on student development during the initial days of instruction in an urban first-grade classroom. The purpose of this article is to share a 21-day curriculum model designed to promote rapid language acquisition through mathematics routines and activities.

The framework outlined below is recommended for settings such as the one illustrated in this article, where it has been successful, although not formally tested. The first-grade Spanish DLI program where English is the home language (L1) of half of the students and Spanish the L1 of the other half presents a unique need for this approach. All of the students come from kindergarten classes where instruction was delivered only in English, requiring intentional preparation for first grade mathematics content instruction in Spanish. Effective beginning-of-year routines that simultaneously review preliminary mathematics content and present linguistically-rich opportunities to engage with

increasingly complex curriculum help facilitate this process.

### Framework: A Nurturing Mathematics and Linguistics Incubation Period

“*Los pollitos dicen pío, pío, pío . . .*” (Orozco, 1999). This song about peeping chicks is introduced on the first day, along with the counting sequence “*uno, dos, tres . . . diez*” (one, two, three . . . ten). Neither the song nor the numbers are familiar to most students as a result of their prior educational experience. Skillful mathematics instruction serves as the context and catalyst for rapid language acquisition in the days and weeks that follow.

The song about the *pollitos* further explains how the hen provides food, shelter, and warmth, elements that offer nurturing for the chick in the same way that skillful pedagogy affords appropriate student growth. Like the hen with her chicks, caring expert-educators instinctively identify the needs of their students and use best practices to meet these. With the pacing and emphasis of this model, both novice and expert DLI teachers benefit from an explicit awareness of the interdependence of linguistic and mathematical development in the first weeks of DLI Spanish math instruction.

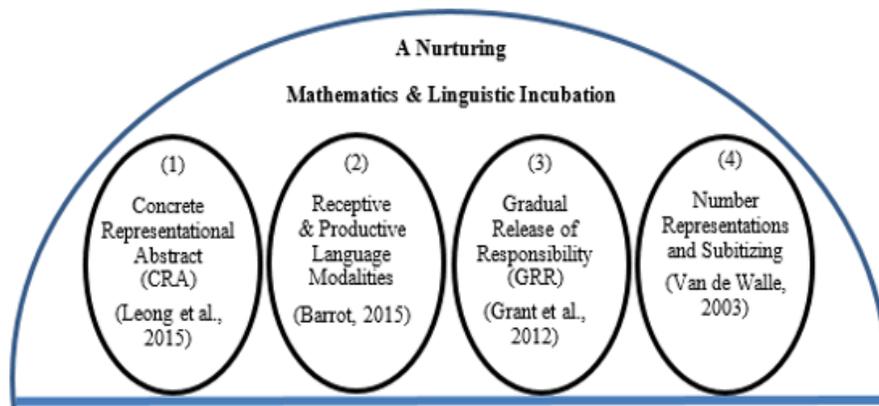


Figure 1. Progression of nurturing mathematics and linguistics incubation.

Four elements of research-based pedagogy (Figure 1) interact to ensure comprehensible input and contribute to essential mathematics and linguistics development. Each “egg” in the framework represents a continuum of progression that accelerates student development, particularly in the incubation period of first grade DLI. The four “eggs” demonstrate how mathematics and linguistic developmental components are unique, as well as interdependent, in the context of mathematics instruction. They each play a role in the first 21 days of first-grade Spanish DLI, similar to the food, shelter, and warmth offered to the *pollitos* as a prescribed incubation.

### **Research Foundations of the Nurturing Incubation Framework**

The following descriptions of research foundations for the progression are intentionally brief, focusing on the pedagogical practices outlined in the subsequent sections.

#### **Concrete-Representational-Abstract (CRA) Instructional Sequences**

**Mathematics acquisition.** Bruner’s (1966) modes of representation: enactive (action-based), iconic (image-based), and symbolic (language-based) created the foundation for what is now the CRA approach to mathematics instruction. The concrete level includes actions, experiences, and tangible manipulatives. This is followed by a transition to representational use of picture and two-dimensional models. Finally, the abstract level incorporates the use of symbols and formulaic terms (Leong et al., 2015). Attention to this sequence accelerates progress in both linguistics and mathematics learning.

#### **Receptive and Productive Language Modalities**

**Language acquisition.** The four language modalities are classified as receptive (listening and reading) and productive (speaking and writing), with the linguistic

complexity increasing in productive mode (Barrot, 2015). Attention to the developmental sequence and linguistic demands affords sheltered instruction and scaffolding for language development during mathematics instruction.

#### **Gradual Release of Responsibility (GRR)**

**Instructional model.** I do – We do – They do – You do. The GRR framework captures the theory that the teacher must model explicitly what she wants students to do before expecting students to perform the task (Grant, Lapp, Fisher, Johnson, & Frey, 2012). Following the teacher model, a pair or group of students demonstrates the task, and then individual students repeat the example. This standard in DLI instruction helps mediate the lack of language that exists at the beginning of the school year.

#### **Number Representations and Subitizing**

**Acquisition of number sense.** There is unique power in ten-frames, dot patterns, tally marks, and other number representations to facilitate subitizing in order to build number sense and foster future learning. Multiple representations offer valuable exposure to and familiarity with number as a foundation for addition and subtraction models – part, part, whole (Van de Walle, Karp, & Bay-Williams, 2015).

### **Use and Implementation of the Nurturing Incubation Framework**

The framework provides the theoretical foundation for the first 21 days of school curriculum for promoting language development through mathematics instruction. The routines and formal instruction are woven together seamlessly in a daily exchange that balances the familiar with the new in order to maintain student engagement and facilitate linguistics and mathematics growth. Figure 2 outlines the framework as a simultaneous interaction of the four elements of the incubation period.

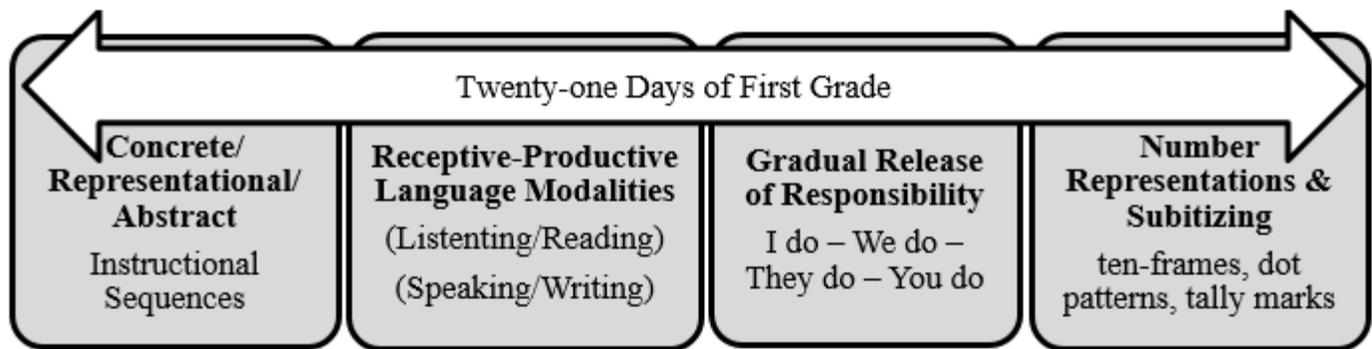


Figure 2. First 21 days of first grade.

**Preliminary First Grade Mathematics in Spanish  
DLI**

First grade mathematics curriculum begins with identifying and representing numbers. In Spanish immersion, students are initially exposed to counting and number through whole-class mathematics and management routines such as calendar, seating arrangements, distribution of supplies, and transitions. Fingers afford a familiar beginning to number sense, followed by a variety of objects and visuals such as ten-frames, drawings, and dot patterns which lead to symbolic representations. Additional experiences are presented through the context of formal mathematics instruction, modeling, and practice. These two approaches represent a practical implementation of the framework (Table 1).

- Informal Mathematics Exposure to Number: Mathematics and Management Routines – calendar, counting, seating configurations, students’ alphabetical number assignment, time awareness, and quantity comparison

- Formal Mathematics Instruction of Number Sense:
  - Number Representations (models) – fingers, digits, dice dot patterns, tally marks, ten-frames, pictures, words
  - Mathematics Tasks – matching, sorting, writing, speaking

The following sections provide examples of practices embedded within the framework.

**Mathematics and Management Routines**

Informal exposure to number takes place through daily routines that establish both a linguistic and mathematical foundation. Some of the routines are intended for explicit exposure to linguistics content while others have an organizational purpose to reinforce management expectations. Both kinds of routines present motivating and meaningful opportunities to reinforce mathematical content.

Table 1  
*Framework Implementation*

Incubation Period	Informal Exposure to Number Mathematics and Management Routines			Formal Mathematics Instruction of Number	
21 Days	Calendar Routines	Seating Arrangements & Expectation Reinforcement	Organization & Pacing	Whole Class Small Groups Independent	Matching Activity Sorting Activity Modeling Sequence Writing Activity

## Calendar Routines

Sitting on the carpet, eyes directed toward the calendar, the student voices repeat in a *call and response* exchange, echoing each phrase of the pattern “*primer día, segundo número, tercer mes, cuarto año*” (day first – number second – month third – year fourth). They follow this four-part sequence, reciting in Spanish the days of the week, days of the month, months of the year, and the year’s number and combining them for the current date (i.e. martes, 27 de agosto de 2019).

**Day of the week.** Cheerful little voices reflect the teacher’s enthusiasm as they sing to the tune of *Are You Sleeping?* “*Siete días, siete días, de la semana, de la semana, lunes – martes – miércoles – jueves – y viernes – sábado – y domingo.*” (Seven days, seven days, of the week, of the week, Monday – Tuesday – Wednesday, Thursday and Friday, Saturday and Sunday). They raise their fingers until they have five and two, concluding the song “*Cinco días sí hay escuela, dos días no hay escuela*” (Five days there is school, two days there is not.)

**Days of the month.** The same voices mimic the teacher model singing “*uno – dos – tres, cuatro – cinco – seis, siete – ocho – nueve – diez . . . veintitrés*” (August 23, the first day of school).

**Months of the year.** Another tune that the teacher models helps guide the student voices as they sing “*enero – febrero – marzo . . . diciembre – doce meses en el año*” (January – February – March . . . December – twelve months in the year).

**Year.** Finally, with a rousing cheer, the voices call out “*dos mil diecinueve, dos mil diecinueve*” (two thousand nineteen – two thousand nineteen).

## Seating Arrangements and Expectation Reinforcement

The following configurations are designed for organizational and management purposes, but also offer essential exposure to number.

**Numbered carpet squares.** Once students choose their spot at the carpet they await the chance to be recognized through positive reinforcement. The thirty small, colored paper squares elicit excitement from the students as they

eagerly observe the color and then the number, and call out together “*azul . . . dos*” (blue . . . two) identifying the carpet square where a particular student is seated. Five sets of colored papers are numbered 1-6 corresponding with the rows of the carpet and are used to reinforce student expectations as well as to draw attention to linguistic and numeric detail in the environment. In this case, a DLI routine is employed, and reinforced, to elicit language from all individuals. Students place a hand on their head to show they know the answer and when the teacher lifts her hand from her head all students are cued to respond chorally, with the benefit of peer models.

**Counting for task completion.** A brief transition for a few students provides an opportunity for class-wide counting that keeps everyone engaged and holds all accountable to ensure timely task completion. The students who are asked to distribute or collect supplies do so as their peers count “*Uno, dos, tres . . . once, doce, trece, catorce, quince, dieciséis, diecisiete, dieciocho, diecinueve, veinte.*” Students may struggle to coordinate unfamiliar words with finger models as they count 0-20. This offers exposure to the often-challenging numbers 11-20, before individual production is expected.

## Organization and Pacing

**Assigned student numbers.** Initial ownership of assigned student numbers comes both through proper identification of backpack hooks and cubbies and the daily rotation of leadership roles. Most students quickly identify that if student 7 had a turn the previous day it will be leader 8’s turn, and they confidently place their hands on their heads ready to call out together “*Marcela, te toca*” (Marcela, it is your turn). The rotation of daily leadership roles presents a model of sequential numbers (one after another, or by twos) demonstrated with sums and differences as well as skip counting. Table groups are numbered as well, giving students the opportunity to respond to that label using variations of evens/odds and sums/differences as they are linguistically and mathematically capable.

**Time awareness and quantity comparison.** A class-wide number fluency task offers students the opportunity to see, think, wait, and respond in unison to correctly identify numbers. This process presents a motivating opportunity when timed and repeated to determine if the

whole class can correctly identify more or less than ten (or twenty) numbers in one minute. The same procedure is used in small-group fluency practice to increase accountability for Spanish number recognition.

### Formal Mathematics Instruction

Explicit mathematics instruction is introduced through gradual release of responsibility (GRR). Whole class matching games allow for introduction to conceptual and procedural components. Teacher and peer models that precede the individual student sorting activity lay the foundation for expectations that emphasize correct Spanish language use and mathematical practice. Finally, students write and produce their own number representations according to the specified and well-practiced models.

### Whole Class Matching Activity

A matching game works well to introduce the concept of multiple representations of numbers. Prior to placing the cards face down in a concentration game arrangement, they are placed face up in sequential order side-by-side, each with its match (pairs of pictures and digits), thus familiarizing students with the goal of pairing representations that are identical in quantity. The students are then shown a 5-by-4 grid arrangement of 20 cards, where rows are labeled with numbers 1 to 5 and columns are labeled with letters A-B-C-D. They are instructed how to select a card orally by identifying a number and letter combination, such as *cuatro-C* (4-C) from the rows and columns arrangement.

The whole-class matching activity provides the initial model for the small-group sorting activity students engage in afterward, which in turn prepares them to independently represent the numbers in writing. The students do not create their own representations of the numbers in the matching and sorting activities, but utilize receptive modes that prepare them for later production.

### Small Group Sorting Activity

Six sets of sorting cards each representing numbers 0-10 differently are used for practice: 1) fingers, 2) digits, 3) dot patterns (like dice), 4) pictures, 5) tally marks, and 6) ten-frames. Three sets at a time are practiced so that 33

cards are sorted into three piles. The cards are placed face down in a central pile where students turn them over one at a time, and together state both the number and form of representation and then model it with counters, a concrete representation of the pictorial exmple. After students model numbers and group the cards. they arrange each set sequentially from zero to ten.

For example, at one table, Scott turns a card over and he and his peers quickly touch their heads to show that they know the answer before they complete the prompt together, “*El número es . . . siete*” (The number is . . . seven). The teacher asks a follow-up question “*¿Muestra siete dedos, el dígito siete, o siete marcas de conteo?*” (Does it show seven fingers, the digit seven, or seven tally marks?) and the four students answer accurately, “*siete marcas de conteo*” (seven tally marks), after which Scott places the card on the pile with tally mark representations. Procedural and oral precision work reciprocally to reinforce correct numbers and representations.

**Modeling sequence.** Scott’s group is selected to model the activity only after the teacher provides the first example, modeling correct linguistic forms (using sentence frames) and mathematical terminology. The whole class gathers around Scott’s group in a fishbowl model (a surrounding circle of observers) to reinforce the expected pattern they will follow in their own groups. The teacher supports the students as they implement the procedures and reinforce mastery of the learning objectives.

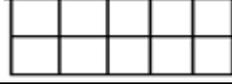
### Written Number Representations Activity

The written activity involves a table with six columns, each labeled with a different form of representation and a number per row to be modeled in the six ways (Table 2). The digit is provided in the first column, and blank boxes invite supplying the word, tally marks, ten-frame, and pictures.

**Modeling sequence.** After the teacher completes the first row of the task, she invites individual students to finish additional rows with coaching and suggestions from the class. Oral modeling and participation help reinforce the language associated with the content. Each demonstration also involves concrete counters to represent the numbers displayed pictorially and abstractly on the activity page.

Table 2

Excerpt of Written Number Representations Activity

Nombre: _____ REPRESENTACIONES DE NÚMEROS 1-9				
dígitos	palabras	marca de conteo	tabla de diez	dibujos
3	tres			
5				
8				

A small group then models further completion of the table by taking turns where each person completes a row and passes it on. The observing students retain their focus in order to correct, commend, and coach one another through the process. The teacher and the students model—draw in the air the correct letters and the tally marks (placing emphasis on the connecting position of the fifth one) while they describe aloud the process “c-i-n-c-o; uno, dos, tres, cuatro y cinco.” Finally, student groups cooperate to follow the example and together complete additional rows in the table of numbers.

All students are accountable to complete their own work after the benefit of multiple models and group practice and prior to engaging in more game-configurations that rely on these number representations. The preparation process increases students’ eagerness for independent effort, similar to the effort a chick makes to leave its shell when the right time arrives. Like chicks, the strength the students gain through the process (including some productive struggle) prepares them to be successful in the first grade Spanish DLI mathematics classroom.

### Conclusion

A newly hatched chick has fledgling, but fully functional, capabilities following a successful incubation period. Students in a DLI program likewise can become capable, long-term learners, contingent on nurturing pedagogical

practices that foster development of both mathematical and linguistic skills. Like the incubation period of a chick, the first 21 days of first grade Spanish DLI ensure that students develop fledgling abilities and are on a course toward further advancement. This is achieved through a four-part *Nurturing Mathematics and Linguistics Incubation Framework*, including 1) CRA instruction, 2) skillful implementation of language modalities, 3) GRR sequencing, and 4) a sound foundation in number representations.

Classroom routines designed to reinforce content and management complement formal mathematics instruction and together provide the context for implementation of the framework. Many of the same practices implemented during the 21-day incubation will continue to offer nurturing throughout the school year as students gain familiarity with and confidence in using them. Successes come as individuals begin to engage meaningfully with number in Spanish, often following the lead of their peers. Students with the highest levels of Spanish proficiency quickly assume the role of language models. This contribution expedites learning for the whole class and reinforces the expectation that all students are accountable for production. On the twenty-first day of first grade, all of the students independently count *uno-dos-tres . . .* and sing “*los pollitos dicen pío, pío, pío . . .*” (Orozco, 1999), happily demonstrating their fledgling successes, fully confident in their future progress.

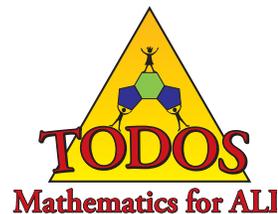
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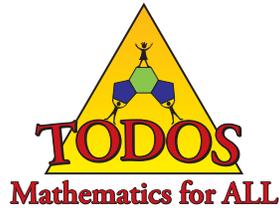
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## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are the four research foundations that shape the Incubation Framework?
2. Describe a couple of the mathematics and management routines explained in the article.
3. How does the formal mathematics instruction described in the article illustrate a gradual release of responsibility (GRR)? How do you incorporate GRR in your instruction?
4. How do concrete models help convey meaning for students who are still developing proficiency in the language of mathematics instruction?

“DARE to Reach ALL Students!”





## Experiencing Multilingualism in the Math Classroom: A Conversation Starter with Teacher Candidates

**Laura Kennedy**  
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### Abstract

This article invites readers into an abbreviated version of the “Experiencing Multilingualism in the Mathematics Classroom” (EMMC) activity as they read about teacher candidates’ responses and reflections. The EMMC activity was designed as a conversation starter for teacher candidates to build their awareness and help them consider how to support emerging multilingual students in the mathematics classroom, including sensory, graphic, and interactive support strategies. Our teacher candidates’ reflections demonstrate the importance of building awareness and understanding of linguistic diversity in the mathematics classroom. They also suggest the need for more conversations and supports for teacher candidates to move beyond awareness.

### Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Have you ever been in a situation where you had to navigate an unfamiliar language or culture? What resources helped you communicate in those situations?
2. What are some instructional strategies that you have found effective for teaching emergent multilingual students in the mathematics classroom?
3. Reflect on the following statement by a teacher candidate. What does it communicate about emerging multilingual students? What kinds of experiences would support this teacher candidate’s learning about the roles and responsibilities of the mathematics teacher?

*“The only ELL student [...] does not speak at all. He keeps to himself and copies his neighbors’ notes because he cannot keep up. My mentor teacher says that you can only do so much, which I completely agree with.”*

**Laura M. Kennedy** ([kenne420@msu.edu](mailto:kenne420@msu.edu)) is a PhD candidate in Michigan State University’s Curriculum, Instruction, and Teacher Education program. Prior to beginning her doctoral studies, Laura worked as an English as a second language (ESL) teacher in South Korea teaching conversational and academic English for many years. Her research interests foreground novice ESL teachers as well as linguistically and culturally diverse students. Laura work explores topics such as teacher identity and translanguaging through collaborative research methods.

**Sandra Crespo** ([crespo@msu.edu](mailto:crespo@msu.edu)) is a professor of mathematics education in the Department of Teacher Education at Michigan State University. Her experiences as a student, teacher, and teacher educator across three different countries (Dominican Republic, Canada, and the United States) drive her commitment to educational equity. Her research focuses on supporting teachers to transform their classrooms into collaborative and equitable spaces for learning mathematics.

## Experiencing Multilingualism in the Math Classroom: A Conversation Starter with Teacher Candidates

Laura M. Kennedy and Sandra Crespo

As educators committed to supporting students with diverse language backgrounds, we [Laura and Sandra] have been working to design experiences that help teacher candidates (TCs) at our institution, who tend to be white, monolingual, middle- and working-class young women, explore the linguistic resources of emergent multilingual<sup>1</sup> students. Our goal is to help our TCs recognize students' linguistic diversity as an asset to their learning of mathematics rather than an issue that has to be overcome.

Our assumption is that taking an asset orientation towards multilingualism in the mathematics classroom entails (1) understanding that mathematics is not language free, nor is it a universal language, (2) the language of instruction will have different language demands at the word, sentence and discourse levels for all students (WIDA, 2012), and (3) the language demands of a lesson exist on a continuum based on how similar the student's primary language is compared to the language of instruction. This work is consistent with the National Council of Supervisors of Mathematics (NCSM) and TODOS-Mathematics for All (2016) joint position statement's call for eliminating deficit thinking. Our experiences working with TCs have taught us that some of them bring deficit thinking about students who are different from themselves into our teacher preparation program and their field placements. The TC statement we offered in the pre-reading is an all-too-common perspective we have encountered every year in our program.

Knowing that in the U.S. the primary language of instruction privileges students from White, middle class, English-speaking backgrounds, we have designed a simulation activity to help our TCs engage with issues of inequity in the mathematics classroom. Specifically, how English monolingualism in the mathematics classroom

disenfranchises students of diverse linguistic backgrounds. We seek to disrupt the invisibility of language in the mathematics classroom and the role it plays in creating categories of students' mathematical ability that shortchanges emerging multilingual students. As the NCSM-TODOS (2016) position statement explains:

Deficit thinking implies students lack knowledge and experiences expected by the dominant group. Deficit thinking ignores, dismisses, or casts as barriers mathematical knowledge and experiences children engage with outside of school every day. A social justice approach to mathematics education assumes students bring knowledge and experiences from their homes and communities that can be leveraged as resources for mathematics teaching and learning (Civil, 2007; Gonzalez et al., 2005; Leonard & Martin, 2013; Turner et al., 2012). It also means broadening participation and engagement of children in light of the varied cultural, linguistic, and mathematical competencies they bring to the classroom (p. 2).

In this article, we will first take our reader through the activity, sharing in detail how we launch and facilitate this discussion. Then we share our TCs' insights and wonderings that emerged when interacting and reflecting on the EMMC activity.

### The Simulation Activity

As emergent multilinguals ourselves, we have experienced learning in multiple languages firsthand. As educators who have studied and taught in non-English speaking parts of the world, we recognize the importance of experiencing language diversity as critical for English

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<sup>1</sup> While labels of "English Learner" or "English Language Learner" are commonly used in education research and policy, we are choosing to use the phrase "emergent multilingual" here, as others have done (Association of Mathematics

Teacher Educators, 2017), because "emergent multilingual" de-centers English as a dominant language, recognizes linguistic diversity as a strength in the mathematics classroom, and acknowledges that students often speak two or more languages.

monolingual TCs who strive to teach emergent multilingual students with awareness and from an asset-based perspective. This work has been a collaboration between a mathematics educator [Sandra] and an English language educator [Laura] since 2016, making it possible to address and sustain this very important domain of access and equity with many cohorts of TCs.

The “Experiencing Multilingualism in the Mathematics Classroom” (EMMC) simulation activity is an innovation for the teacher preparation program at our institution, which is similar to many other programs in that it has tended to focus more on developing content knowledge for teaching than instructional methods for supporting emerging multilingual students. The EMMC activity draws from work that our institution’s world language and TESOL programs have been doing with world language TCs. We see this work as contributing to the ongoing conversation in the field and to the growing literature of mathematics teacher educators committed to asset-based mathematics teaching and to supporting linguistic diversity in the mathematics classroom (e.g., de Oliveira, 2011; Ewing, 2017; I & Stanford, 2018; Kenney & de Oliveira, 2015; Roberts & Truxaw, 2013).

The way we launch the EMMC activity is by asking TCs to complete an open response online survey (1) indicating what languages they speak, (2) rating their level of fluency with each of these languages, and (3) naming the home languages of the students within their field placement classrooms. There are multiple reasons for launching this activity with a language survey. First, we wanted to trouble the myth of monolingualism both within our own teacher education classroom as well as their field placement classroom. Unless prompted to the contrary our TCs identify primarily as English monolinguals. This framing, and how the literature in teacher education often speaks about the lack of language diversity in the teaching force in the U.S., fails to recognize classroom teachers’ potential for leveraging prior formal and informal language learning experiences to support emergent multilingual students. Thus the second reason for the survey was to help TCs begin to leverage their own language learning experiences and take an asset perspective when they frame their own language abilities. And finally, the third reason was a logistical one: the survey allowed us, as we will discuss later, to form small groups during the EMMC activity.

In the following class, we then engage in the EMMC activity. Before sharing the rest of the EMMC activity, we invite you to work on the following task in three languages: first Korean, then German, and finally Spanish. Pay attention to the language demands as you work on the same task across these three languages. What are you able to figure out and not figure out on your own?

다음 방정식이 해가 없도록 하는 값을 찾아 빈 칸을 채우세요:

$$8x - 3x + 2 - x = \underline{\quad} x + \underline{\quad}$$

Now consider the task in German:

Schreiben Sie eine Nummer in jeder Zeile das erstellt eine Gleichung mit keine Lösung:

$$8x - 3x + 2 - x = \underline{\quad} x + \underline{\quad}$$

The cognitive demand of the task (Smith & Stein, 2018) remains the same, but it is important to consider whether your strategies varied when the task was posed in multiple languages. How are the language demands of these two tasks different? Did you attempt to solve either of these problems? If so, what linguistic resources did you use? And now attempt the same problem in Spanish:

Escribe un número en cada raya para crear una ecuación sin soluciones:

$$8x - 3x + 2 - x = \underline{\quad} x + \underline{\quad}$$

Regardless of whether or not you are familiar with the three languages above, you have probably already worked out a solution to the problem. But, would your response be considered correct? Without being able to understand the directions, you cannot be sure, can you? This hesitation, this feeling of uncertainty that you are now experiencing is the same one that we worked to create with the EMMC activity.

## The EMMC Activity with Teacher Candidates

The EMMC activity we use with TCs is similar to the one we have shared above but consists of five algebra tasks—sample standardized 8<sup>th</sup> grade mathematics test questions provided by the Michigan Department of Education. The five tasks were chosen because they represented different mathematical content (e.g., slope, (ir)rational numbers) as well as different question forms (e.g., multiple choice, fill in the blank). We present the activity in three phases as detailed next.

### Phase I

In the first phase of the activity, we create three groups of four or five TCs (depending on the size of the class) each based on their linguistic expertise. This of course varies depending on the TCs' linguistic repertoires. For example, we might organize the class as follows: Group 1 is familiar (to varying degrees) with Spanish; Group 2 has studied other European languages (e.g., German, French, Portuguese, etc.); and Group 3 is a combination of candidates who identify as English monolingual or familiar with one of the myriad Asian languages (e.g., Japanese, Mandarin, Tamil, etc.).

Each group receives the same five math questions. The only difference is that the language of the test is different for each group. Continuing with the example above, Group 1 receives the German version of the test; Group 2 the Korean version; and Group 3 the Spanish version. Candidates are instructed to attempt each problem and treat this activity as a quiz, working independently for the first five minutes.

When the TCs are asked to silently and independently work on the five math problems in their assigned language, invariably all of the groups we have worked with over the years—despite being directed explicitly to work on the task as if it were a quiz—have started talking with one another almost immediately as they (1) try to figure out what the questions are asking, (2) express confusion, and/or (3) look to their classmates for clues.

Initial reactions from TCs have included to seemingly freeze, put their pencils down, push away from the table, and/or flip the two-sided handout over as they quickly and repeatedly scan the five questions. “I should know how to do this!” is a common statement TCs make as they recognize that these are 8<sup>th</sup> grade algebra questions.

Regardless of the language mediating their task, TCs have tended to gravitate towards the questions that include multiple representations such as graphs, tables, and equations. They also tackle the multiple-choice questions before they attempt the more open-ended tasks.

When time is called, members of the groups are invited to debrief with one another. While the discussion often begins with candidates commiserating about the challenge of the activity, they are asked to also consider three questions:

1. What was your level of access in this activity?
2. What strategies did you use when attempting to solve the problems?
3. How did you physically and/or emotionally react to the activity?

### Phase II

Following this small group discussion, we have the TCs reconstitute groups and now form groups that include TCs who attempted the task in all three languages. Posing the same three questions to the new groups, candidates are given time to share their experiences, hear from those who had engaged in the activity in other languages, and compare strategies across languages.

Once the TCs have time to talk across languages in small groups, we invite the class to come back together. Based on their reading of Celedón-Pattichis and Ramirez's (2012) “Consejos from ELL students” as well as their experiences in the EMMC activity, we launch into a discussion of the following:

1. What, if anything, made you successful when attempting the five questions in an unfamiliar language?
2. What, if any, other forms of support could have made you more successful in these tasks?

Over the years, these debrief sessions have taught us that a main takeaway for TCs is the realization that it is impossible to consider mathematics as either language-free or as a universal language (Arnold & Davis-Wiley, 2015). It is equally impossible to separate math learning from language learning (Moschkovich, 2016). TCs also discuss the importance to provide emergent multilingual students with opportunities to use their dominant language as a resource and to make connections across languages when learning mathematics.

In the debrief session, TCs discuss the wide variety of strategies they tried and the varying degrees of success for each. For example, those TCs who work on the German and Spanish tests—languages with which our TCs are generally more familiar—tend to look for and recognize cognates (similarly written and/or pronounced words) such as the German word for square (quadrates) and the Spanish words for rational or irrational (racional ó irracional). In one instance, a TC who was working on the Korean test asked Laura, who studied Korean for a number of years, to read the Korean questions aloud in hopes that she [the TC] would hear cognates—a strategy that proved successful for the word problem asking TCs to calculate the area of a square given that the Korean word for centimeter is 센티미터 [sentimiteo].

We connected our TCs' conversations of support strategies with some of the tenets of the WIDA English Language Development Standards (2012), which speak to emergent multilingual students' academic language development and teachers' instructional supports in content area classrooms. In the above example, the WIDA standards helped frame this instance as an illustration of how attention to cognates can support academic vocabulary use and comprehension while communicating that a students' primary language can serve as a resource for learning mathematics. In addition, we also discussed other categories of support aligned with the WIDA framework.

### Phase III

The in-class debrief of the EMMC activity is then continued through an online discussion forum with a focus on relating the in-class experiences with their field placement observations. In particular, TCs are asked to focus on the norms and practices that support and challenge emergent multilingual students in the mathematics classroom.

### Teacher Candidates' Insights and Wonderings

*I think it is important to understand that as explained in class last week, every student is an English language learner [...] Even if it is their first language, there may be diversity in their understanding of English that could help or hinder their mathematical understanding.*

As the above quote from one of our TCs illustrates, there are many insights and wonderings that are made possible by participating in the EMMC activity and the subsequent discussions. These include, but are not limited to, building awareness, understanding language demands, and developing strategies that support emergent multilingual students and are illustrated and discussed next.

### Building Awareness

One of the critical questions for math (teacher) educators today is how to build awareness of emergent multilinguals' experiences in the mathematics classroom because without explicit attention to linguistic diversity in the mathematics classroom, chances are that teacher candidates and math educators in general would not pay much attention to the language demands of mathematics teaching and learning (Celedón-Pattichis & Ramirez, 2012; de Oliveira, 2011). This was also true in our course. Although there were some conversations early on that focused on issues of access and equity, none were specifically focused on supporting emergent multilingual students in the mathematics classroom. For example, when our TCs were asked to reflect on their first impressions after their initial week in field placement classrooms, their reflections focused on classroom structure, technology, grading practices, and the teaching of mathematical content. It was only after experiencing the EMMC simulation activity followed by specific prompts to look for linguistic resources and instructional strategies in their field placement classroom that TCs began to notice the myriad ways in which their mentor teachers were supportive of emergent multilingual students.

The discussion prompt—posted on the online discussion board one week prior to the simulation activity—directing TCs to investigate and list the languages other than English that their students were familiar with or had as their first language generated a rich opportunity for TCs to develop awareness about students' linguistic diversity and how it can be a resource in the mathematics classroom. As one candidate recounted in his weekly reflection on the discussion board:

*The first thing I noticed while I was walking around my teacher's classroom was a world map in the back of the classroom. Being that it was a mathematics*

*classroom, I found it strange that this would be there. Looking closer I realized there were pins all over the map, recording where students call home. This gave me an idea of the diversity of the school and the various languages that are circulating [in] the school.*

Even we were surprised when our TCs reported that students within their school placement classrooms spoke 19 different first languages, including Hmong, Somali, Spanish, Urdu, and Vietnamese. Additionally, TCs began noticing emergent multilingual students' physical location within the math classroom and the (lack of) interactions between emergent multilingual students and their English monolingual classmates and/or teacher. For example, one TC, connecting his experiences in the field placement classroom to his reading of Celedón-Pattichis and Ramirez (2012), noted that similar to the student in the reading, an emergent multilingual student in his placement classroom “*sit(s) in his chair without talking or with his head down while the rest of the class is discussing problems and figuring things out.*”

### **Understanding Language Demands and Resources**

Another critical question for math educators is how they can better understand and assess the language demands within their classroom. The EMMC activity brought this question to the forefront and as the TCs carried this lens into their field placement they began to ask questions about the language supports and linguistic resources that were (or were not) made available to the students in the math classroom. For example, in response to another TC's post about the ways in which their mentor uses visual representations to teach definitions, one TC simultaneously (1) recognized that teaching definitions is a high language demand activity in the math classroom, and (2) wondered whether the potential benefit of visual resources would be enough to support emergent multilingual students. He writes:

*I can see how [your mentor teacher's use of] visual representations could help someone who did not understand a verbal explanation, but how does your mentor teacher's use of a lot of definitions assist in an ELL student's understanding?*

Another TC questioned the strategy of grouping students based solely on shared linguistic backgrounds, given her experiences in the simulation activity. By noticing that she had been unsuccessful when attempting to use similar strategies, this TC also wondered about other opportunities that are not afforded to the emergent multilingual students in her field placement classroom. She writes:

*Our teacher allows our two Spanish-speaking students to sit next to each other in class so that they can support each other's learning and understanding with help from their home language...I notice that these two learners employ strategies such as collaborative studying and learning, note-taking, and using procedural math knowledge to get them through problems whose context they may struggle to understand (much like we tried to do, unsuccessfully, with several of the problems on the [simulation] test during last Wednesday's seminar). Unfortunately these students don't often get the opportunity to communicate their ideas about mathematics during class time instead they wait until after the lesson is finished to ask our teacher for any additional help.*

### **Strategies that Support Emergent Multilinguals**

*To assist the ELLs in the classroom, my mentor teacher tries to provide as many visuals and definitions as possible. This allows these students to look back and see the definitions and make connections with the pictures instead of having to ask the teacher every time they get tripped up.*

As the reflection above shows, the TCs also engaged with a third critical question for math educators—what strategies can they use to support emergent multilingual students (Chval & Pinnow, 2016; Kalinec-Craig, 2016; Roberts & Truxaw, 2013)? The TCs we have worked with over the years have noticed many resources and supports in their mentor teachers' classrooms. These resources align well with the WIDA standards, which outline three categories of instructional support for language development: sensory, graphic, and interactive. In Table 1, we have mapped onto the WIDA categories our TCs' online discussion posts and the strategies of support they were identifying within their field placement classrooms. This mapping illustrates our TCs' ability to see the

mathematics classroom from an emergent multilingual student’s perspective, and their awareness of various instructional support strategies that can be employed in the mathematics classroom. Because of time constraints we did not share this mapping with our TCs, however, we can see the potential it offers for reflection and deepening their understanding. We can also imagine asking our TCs to use the WIDA categories to make sense of their mentor teachers’ instructional moves and to support their future conversations about teaching linguistically diverse students in their mathematics classrooms.

### Final Reflections

We argue for the importance of experiences like the EMMC simulation activity to help TCs consider the

important role of language in the mathematics classroom. We also discuss how continuing the conversation online after the EMMC simulation activity was important to help our TCs build further awareness of how language matters in students’ mathematics learning and connect the university classroom content with their field placement classroom. Their willingness to engage with these ideas suggests that the simulation activity is a good way of starting the conversation about multilingualism in the mathematics classroom, which, as de Jong and Harper (2011) noted, is a conversation that needs to become more common in teacher preparation and teacher professional development work more broadly.

**Table 1**  
*Teacher Candidates’ Discussion Posts Mapped onto WIDA Categories of Support*

<u>WIDA Categories</u>	<u>TC Discussion Board Excerpts</u>	<u>Strategies Named in TCs’ Posts</u>
<p><b>Sensory Support:</b> Ways of representing information</p>	<p><i>“Visual representations that you mentioned are a huge piece, as those can help students to ascertain the meaning of words without explicitly defining them. These visuals are also beneficial to native English speakers, so it should be a net positive for everyone involved.”</i></p>	<ul style="list-style-type: none"> <li>• Posters</li> <li>• Multimodal instructions</li> <li>• Definition word walls</li> <li>• Drawings and illustrations</li> <li>• Gestures</li> </ul>
<p><b>Graphic Support:</b> Ways of organizing information</p>	<p><i>“All of our lessons are taught from printed note packets, which provide fill-in-the-blank style definitions and exercises for students to also use as a resource to access mathematical language.”</i></p>	<ul style="list-style-type: none"> <li>• Graphs</li> <li>• Number lines</li> <li>• Graphic organizers</li> <li>• Tables</li> <li>• Guided notetaking</li> </ul>
<p><b>Interactive Support:</b> Ways of communicating</p>	<p><i>“Our teacher allows our two Spanish-speaking students to sit next to each other in class so that they can support each other’s learning and understanding with help from their home language, and she often lets them take tests and quizzes to the school ESL teacher’s room during lunch or after school if they need more time to work on them.”</i></p>	<ul style="list-style-type: none"> <li>• In pairs or partners</li> <li>• In small groups</li> <li>• With the math teacher</li> <li>• With an ESL educator</li> </ul>

Although the EMMC activity started many productive conversations, it is also clear that one experience is not enough. Some of the TCs' post-activity reflections still reflected deficit perspectives and problematic attitudes towards emergent multilingual students. In the third pre-reading question, for example, we asked you to reflect on a TC's statement (let's call the TC Chris): "*the only ELL student [...] does not speak at all. He keeps to himself and copies his neighbors' notes because he cannot keep up. [My mentor teacher] says that you can only do so much, which I completely agree with.*" This is a statement that one of our TCs made after participating in our EMMC activity. This points to the challenge that lies ahead for TCs as they continue learning to teach across difference (e.g., Zavala, 2016), as well as the challenge for teacher educators facilitating these kinds of experiences with TCs.

In reviewing the discussion board posts, we noted moments where TCs were ever so slightly pushing back on other TCs' use of deficit language regarding emergent multilingual students and what they were (not) able to do in the mathematics classroom. Returning to Chris' reflection specifically, we see their post as a missed opportunity and a place where we, the teacher educators, needed to intervene more explicitly and draw attention to the importance of troubling deficit perspectives and the need for an asset-oriented lens on language diversity in the mathematics classroom. We could have, for example, brought back to class statements made that reflected deficit or asset-oriented perspectives on language diversity and asked our students to engage in a discussion about the importance of and strategies for challenging deficit perspectives for themselves and their peers. We can imagine that this conversation could then lead to a deeper layer of discussion about language privilege, oppression, and suppression in U.S. schools today. We can also imagine bringing into this conversation the TCs' mentor teachers to widen the circle for professional learning about how to not only support emergent multilingual students but also position their linguistic diversity as a resource in the mathematics classroom.

Although the EMMC activity extended beyond the one classroom activity to include connections with course readings and field placement activities, a single class experience is still not enough. Many more of these conversations are needed in order to support candidates'

and even experienced teachers' learning of new strategies and new ways to support emergent multilingual students. We invite readers to continue the conversation with colleagues using this activity and the DARE questions below. To close, we share the English version of the initial task to help readers reflect on what we shared in this article. Did you have a clear understanding of the math task? Would you stick to your initial response or would you change it? Although the symbolic equation remains the same across the tasks, reading the task in a more familiar language makes visible aspects of the question that may not have been accessible or clear in the other languages. Finally, we now invite you to take a moment to imagine yourself as the teacher of an emergent multilingual middle school student. We hope that the ideas we have shared in this article can help you to be more aware of the language demands of this task and to consider the sensory, graphic, and interactive support strategies that can help you support emergent multilingual students' learning of mathematics.

Write one number on each line to create an equation that has no solution:

$$8x - 3x + 2 - x = \underline{\quad} x + \underline{\quad}$$

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## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How would you have responded to Chris?
2. How might we extend the conversation about linguistic diversity in the mathematics classroom in a way that builds on the simulation activity that is shared in this article?
3. What challenges and opportunities might you anticipate when using the “Experiencing Multilingualism in the Mathematics Classroom (EMMC) activity with either teacher candidates or practicing teachers?
4. Consider the following two examples from the original EMMC activity and reflect on the language demands of each. What kinds of resources and strategies might students use in order to engage meaningfully and mathematically with these different tasks?

### EMMC Task #1

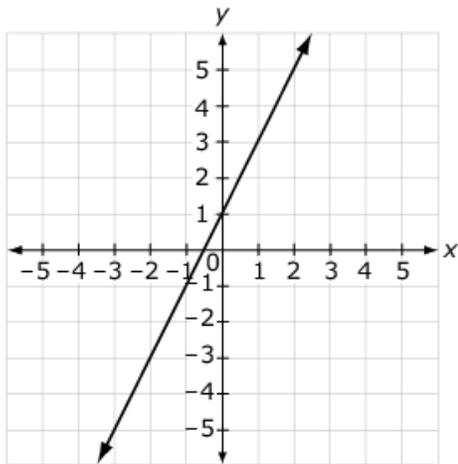
측면의 길이가  $s$  인 정사각형의 면적이 324 제곱 센티미터이다. 이 정사각형의 측면의 길이는 몇 센티미터인가?

Ein Quadrat mit einer Seitenlänge  $s$  hat eine Fläche von 324 Quadratzentimeter. Was ist der Seitenlänge des Quadrates in Zentimetern?

Un cuadrado con un lado de longitud  $s$  tiene una área de 324 centímetros cuadrados. ¿Cuál es la longitud del lado de dicho cuadrado en centímetros?

A square with side length  $s$  has an area of 324 square centimeters. What is the side length of the square in centimeters?

### EMMC Task #2



다음 방정식 중 그 기울기가 주어진 그래프의 기울기보다 큰 것은 무엇인가요?

Welche Gleichung hat eine Veränderungsrate größer als die Veränderungsrate für diese Linie gezeigt?

¿Cuál ecuación tiene un ritmo de variación que es mayor que el ritmo de variación en la línea mostrada?

Which equation has a rate of change greater than the rate of change for the line shown?

A.  $y = 3x - 1$

B.  $y = x/2 + 4$

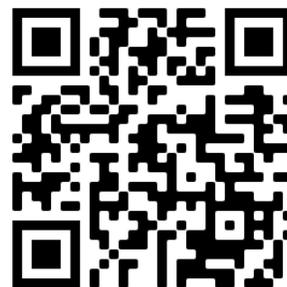
C.  $y = 2x + 2$

D.  $y = x/3 - 3$



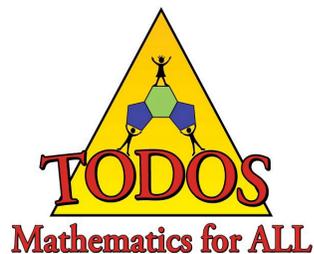
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## TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

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“To advocate for equity and high quality mathematics education for all students—in particular, Latina/o students.”

The journal aims to address topics involving excellence and equity (simultaneously) in the teaching and learning of mathematics, in a way that connects research to classroom practice and can directly inform practice. To this end, manuscripts are welcomed that relate to:

- ALL students learning mathematics as problem solvers beyond computational abilities;
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The journal defines equity broadly, including (but not limited to) issues of language, gender, ethnicity, and culture. *TEEM* welcomes manuscripts on issues of language, culture, access, equity, and quality from diverse viewpoints.

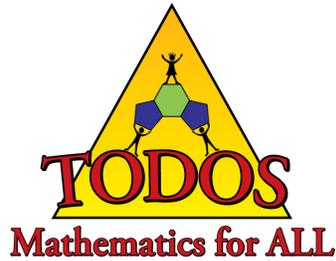
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We encourage the submission of manuscripts, including applied or action research, literature surveys, thematic bibliographies, commentary on critical issues in the field, professional development strategies, and classroom activities and resources. While contributions in English are recommended, *TEEM* will also consider contributions in languages such as Spanish. The *TEEM* Editors welcome query emails about the suitability of proposed topics: email at [teem@ todos-math.org](mailto:teem@todos-math.org).

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- A focus on some aspect of the TODOS mission and related goals:
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  - to implement lessons and programs that incorporate the role that language and culture play in learning mathematics;
  - to inform the public, including parents, and influence educational policies in ways that enable students to become mathematically proficient: and
  - to inform teacher education programs.

For more details on the guidelines for papers, see <https://www.todos-math.org/teem>.



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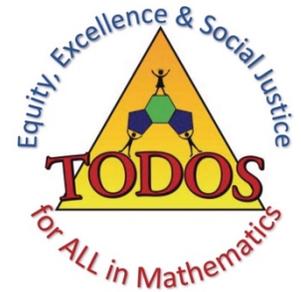
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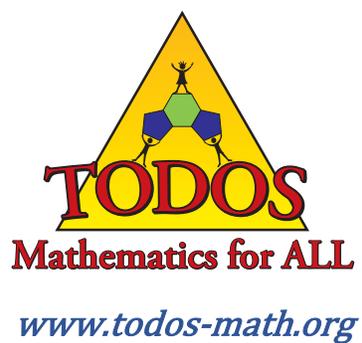
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