Learning Mathematics in the 21st Century

ADDING TECHNOLOGY TO THE EQUATION

EDITORS | **Elena Arias Ortiz**

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IDB

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INTER-AMERICAN DEVELOPMENT BANK

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Preface

I t is the year 2020, and in Latin America and the Caribbean, 154 million students are learning from home, their schools closed because of Covid-19. Overnight, teachers with 20 or 30 years of experience have had to learn how to teach virtual classes. Along with them, all actors in the education system have had to make a leap towards online education, revealing the low level of technology integration and gaps in student access to connectivity and devices at home.

Outside the region's classrooms, the world has been undergoing intense technological ferment for years. While teachers and students in our countries adapt to their new digital environment, an army of robots dances without music in Baltimore, U.S., preparing orders that just arrived over the Internet at one of Amazon's 177 distribution centers. At the same time, in Cologne, Germany, a group of computer science experts are putting the final touches on a new version of the DeepL translation engine, which is revolutionizing the field of artificial intelligence-based translation. Meanwhile, in Zhongwei, China, the sun is rising on 43 square kilometers of solar panels located in the Tengger desert, which produce enough energy to meet the needs of millions of people.

Technological developments are revolutionizing markets for goods, services, and energy worldwide. The big question is, how will these technological changes affect labor markets? Experts differ in their views, but they do tend to converge on one central policy recommendation: It is crucial to prepare present and future generations for the changes being brought by the fourth industrial revolution, which is already underway.

In Latin America and the Caribbean, the good news is that education has improved notably in recent decades, moving from low levels of access and high levels of illiteracy to almost universal basic education and increasing access to higher education. However, time and again, it has been found that school attendance does not necessarily mean acquisition of knowledge and basic skills. National, regional, and international

evaluations of student learning have found that in many countries of the region, at least half of students cannot understand a simple text or solve a basic math problem. This is clearly a problem in itself, but these deficiencies also make it harder to develop the 21st-century skills people need to act as committed citizens and efficient workers in the new economic and social environment.

In this context, the mandate for education ministers in the region is two-fold: First, they need to resolve the learning crisis in traditional areas like mathematics. Second, they must promote new ways of teaching and learning to develop the critical skills people need. The key questions that emerge are how should children learn mathematics? What are the new teaching practices that foster the development of mathematical thinking among students, rather than simply transmitting knowledge? What are the areas where our region faces its biggest challenges? Which models for technological innovation seem most promising? This book addresses these and other questions with the aim of offering a roadmap for countries wishing to use technology in education effectively.

These issues are even more relevant in the current context of the Covid-19 pandemic. In order to educate our young people despite their confinement and get them the knowledge they will need in the labor market of the future, the new mandate is to accelerate the digital transformation of our education systems guided by evidence. Developing effective hybrid education models—part-time at home, part-time in the classroom—for the months following reopening will be crucial for keeping their learning apace while we look for a permanent solution to the health crisis. Progress in this regard would contribute not only to improving learning but also to promoting more robust and flexible education systems.

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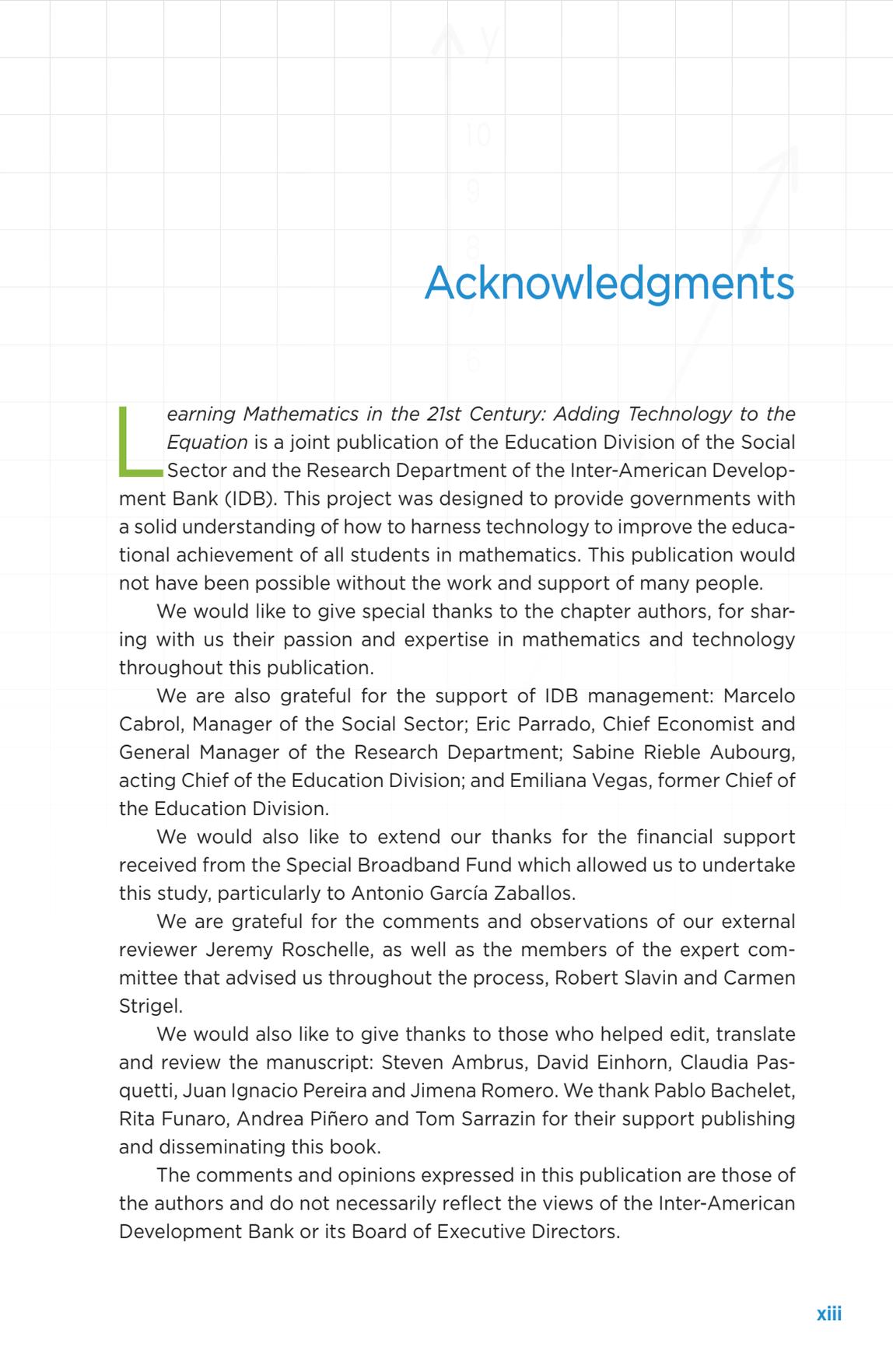
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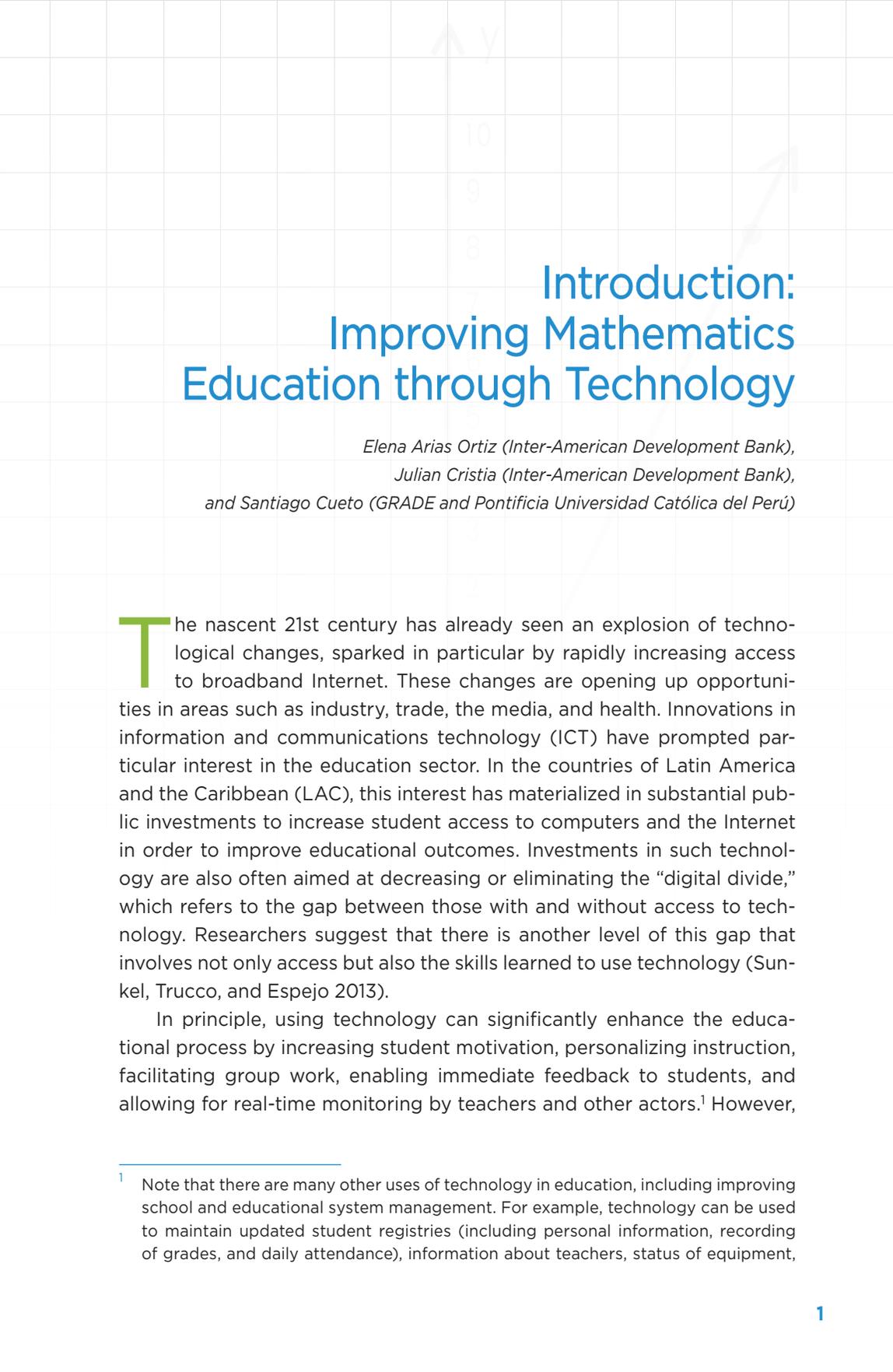
Introduction: Improving Mathematics Education through Technology

Elena Arias Ortiz (Inter-American Development Bank),
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The nascent 21st century has already seen an explosion of techno-logical changes, sparked in particular by rapidly increasing access to broadband Internet. These changes are opening up opportuni-ties in areas such as industry, trade, the media, and health. Innovations in information and communications technology (ICT) have prompted par-ticular interest in the education sector. In the countries of Latin America and the Caribbean (LAC), this interest has materialized in substantial pub-lic investments to increase student access to computers and the Internet in order to improve educational outcomes. Investments in such technol-ogy are also often aimed at decreasing or eliminating the "digital divide," which refers to the gap between those with and without access to tech-nology. Researchers suggest that there is another level of this gap that involves not only access but also the skills learned to use technology (Sun-kel, Trucco, and Espejo 2013).

In principle, using technology can significantly enhance the educa-tional process by increasing student motivation, personalizing instruction, facilitating group work, enabling immediate feedback to students, and allowing for real-time monitoring by teachers and other actors.[1] However,

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[1] Note that there are many other uses of technology in education, including improving school and educational system management. For example, technology can be used to maintain updated student registries (including personal information, recording of grades, and daily attendance), information about teachers, status of equipment,

the large investments in technology in the education sector in LAC have been the subject of heated debate. There is a gulf between the expected impact of technology and the actual results. Indeed, the few rigorous evaluations conducted to date suggest that many educational technology programs have had limited effects on student learning (Lubin 2018).

A typical example of the mismatch between promise and reality is the One Laptop per Child (OLPC) Program, which sought to improve education in the poorest regions of the world. The program was implemented worldwide but was especially popular in LAC. In fact, about 80 percent of the 2.4 million laptops distributed worldwide under the program were distributed in the region. In Uruguay, for example, all students in the country received a laptop under the program. Peru also participated, with over 800,000 laptops purchased. Unfortunately, a rigorous, large-scale evaluation of the OLPC Program in Peru showed that, although the program had some positive effects on general cognitive skills and digital skills, it did not have measurable effects on mathematics or reading comprehension, which had been one of the government's objectives (Cristia et al. 2017).

Promoting the Use of Guided Programs

The public discussion generated by Cristia et al. (2017) reveals a strong demand among governments and other stakeholders for high-quality evidence. In particular, people want to know how educational benefits from technology can be increased. Cristia and his colleagues used surveys of students and teachers, computer logs, and a parallel qualitative evaluation to show that the lack of academic results can partly be explained by the limited use of computers in activities directly related to mathematics learning and reading comprehension.

There are few rigorous evaluations of other large-scale programs in LAC, leaving open the question of how far technology investments improve academic outcomes in the region. Indeed, pioneering national ICT and education programs, such as Enlaces in Chile and Plan Ceibal in Uruguay, have made great strides toward closing the digital gap in their countries.[2] However, evaluations of how the various components of these

and communications between schools, parents, and other educational institutions, including the Ministry of Education. However, the analysis of these and other potential uses goes beyond the scope of this book. For more information on the digital transformation of education management, see Arias Ortiz et al. (2019).

[2] Enlaces, the Center of Education and Technology of the Ministry of Education, was launched in 1992 to further educational quality in Chile by (1) improving access to technology in public schools, (2) training teachers in the use of ICT in the classroom,

ICT programs affect student learning are scarce. Although national policies to support ICT in education may not have a direct impact on students' learning achievement in their early phases, it is crucial to establish a causal effect given the significant investments involved. Moreover, evaluations offer important lessons that may be used to improve the design of programs across the region.

One review of educational technology evaluations (Arias Ortiz and Cristia 2014) sheds light on this debate. In particular, this review found that programs that clearly guide participants on how to use the technology resources at hand foster better academic outcomes than those that do not guide technology use. A program is considered as "guided" if it specifically defines the target subject, the software to be used, and the weekly duration of use. That is, a guided program clearly defines the three "S"s: **s**ubject, **s**oftware, and **s**chedule. In contrast, "nonguided" programs provide access to technological resources, but the user (teacher or student) must define the learning objective, the software involved, and the frequency of use.

By this definition, the OLPC Program in Peru was nonguided. Through this program, the government of Peru aggressively distributed personal laptops to students in primary schools in rural areas. Teachers were trained for one week but received little guidance on how to integrate computers into pedagogical practices. In contrast, in a program implemented in primary schools in India (Banerjee et al. 2007), students used computers for two hours every week, the difficulty level of mathematical exercises was personalized, and the program generated significant improvements in students' mathematics achievement.

The effects of guided programs also vary across a wider range of outcomes than do the effects of nonguided programs (Arias Ortiz and Cristia 2014). That is, while some guided programs generate large positive effects, others generate few or even negative effects. This dispersion suggests high returns from experimenting with different models of guided programs to identify the most effective ones. Moreover, the review also

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and (3) helping students develop 21st century skills. Plan Ceibal was created in 2007 to support Uruguayan education policies through technology, with the aim of fostering inclusion and equal opportunity. Since its implementation, every child who enters the public education system throughout the country has been able to access a computer for personal use with a free Internet connection from the school. In addition, Plan Ceibal provides a host of other services, including educational resources and teacher training. Recent initiatives by Plan Ceibal seek to support student learning more directly and use evaluations to guide decision-making. In particular, the Inter-American Development Bank approved a $30 million loan to Plan Ceibal in 2017 to enhance student learning in basic education by promoting better use of technology in the classroom, the creation of digital educational content, and the training of teachers.

documents that guided programs are among those educational programs with the greatest impact on academic achievement, proving the great potential of technology to improve student learning. This is particularly true in the case of mathematical skills, where the effects seem to be greater than for reading comprehension.

However, in spite of the limited evidence of effective technological programs that have been implemented on a large scale, it should be recognized that technology in general and computers in particular are here to stay. The technological changes of the 21st century require that young people leaving the education system have mastery of several key technologies to perform well in the labor market. As the presence of computers and the Internet becomes increasingly integral to the education process, governments will continue to invest in it. The question of how to use technology in a cost-effective fashion is thus of utmost importance.

At the same time, it is important to highlight the major educational challenges faced by the countries of LAC. To start with, average levels of academic achievement are low across the region (Bos et al. 2016a). This is problematic, because weak average performance on standardized tests has been clearly linked to poor economic performance at the country level (Hanushek and Woessmann 2009, 2012). On top of that, there are large skill gaps between individuals from low- and high-income households and from urban and rural areas (see Chapter 4).

Mathematics is a particularly critical learning area, and most students in LAC do not attain the most basic levels of proficiency. Overall, students in the region display a low level of performance in math, reading, and science, and, of the three subjects, their performance in math is consistently the worst. Sixty-three percent of 15-year-old students in the region have not reached a level 2 (basic level) of proficiency in math, compared with 50 percent in science and 46 percent in reading (Bos et al. 2016b). Yet mathematics proficiency is critical to occupations in science, technology, and engineering, which are expected to be in increasing demand in the coming years.

How to Improve Mathematics Learning in Latin America and the Caribbean Using Technology

Against this backdrop, this book seeks to answer one question: how can governments in Latin America and the Caribbean improve mathematics learning using technology? To answer this question, the book identifies, reviews, and synthesizes knowledge relevant to designing education programs that utilize technology to improve mathematics learning in primary schools in the region. Around the world, researchers, policymakers, and practitioners have

been experimenting and generating knowledge about effective ways to use technology to enhance mathematics learning. Unfortunately, this knowledge is dispersed in a multitude of studies and reports, known only by a range of specialists from the education, economics, psychology, and computer science fields. By describing in detail promising programs and policies that incorporate technology into the classroom, and analyzing their impact, this book provides a deep and hopefully useful review of the state of the art, especially relevant for directors of technology and their technical teams in education programs in LAC, for education specialists from multilateral organizations and nongovernmental organizations, and for other actors involved in the implementation of projects in this area.

In preparing this book, a major challenge was quickly recognized: effective programs may vary by grade level and context. For example, programs that may be effective in primary education may not work well in secondary education. Also, programs that may work well in urban areas may be less effective in rural areas. This study focuses specifically on programs that foster mathematics learning in primary schools in urban areas of LAC.

Analyzing primary education is critical to help countries improve the educational outcomes of disadvantaged students who frequently lack the basic skills necessary to attain secondary and higher levels of education. Indeed, evidence suggests that policies to expand primary education in LAC have been successful: enrollment is now nearly universal. Yet the quality of education remains low (Ganimian and Murnane 2016). Exploring in depth how educational technology can best address this challenge will provide a great opportunity for countries in the region to leverage the use of recent investments that expand technology access at the primary level. This book's focus on mathematics has a further advantage: learning expectations in math, as expressed in national curricula and international evaluation frameworks, are quite similar across countries in the region and worldwide, facilitating the adaptation of solutions from one country to another.

The book focuses on urban areas, as LAC is a highly urbanized region and hence the vast majority of students are concentrated in cities. However, while examining effects on the largest number of people makes good sense for policy, the limited scope of this book will, it is hoped, be expanded by future studies of rural areas. Many of the lowest-achieving students in the region live in rural areas. These students face significant educational challenges linked to their socioeconomic characteristics (e.g., poverty and ethnicity) as well as lower and unequal access to infrastructure and public resources (e.g., electricity and the Internet).

Finally, throughout the book, the meaning of the word "technology" is restricted to computers (desktops, laptops, netbooks) and tablets. These

are powerful tools that facilitate a variety of possible ways to search and process information. The technological tools commonly used in distance education programs, such as television and radio, do not foster such rich interactions among students and teachers, and thus are not included in the analysis in this book.

A Review of the Challenges of Mathematics Education

This book is divided into two parts. The first, which includes Chapters 1 to 4, aims to document the main challenges to mathematics learning in Latin America and the Caribbean. In particular, these chapters seek to identify effective instructional processes that are important to learning but not prevalent in the region. In other words, the first part of the book intends to provide a thorough diagnosis of the main challenges to mathematics learning in the region. The second part of the book, which includes Chapters 5 to 8, highlights how these processes can be potentially strengthened using technology, and describes the main types of programs or models of technology use that are relevant to the instructional challenges in mathematics that LAC faces today. The models presented in this second part have the potential to produce large effects on mathematics learning. Effects on socioemotional outcomes such as motivation, attitudes, and teamwork skills will also be considered as potential mediating factors to gains in academic achievement in mathematics.

The paragraphs below briefly summarize the contents and main ideas of each chapter. Most chapters include a summary of policy recommendations in their final section.

In **Chapter 1**, Lindsey E. Richland, Kreshnik N. Begolli, and Emma Näslund-Hadley synthesize educational research to provide a definition and key aims for mathematical proficiency in the 21st century. The chapter provides a solid theoretical foundation for the book by outlining key developmental changes in children's mathematical thinking over time, and by providing a contemporary definition for mathematical proficiency. The authors highlight how children's minds are uniquely ready to develop mathematical concepts, but also how instruction adapted to their age and background knowledge will have the greatest impact. The main message for educators is to design programs in which instruction and technology are based on how children think, rather than on pedagogical techniques per se. While this sounds straightforward, what the authors describe here implies a major shift in orientation, away from focusing on instruction (that is, what the teacher or technology is doing), to focusing on how to best respond to and foster children's thinking.

The chapter also provides a guide for how to use standards to reach developmental goals in mathematics. Learning standards provide common norms for everyone involved in the decision-making process of designing and implementing mathematics educational technology. The chapter considers the example of educational reforms in the United States and the development of high-quality standards for mathematical proficiency throughout primary school, which are known as the U.S. Common Core State Standards for Mathematics (CCSS Initiative 2010). This experience is relevant to LAC because many countries there are still advancing in this area. Throughout the chapter, the authors argue that the effectiveness of technology in education programs will critically depend on how its use helps children build key foundational skills and overcome learning challenges.

Building on the case made in the first chapter about the importance of mathematical proficiency in the 21st century, Aki Murata, Karen C. Fuson, and Dor Abrahamson provide a framework in **Chapter 2** for understanding how teachers can help students develop understanding and fluency in mathematics. The balanced teaching framework discussed in the chapter offers a three-phase model for how teachers can help their students' progress from (1) exploration to (2) understanding to (3) fluency in each new math topic. Phase 1 aims at developing mathematics structure and sense-making by encouraging students to use their intuition to explore new concepts. In phase 2, the heart of the process, the class engages in discussion as students talk through their mathematical reasoning processes, with the help of visual supports. Once a certain level of understanding is reached, teachers introduce formal methods and seek to develop mathematical fluency in phase 3.

The chapter presents an alternative view to the dichotomy between traditional instruction that has emphasized procedural fluency and practice versus new trends that promote children's exploration. The authors highlight connections between different types of student thinking and visual representations to illustrate a learning process that moves through the three phases and uses "math talk" to support the connections.[3] Finally, the authors outline the advantages of using technology to support balanced teaching by describing how it can guide national decisions about teaching and learning, including choices regarding what type of technologies to use and consideration of those that are already available.

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[3] As explained by the National Council of Teachers of Mathematics, "math talk" is an instructional conversation directed by the teacher, but with as much student engagement as possible. The idea behind it is that if students take time to explain their mathematical thinking, this will increase their understanding.

After this review of effective instructional strategies, Gilbert A. Valverde, Jeffery H. Marshall, and M. Alejandra Sorto provide a detailed assessment in **Chapter 3** of the mathematical content that children in LAC know, using available standardized test results and contrasting these results with the goals set by national curricular policies. The authors rely on data from the Trends in Mathematics and Science Study (TIMSS), an international student assessment, and the Third Regional Comparative and Explanatory Study (*Tercer Estudio Regional Comparativo y Explicativo* – TERCE), a regional student assessment. In addition, original data on national curricula from the University at Albany's International Curriculum and Textbook Archive are used. These data include the topics outlined in an intended curriculum for primary school mathematics (and reading) in developing countries, defined as the official expectations regarding mathematics learning promoted by ministries and national education agencies.

What do data and research findings in LAC tell us about the current status of student achievement in mathematics education? The authors show that students in the region have consistently low to average achievement in mathematics compared with students from other regions of the world. Evidence from regional student assessments also suggests that average levels of mathematics achievement are seriously low across LAC. According to the TERCE, student performance in third-grade mathematics was critically low (UNESCO-OREALC 2016), even in content areas specifically covered in national curricula. The authors also find evidence of persistent gaps in educational attainment between subpopulations of students— inequality that favors urban students and those in private schools.

Finally, the authors find gaps in national mathematics curricula across LAC. Missing as an explicit goal in some cases is knowledge of key content such as integers, rational and real numbers; proportionality problems; patterns, relations, and functions; and the performance of mathematical reasoning. The authors consider these gaps cause for concern: unless these topics are explicitly addressed in a national curriculum, few students will have the opportunity to learn them. Beyond curricular policy, the authors discuss persistent structural and implementation factors that require attention. Indeed, in even the highest-achieving countries where key content knowledge is covered in the class curriculum, most students can solve only the most routine problems, and at the lowest levels of cognitive demand.

In **Chapter 4,** Jeffery H. Marshall and M. Alejandra Sorto explore which inputs and classroom practices are associated with the highest academic achievement, using databases from the TERCE as well as the Second Regional Comparative and Explanatory Study (*Segundo Estudio Regional Comparativo y Explicativo* – SERCE). The authors review empirical studies

describing the educational opportunities available to children in particular countries or locales. In particular, they analyze three factors: (1) school and teacher observable characteristics, (2) teacher capacity or knowledge, and (3) teaching processes. Their results reveal that there is a major difference between what an effective mathematics class should look like and what classrooms actually look like in LAC. Significant deficiencies in teaching and learning environments exist: teachers exhibit low levels of pedagogical content knowledge and students spend much time memorizing and applying algorithms instead of engaging in high-level cognitive tasks. In addition, large gaps in inputs and classroom practices are documented between schools attended by low- and high-income students. Moreover, few classrooms report using manipulatives, pointing to a lack of materials. In summary, classrooms in the region depend on lessons with low cognitive demand that do not challenge students to really learn mathematical concepts in a profound way to achieve proficiency.

What explains the general lack of quality observed in primary school mathematics classrooms across the region? The authors identify and analyze various factors, including limited teaching materials, little support for students outside the classroom, and inadequate mathematics knowledge among teachers. In this context, computer-assisted learning could help both students and teachers. Yet, the authors warn about the dangers of a simplistic reliance on technological solutions that will not automatically improve learning. Mathematics classrooms in LAC need to expose students to learning tasks that promote reasoning and thinking, and technology can serve as a catalyst for reaching this goal but should not be a goal in itself.

Examples of the Use of Technology to Improve Learning

Based on the challenges identified in the first part of the book, part two provides concrete models of how technology can be used to improve mathematics learning in Latin America and the Caribbean. The goal is to identify programs that are effective—or at least promising, given their design features—in improving mathematics learning in primary school. To do this, **Chapter 5** analyzes alternative models of technology use up until the second grade. **Chapters 6, 7,** and **8** analyze the potential uses of technology for mathematics learning between the third and sixth grades.

In **Chapter 5**, Julie Sarama and Douglas H. Clements provide an overview of models for learning mathematics using technology between pre-primary years and the second grade, including technology-assisted instruction that encourages students to practice in order to gain fluency;

tutorials and learning games; technology-enhanced management tools to track children's progress and individualize instruction; and technology-based manipulatives that encourage cognitive play.

Their theoretical foundation is based on learning trajectories that offer a conceptual framework for constructive-based learning and teaching. Each learning trajectory has three parts: (1) a goal, (2) a developmental progression, and (3) instructional activities. In this framework, to attain mathematical competence in a given mathematical topic or domain (the goal), students master each successive level (the developmental progression), aided by tasks (instructional activities) that facilitate thinking at that level. The authors describe how technology can make substantial contributions to early childhood mathematics education if their applications are consistent with expected learning progressions.

The authors also discuss the requirements of teacher training and the resources needed for these models to be successful in LAC. For example, although the benefits of computer programming are promising, especially in an increasingly complex technological age, there are significant requirements for the effective use of coding. If hardware is scarce and teachers have not received considerable professional development and support, this may be an unproductive and frustrating model to implement. In contexts of limited resources, the authors recommend simple applications, such as technology-assisted instruction tutorials and practice applications explicitly aligned to extant standards (goals) and curricula. But even for these simpler applications, teachers must receive training and in-class support, and the software chosen must fulfill certain requirements such as moving children through learning trajectories, featuring introductory exploratory activities, and including technological manipulatives.

In **Chapter 6**, Roberto Araya and Julian Cristia analyze guided programs that seek to promote student math practice. These types of programs involve students performing exercises on computers and promote student engagement through games and tournaments to motivate children to practice. This type of model can be implemented to supplement regular mathematics instruction, and it does not require close coordination between technology-based and traditional instructional activities. Building on the experience of a pilot project in Santiago, Chile, the chapter analyzes 10 key design decisions that maximize impacts. These design decisions center on clearly defining the objective of the program (i.e., which mathematical skills will be developed), how computers are expected to be used during the technology sessions, and which inputs are directly provided by the program, including how teachers and lab coordinators are to be trained and supported. For each of these key decisions, different options

are analyzed using theoretical arguments and empirical evidence, and also taking into account the actual choices made in a number of effective programs that guide use and are focused on promoting student practice.

Two main findings emerge from the analysis presented in the chapter. First, many of the analyzed decisions involve difficult trade-offs that need to be considered but which, a priori, may not be recognized. To make decisions regarding how to tackle these trade-offs it is important to analyze potential options in a careful manner considering not only the potential benefits of each option but also the potential challenges to be faced during implementation. Second, the decisions to be made when designing these models should make sense when considered not only in isolation, but also in conjunction with all the other decisions made. That is, the chapter emphasizes the critical need to ensure coherence across design.

Chapter 7 is devoted to analyzing a comprehensive model that emphasizes the comparative advantage of technology in the visualization and exploration of complex mathematical concepts. In this chapter, Nicholas Jackiw discusses in detail mathematically open learning technologies (MOLT) that engage students in the active pursuit and construction of knowledge of mathematics topics and best practices that can be used across grades. These technologies are defined by three characteristics: (1) a student-centric design and user model, (2) an open-ended activity structure, and (3) an innovative application of technology to mathematical representations and practices. Two practical experiences are highlighted: dynamic geometry manipulatives and mathematically embodied number environments. The chapter recommends that policymakers focus on the professional development of teachers (in mathematics and pedagogy more than in technology) as well as on incrementally staged implementations to effectively adopt mathematically open learning technologies at scale.

Finally, in **Chapter 8**, Ana Díaz and Miguel Nussbaum review the concept of orchestration within the context of teaching mathematics using technology. Orchestration is the coordination of pedagogy, curriculum, and technology in a student-focused setting. The authors argue that children are not learning and that computers in education systems are not being used effectively because there is a lack of pedagogical support for teachers to integrate technology and student needs into their teaching practices. Thus, to implement the use of technology in the classroom, under the orchestration model teachers are provided with a detailed set of guidelines for how to implement new teaching strategies.

An orchestration can either be provided from the outside, or developed internally by schools, but in both cases certain social and infrastructural conditions need to be met to help teachers overcome the challenges they

face. A series of guiding questions and a diagnostic of a school's specific context can help school communities develop their own orchestrations. Lessons can either be completely or partially orchestrated, depending on the teacher's tools, knowledge, and skills. The authors demonstrate that an orchestration can be an effective tool, not only when teaching mathematics, but also in other areas of the curriculum.

A Range of Promising Options and Caveats for Policy

In summary, the analysis presented in this book underscores some of the options that policymakers and educators can explore when deciding which educational technology model to implement in a specific context. Thus, instead of generating a one-size-fits-all recommendation for how to incorporate computers into mathematics lessons, the book's authors consider several program models that have been found to impact teaching practices and, hopefully, students' knowledge and mathematical thinking. Based on the discussions, materials, and references presented in this book, effective programs are seen to have several key characteristics:

1. Technology, including access to computers and Internet, is not the main objective but merely an instrument used to introduce effective pedagogical practices that can build students' mathematical knowledge and thinking. Technology is not here to replace teachers.
2. All the models require that teachers receive considerable professional development and training to be successful. In particular, it is critical that teachers be guided in the pedagogical use of technology, and not only in operating the equipment, in order for a program to be effective.
3. Computers do not need to be provided to each student; physical resources can be shared.
4. Programs need to be adapted to the context of the school and the education system in which they will be implemented. For example, in contexts where teachers' experience or skills with technology is limited or where infrastructure conditions are not ideal, relatively simple approaches need to be implemented.
5. Successful programs require that local stakeholders, including school principals and teachers as well as parents and students, have a positive attitude about the program in order to foster and follow through on its implementation, overcoming sometimes inevitable constraints or criticisms that often arise when a program begins.
6. A support system for schools is needed to solve any problems with operating, fixing, or replacing equipment, gaining access to the Internet,

and providing pedagogical support about the resources available and how best to use them locally.
7. Interventions requiring technology need to be aligned with other interventions at the national or regional levels, including curriculum and teacher policies (both pre-service and professional development), as well as other interventions in other sectors (in particular, school infrastructure and access to electricity and the Internet).

Hopefully, these design principles and the lessons presented in this book will contribute to better policy decisions regarding how to use technology to enhance mathematical learning in Latin America and the Caribbean. Countries in the region have made large investments in educational technology, and access to computers and tablets is widespread across urban public schools. It is important to design and implement models that can make effective use of available technologies, generating significant benefits at a low cost. Moreover, governments in the region are increasingly receptive to evidence as an input into policy decisions, especially when it can inform how to optimally structure a program, rather than decisions related to whether or not to launch a program in the first place. However, there have been examples of governments in the region embarking on massive access to technology programs only to realize later on that these programs were not fully developed in their aims and procedures (e.g., no theory of change for the program) or that they did not have sufficient professional or monetary resources to implement them over time. As suggested earlier, it is critical to have efficient systems to monitor the implementation of programs and have plans for qualitative and quantitative evaluations embedded in the design of the intervention. However, this has often not been the case in the region.

Multilateral organizations like the Inter-American Development Bank (IDB) promote the use of evidence in operational and policy dialogue, hence there are clear channels to disseminate the generated knowledge. The IDB expects to remain a relevant actor in this area and to continue supporting a network of specialists in the development, implementation, evaluation, refinement, and scaling-up of interventions using technology to improve mathematics learning (and eventually other areas). Thus, this book can be considered an additional step in a comprehensive and ongoing initiative involving multiple actors and views from different disciplines such as psychology, education, and economics, and drawing on experiences from regions around the world that can provide multiple and complementary perspectives. The ultimate goal is that these concerted efforts contribute to making the promise of technology in education a reality for all students in Latin America and the Caribbean.

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The Development of Mathematical Thinking in Children

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The aims for mathematical proficiency in the 21st century have changed, leading to educational reforms in mathematics education around the world. Economic success increasingly depends on building a workforce of mathematically proficient students able to apply learned mathematics to real-world problems, and to innovate, think creatively, and adaptively participate in continually shifting economies. This type of mathematical proficiency requires a more conceptual, flexible understanding of mathematics than is traditional in classrooms in Latin America and the Caribbean (LAC)—and it is argued that the lack of such an understanding has direct economic consequences (Hanushek and Woessmann 2012). Educational reforms are thus essential. They are also challenging, since most educators tend to teach the way they were taught, and students' parents are often uncomfortable with new instructional modes. Reforming mathematics instruction involves a major cultural shift both in the aims for student learning (i.e., deciding what counts as mathematical proficiency) and in the pedagogical techniques used to teach mathematics.

This chapter synthesizes a large body of mathematics educational research to provide educators and administrators in LAC with broad information about how children's mathematical thinking develops, a framework for mathematical proficiency goals for the 21st century economy, and key ideas to consider when adopting educational technologies to support mathematical thinking.

The chapter begins by describing how children's developing brains make them open and ready to learn mathematical concepts, and also how instruction (via both teachers and technology) must consider the ways that

children might need additional support due to their age or background knowledge. At the same time, anxiety and feelings of pressure—or discriminatory stereotypes—may contribute to serious achievement gaps, such as those observed among students in LAC. These stereotypes may lead girls or children from minority backgrounds to learn less, or to perform below their actual ability on tests, due to social perceptions that these populations are poor at math. Throughout the chapter, tables summarize key points that, while not exhaustive, constitute some of the primary curricular and brain developmental considerations to keep in mind when evaluating educational technology or instruction.

The chapter revolves around a key message: that the blueprints used to design instruction and tools should be based on how children think, rather than on pedagogical techniques per se. While this sounds straightforward, it actually represents a major shift in orientation away from focusing on instruction and on what the teacher or technology is doing, to focusing on how best to respond to and foster children's thinking. This means organizing instruction around what children already know, what their minds are ready for, and how they may be feeling at the moment (e.g., anxious, curious, engaged, or bored). The successes or failures of mathematics education and the way educational technology is selected and implemented thereby relies on an ability help children build key foundational skills and overcome challenges.

1.1 A Conceptual Understanding of Mathematics

From early infancy, children's minds interpret the world through quantities, space, shapes, and patterns—the building blocks of complex mathematical thinking. Children do this naturally. For example, most babies pay attention to quantity and small number sets before they can walk or talk (Dehaene 1997; Gallistel and Gelman 1992; Lipton and Spelke 2003). Thus, despite the fact that many school-age children—and adults—harbor feelings of discomfort with mathematics, mathematical thinking is an innate part of human life.

Children's minds are biologically organized in a way that allows them to do mathematics, but they cannot learn more than basic quantity comparisons without instruction. Their environment for learning mathematics matters dramatically, and there is much that must be explicitly taught. For children to participate in higher-level mathematics, they must connect their early mathematical thoughts with mathematical symbols, such as numbers and operators. This is the first hurdle children face when learning formal mathematics, and one that is of fundamental importance to their

ability to reason using mathematical calculations, and to think in terms of mathematics instead of seeing it merely as a set of abstract principles to be memorized for a future test.

How are children first introduced to mathematical symbols? And how is their use of these symbols tested? This chapter argues for a new definition of mathematical proficiency, a definition that goes beyond simply ensuring that children know the correct answers to problems to ensuring that they do so with understanding. Consider the following problems (adapted from NRC 2009) as an analogy to how a child might learn to use number symbols that have been memorized as a list, rather than as a set of quantities.

Use the alphabet to solve the following problems (e.g., A = 0, B = 1, etc.):

1. Count onward from "J"
2. F + D = ___?
3. E x C = __?
4. How many fingers is "H"?

Despite our understanding of what it means to add, subtract, multiply, or compare quantities, and our easy knowledge of alphabetical order, the novelty of using these alphabetical symbols in that way significantly compromises our ability to solve these problems. This example demonstrates how challenging mathematics can be if it is known only as a set of rules that must be memorized. In the same way that many of us struggle with the use of alphabetical symbols to solve math problems, encouraging children to memorize the sums or products of calculations may lead them to simply answer particular questions correctly. But without first ensuring their understanding, it will mean that more complex calculations are just as awkward and abstract to think through. Part of the challenge for teachers is to recognize that for their own more developed understanding of mathematics to be shared by students, the teachers must ensure that students develop a deep sense of numbers, grounded in concrete experience, before they are taught how to manipulate numerical symbols in more abstract ways.

Traditional mathematics instruction has expected students to memorize procedures and follow rules to manipulate these symbols without providing conceptual connections to quantity, shape, space, or patterns. Teachers may have learned these symbols so well that they do not realize what a challenge it is for students to make these connections. Their expectation is that by getting students to practice using procedures, symbols, or rules, the students will make the connections needed

to solve everyday problems. Unfortunately, students instead often begin to view mathematics as a discipline comprised of computations within a set of disconnected procedures and rules that need to be memorized. While highly practiced fluency with mathematical symbols together with procedures and rules is essential for developing complex mathematical thinking, the ability to compute procedures quickly is not the same as mathematical proficiency.

Children must develop an understanding of mathematics that is connected to an internal model of quantity, and that enables them to reason through mathematical ideas generatively and in new ways, rather than memorize a set of disconnected rules. To accomplish this, they must understand mathematics as a discipline based on thinking and problem-solving (not just memorization), and mathematical concepts must be presented to them in a holistic, integrated manner (rather than as a list of separate, disconnected topics). This will provide them with the fundamentals needed to solve problems across employment sectors, and for a wide range of purposes, from accounting and financial management to policy decision-making (based on data) and programming technology. Thus, it is important that educational technology decisions begin with a clear definition of mathematical proficiency: namely, what do we want our students to know when they enter the workforce?

The next section describes the development of children's mathematical and cognitive skills in order to provide a framework for thinking about how to support mathematical proficiency over time.

1.2 How Children Learn

Instruction throughout primary school must consider children's developing minds; it must be age-appropriate and build on children's growing capacity. Piaget (1970, 1977) revolutionized child development research with the realization that young children are not less intelligent than adults, though they may view and engage the world in different ways than adults. Piaget then posited a set of stages that all children progress through. Much contemporary research (Demetriou et al. 2013; Fischer 2008; Weiten 1992) has revealed that these are not universal across cultures, and that a child's development does not always plateau at a specific milestone. Even so, Piaget's basic insight is important: adults must be aware that children's minds are not working the same way as their own, and adults must engage in explicit work to recognize children's thinking and identify their learning needs. Technology can facilitate the process of identifying children's developmental progressions in order to provide support that best

meets children's needs, based on each child's developmental stage and cultural milieu.

Adult support for cognitive development is important. As described by Vygotsky (1978), adults play a fundamental role in guiding children's development and must be sensitive to children's current state of knowledge and ability in order to help them reach the next level possible within the scope of their current capacity. This is described as the zone of proximal development—the range between what a child already knows and what he or she can attain with adult support. Technology can also play a similar type of supporting role. Most ideally according to this theory, technology will be able to meet children adaptively based on their prior knowledge and provide support to allow them to be successful at the next skill level. Thus, outside support, whether human or technology-based, must first attend to children's current thinking before it can help them move forward. The following describes key aspects of children's development based on maturation.

1.2.1 Brain Maturation and Cognitive Constraints

Much research states that children's brains continue to develop throughout childhood and into adolescence, and in some regions even into the third decade of life (Mungas et al. 2014). Thus, even throughout the primary school grades, children's brains are still malleable and changing with age and inputs from their environment, including neighborhoods and schools (NRC and Institute of Medicine 2000).

One particular area that continues to develop is the frontal lobe, the part of the brain located behind the forehead, which has serious implications for children's mathematics instruction. The frontal lobe is engaged in many higher cognitive acts; it is in part responsible for problem-solving, reasoning, and planning effortful solutions, as well as for inhibiting one's impulsive behavior or thoughts (Stuss 2006).

Within the frontal lobe, a constellation of mechanisms work together to regulate humans' attention and cognitive processing, known as executive functions. This is a system that takes a limited set of attention resources and distributes those resources to a variety of cognitive subprocesses that in turn regulate the dynamics of human cognition (Diamond 2013; Miyake et al. 2000). One very important part of this system is working memory, which involves the ability to hold information in the mind and actively use it for problem-solving, reasoning, or other purposes. For example, in a classroom, if students have been given class instructions (e.g., "finish this problem and then write your solution on page 7 of

your packet") and then a word problem to solve, they must hold the class instructions as well as the problem numbers and task goal in their mind while also attempting to perform the relevant calculations. If a student does not have enough working memory available to remember all of this, he or she will likely lose parts of the problem and appear to not know how to solve it, or might not write down the solution where the teacher asked, when really the issue may have been the student's ability to simultaneously remember all of the details.

The second primary subcomponent of executive functions in children is inhibitory control, also called cognitive control, in which the reasoner exerts control over his or her immediate impulses and works to ignore irrelevant information (Diamond 2013). As an example, if a student is adding two fractions in a classroom, the student's impulse is to add both the numerators and then the denominators, since that is the way arithmetic has always worked with integers. However, the student exerts inhibitory control to resist this temptation and instead searches his or her mental space for the correct procedure and executes this.

Reasoning mathematically requires a large amount of both working memory and inhibitory control, meaning that learning environments that tax these resources tend to reduce reasoners' ability to make inferential leaps, attend to abstract relationships, and broadly perform higher-order thinking (Tohill and Holyoak 2000; Cho, Holyoak, and Cannon 2007). Thus, teachers and educational technology must not overload children's developing executive function resources so that students do not attend to irrelevant information and can retain adequate information in their mind to solve a problem. Also, overloading will mean that students memorize procedures rather than draw connections and develop necessary conceptual knowledge.

Overloading working memory and inhibitory control resources can happen when children have to do lots of calculations in their heads, such as when teachers give long lists of instructions, or when there are lots of distractions that they have to work to ignore or not respond to. In technology platforms, these distractions such as irrelevant pictures, noises, or game steps that a child needs to either remember or ignore, should be avoided. Similarly, there can be distractions in an everyday classroom, such as having to ignore an alluring misconception, remembering problems without being able to write down or see the steps, or needing to recall a long list of activity instructions that are external to the mathematics concepts themselves.

The reason that working memory is important in mathematical thinking is because it plays an important role in taking mathematical

information and making sense of it, transforming it (e.g., moving from a word problem to a symbolic equation), and, generally, allowing children to think their way through problems. Imagine that a student is listening to two other students describe different ways they solved a problem. This can be an extremely beneficial way to help students realize that most math problems can be solved many different ways, and so they should therefore try to think through a problem, not just use a method taught by the teacher. However, to benefit from this kind of mathematical discussion comparing solutions to problems, each student must create two mental models of these solutions and then line them up in his or her head to consider whether both are correct, how similar (or divergent) they actually are, and whether the student could use these models on another new problem.

Inhibitory control is integral to suppressing irrelevant yet potentially salient misconceptions (Cho, Holyoak, and Cannon 2007; Richland, Morrison, and Holyoak 2006; Begolli et al. 2018). In mathematics, this could include the misconception that dividing by a fraction should lead to a smaller number (as it does in integers), or that 6/10 should be more than 3/5 since the numbers are larger. Thus, variations in executive function cognitive capacity may explain why some students notice and benefit from mathematical learning opportunities, while others do not unless provided with more instructional support.

For example, in the problems mentioned earlier, when A = 0, B = 1, C = 2, and so on, solving the equivalence problem F + B = ——— + D requires that we hold active in working memory each letter (or number symbol in the case of young children), retrieve its correspondence to the number symbol (magnitude) from our long-term memory, and manipulate this information in our working memory to get to the answer. So, understanding this solution in terms of the magnitudes that F, J, and D represent, while focusing on the steps necessary to get the correct answer (D), represents a considerable effort even for adults. Thus it is unlikely that a child will have adequate additional mental resources to consider why and how he or she is doing this manipulation and whether the answer seems correct. However, in LAC, the primary mathematics teaching method continues to be drill, practice, and memorization of procedures (Näslund-Hadley, Loera Varela, and Hepworth 2014). Although some memorization is needed, an almost exclusive focus on procedures and artifacts leaves the child with fewer resources for critical and creative thinking.

Educators and technology designers need to be aware that if children's executive function resources are overtaxed (including by mathematically

irrelevant artifacts such as the requirement to remember complicated instructions or in a game design that requires attending to features that are not mathematically relevant), children may not have adequate resources to deploy for problem-solving, mathematical reasoning, checking solutions, or remembering complex concepts. For example, identifying similarities and differences between mathematical problems or solutions has been deemed as useful for building conceptual knowledge of mathematics (NRC 2001; CCSS Initiative 2010). However, the way that these problems or solutions are presented can have a large effect on student thinking, with the largest gains being when students do not have to remember what their classmates or the teacher said, but rather when they can see these both on a board, a screen, or on paper (Begolli and Richland 2016; Richland and McDonough 2010). If a teacher states that a new problem "uses the same strategy as the last problem," but the students have to spend mental effort remembering what that last problem strategy was, they will have less time to think about how that solution can be applied to the new problem. Educational technology needs to have similar considerations in mind, for example by visualizing previously referenced examples, to ensure that children's resources are not overburdened. Table 1.1 highlights points in children's cognitive development that are important to consider when using technology in the classroom.

1.3 A New Definition and Standards for Mathematical Proficiency

The recommendations of research analysis and international reports on LAC are evident: educational improvements in the region require clear, high-quality standards for student learning and achievable steps for attaining these aims (ICSU-LAC 2010; Board 2006; Puryear and Goodspeed 2011). Learning standards provide common norms for everyone involved in the decision-making process of designing and implementing mathematics educational technology. The standards should involve attainable goals for student thinking, rather than a list of topics to be covered or general learning theories that are difficult for teachers to implement (Zimba 2014). LAC has largely focused on expanding access to education, but few countries have focused reform efforts on developing national learning standards (Board 2006). Many of these standards are still developing, but investments in schools have thus far not resulted in increased learning outcomes (Puryear and Goodspeed 2011).

As in other higher-achieving countries, educational reforms in the United States have taken the approach that a necessary first stop to creating coherent and effective learning experiences for the nation's youth

TABLE 1.1

KEY DEVELOPMENTAL POINTS TO CONSIDER WHEN EVALUATING AN EDUCATIONAL TECHNOLOGY OR CURRICULUM

Cognitive or Curricular Factor	Implication for Learning	What to Look For
Primary school children's brains are still developing the capacity to control their attention, focus on the relevant parts of incoming information, and purposefully not pay attention to irrelevant information. This capacity is referred to as their executive functions.	Instructional designers may be surprised that children have trouble identifying the key information they are supposed to be attending to within a lesson or informational display, and are easily distracted. This distraction can lead to not paying attention to crucial lesson content, or to remembering irrelevant or sometimes misleading information.	Informational displays should limit irrelevant information (even if intended to increase interest), use movement sparingly but intentionally, and at the same time use cues to draw attention to key information (e.g., brighten information, or show multiple representations of the same concept together).
Children's brains are also developing an ability to hold several pieces of information active simultaneously. This is called working memory.	Holding multiple steps of problems in their memory, remembering how one problem is related to another problem, thinking about task instructions while planning a multistep solution—all of these are challenges for primary school children.	Activities should not require remembering lots of instructions or steps while planning, thinking through, or executing complex problem-solving. If children are working to control their attention (see the discussion of executive functions in the top left panel of this table), they will have less working memory available to simultaneously think about many steps, pieces of information, or plans.
Transfer: Children who learn a concept or problem-solving strategy for one problem often do not notice that they can use it for other problems or in new contexts.	Children's mathematical knowledge becomes inflexible and unlikely to be used in everyday contexts, or they do not make connections from one concept to another. This requires relearning and memorizing many separate topics rather than making sense of mathematics in a more coherent manner.	Mathematical ideas should be taught in relation to other ways that they can be used. Activities should be very explicit about the connections between mathematical ideas or concepts, using comparing or contrasting language.

Source: Prepared by the authors.

is to develop high-quality standards for mathematical proficiency. The U.S. standards stem from a comprehensive research base and are aligned with educational standards of other higher-achieving countries (Cobb and Jackson 2011). While the U.S. Common Core curriculum has generated controversy and is not without fault, it does provide one strong model for how to use researcher-practitioner partnerships to build a coherent set of standardized goals for student learning. Importantly, these goals are not only about curriculum topics. They also include practice standards, which are goals for student behaviors and approaches to mathematics, as described in more detail below.

The U.S. Common Core Standards were established with a dual goal to (1) provide guidance for educators about key topics and practices to focus on, and (2) allow for common ground so that there could be district, state, and federal testing to measure and compare student progress. The Common Core has been controversial and has been both praised and criticized in terms of both of these goals. With regard to the first goal, the standards are under pressure to ensure that key mathematical content areas are adequately covered and that enough guidance is provided to teachers to ensure that the aim can be implemented as intended. The second goal has been more controversial, with concerns arising in part because testing has grown to replace many instructional days, and in part because these tests are often tied to funding decisions. Educators argue that there are many reasons why students might underperform relative to peers that are not tied to educational quality, such as parent investment or financial security. At the same time, testing can provide insight into where resources must be directed to improve student learning, including professional development of teachers.

Since the focus here is on the important step of developing standards for mathematics instruction, the U.S. Common Core Standards are used as an example that raises some key issues for consideration as Latin American and Caribbean countries develop their own versions of the standards.

1.3.1 An Example of Research-Based Standards: The U.S. Common Core Standards

The development of the U.S. Common Core State Standards for Mathematics (CCSS) derived from the collaborative efforts and expertise of 73 specialists involved in educational reform (Zimba 2014). It took into consideration the learning standards of internationally top-achieving countries on the standardized Programme for International Student

Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) assessments (Cobb and Jackson 2011). While many countries have their own curriculum standards, the CCSS has unique properties that provide recommendations on how to formulate teaching goals that draw on theory and aim for deeply thoughtful learners, but are also practical in a classroom context. The CCSS is not prescriptive in that these goals may be reached using different instructional techniques. But it does provide teachers with goals for practices, as well as a structure for deciding which topics to cover and when. Having teachers all align with these topical sequences helps to ensure vertical integration—coherence between the curricula taught over multiple years of primary education—such that teachers know what their students will have learned the year before. The standards were developed by establishing the following rigorous criteria (CCSS Initiative 2010):

- Fewer, higher, and clearer standards to best drive effective policy and practice
- Alignment with college and work expectations so that all students are prepared for success upon graduating from high school
- Inclusive of rigorous content and applications of knowledge through higher-order skills so that all students are prepared for the 21st century
- Internationally benchmarked so that all students are prepared for succeeding in the global economy and society
- Research- and evidence-based.

These criteria resulted in the creation of a model for standards based on two key components: the development of curriculum content (content standards); and the instructional practices that lead to mathematical proficiency (practice standards), as summarized in Table 1.2. Practice standards describe the particular skills that are expected from students, while content standards are developed on the basis of these expectations. The relationship is reciprocal: knowledge and skill expectations drive the nature of content, and content selection constrains/fosters the expected knowledge and skills. Educational technology design should be informed by standards, and content should feed into expectations. The next section focuses on standards for curriculum content and mathematics practices, drawing attention to not only content but also to specific goals for students' mathematical thinking. The sections that follow then highlight some of the core ideas that run throughout the key standards for primary school mathematics.

TABLE 1.2
U.S. COMMON CORE STANDARDS

A. Practice Standards	B. Content Standards
• Make sense of problems and perseverein solving them • Reason abstractly and quantitatively • Construct viable arguments and critique the reasoning of others • Model with mathematics • Use appropriate tools strategically • Attend to precision • Look for and make use of structure • Look for and express regularity in repeated reasoning	**Elementary** • Whole numbers • Addition and subtraction • Multiplication and division • Fractions and decimals **High school** • Number and quantity • Algebra • Functions • Modeling • Geometry • Statistics and probability

Source: Prepared by the authors based on CCSS Initiative (2010).

Practice Standards

Practice standards are designed to describe and codify critical "processes and proficiencies" needed for students to be qualified users of mathematics in and outside the classroom. Practice standards refer to how students engage with mathematical tasks and content, and are distinct and separate from curriculum topics, which state the mathematical content knowledge students must master. Internationally, national mathematics standards typically include curriculum content skills, but they less frequently contain process skills such as those described in panel A of Table 1.2. The practice standards needed by successful students in the U.S. context were identified by a highly esteemed, independent organization that has been involved in school reform and standard development since 1920, the U.S. National Council of Teachers of Mathematics (NCTM). The NCTM publishes four research journals in mathematics education, including the most influential periodical in mathematics education worldwide—the *Journal for Research in Mathematics Education* (H Index, 60, 2017). The NCTM's synthesis of research resulted in the creation of the "process" strands in problem-solving, reasoning and proof, communication, representation, and connections (NCTM 2000).

These strands derive from a definition of mathematical proficiency developed by the U.S. National Research Council (NRC 2001), which was charged by the U.S. Department of Education with defining what to expect of students exiting high school. The council's landmark 2001 report integrates a large body of mathematics educational research to define mathematical proficiency and guidance on how they should be used to

frame educational goals. The NRC defined mathematical proficiency as a set of five interwoven strands:

- *Conceptual understanding:* Comprehension of mathematical concepts, operations, and relations
- *Procedural fluency:* Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence:* Ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning.* Capacity for logical thought, reflection, explanation, and justification
- *Productive disposition:* Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (NRC 2001).

The CCSS practice standards were developed by building on the NCTM's definitions of mathematics proficiencies—"processes and proficiencies." More detail about each student expectation is as follows:

1. *Make sense of problems and persevere in solving them.* Problem-solving begins with students explaining to themselves the meaning of the problem and analyzing multiple entry points for solutions. Solutions themselves are checked with different methods to ensure their validity.
2. *Reason abstractly and quantitatively.* The student should have the ability to decontextualize a problem by representing it only through abstract symbols, such as numbers and/or shapes (e.g., Juan had some apples. He gave 42 to Maria and is left with 34. $x - 42 = 34$), and the ability to pause and contextualize abstract symbols into their referents. Children should reason about quantity through units and by understanding the meaning of quantity, not just by following the computations.
3. *Construct viable arguments and critique the reasoning of others.* Students should communicate by building logical arguments and justify critiques by breaking down situations into cases. They should reason inductively about data and evaluate plausibility based on the context of the data. Mathematically proficient students should be able to evaluate effectiveness and plausibility to recognize flawed arguments, recognize domains where correct arguments apply, and ask questions.
4. *Model with mathematics.* Students should reason about everyday events and use mathematics to describe these events. They

should be able to quantify practical situations and connect or represent them through graphs, tables, diagrams, flowcharts, and formulas while being able to flexibly traverse between the data and context.

5. *Use appropriate tools strategically.* Students should be familiar with the mathematical tools that are available to them to solve problems, including pencil and paper, calculator, spreadsheets, compass, ruler, dynamic geometry software, or a statistical package. Importantly, students should be able to recognize which tool is appropriate for each situation or problem.

6. *Attend to precision.* Mathematically proficient students should be able to communicate with precision (both in writing and verbally, according to context), determine units of measure and labels, understand the symbols they use, and calculate with efficiency.

7. *Look for and make use of structure.* Students should be aware of structural properties of numbers and shapes. They should understand how to make use of the commutative property (A + B = B + A and A x B = B x A), the associative property (A+ (B + C) = (A + B) + C and A x (B x C) = (A x B) x C)), and the distributive property (A x (B + C) = A x B + A x C). Mathematically proficient students will notice patterns and be able to break problems down into parts that make sense. For example, in the following problem, 5 * (x+3) = 30, students should be guided to notice that 5 * "something" = 30, namely (x + 3) could be thought of as an abstract quantity, and if students know 5 x 6 = 30, then "something" or x + 3 = 6. Students can simplify the problem by using their knowledge of its mathematical structure to solve it on a conceptual level without necessarily following a procedure. Similarly, by knowing that a box is comprised of six sides, students should recognize that the area of a cube is equal to the area of six squares.

8. *Look for and express regularity in repeated reasoning.* Mathematically proficient students should be guided to reason out repetitive mathematical operations and/or results to derive abstract knowledge—including generalizations, such as mathematical formulas and shortcuts. For instance, a teacher could ask students to come up with various ways of representing the number 32 through fractions. Some students may represent 32 as 64 ÷ 2, 96 ÷ 3, 128 ÷ 4, while others may represent 32 as 320 ÷ 10, 3,200 ÷ 100, 32,000 ÷ 10,000, and so on. The teacher can lead them to discover that 32 could be any number, for instance, "x" and that the general pattern is nx/n = x for all n>0.

Content Standards

A second major contribution of the CCSS is to specify big ideas that run through multiple topic areas but that lend coherence to the curriculum. Many national standards have been critiqued as being a "mile wide and an inch deep" (Schmidt, Houang, and Cogan 2002), meaning that they encourage a curriculum with too many topics, such that teachers cannot help students attain a deep conceptual understanding of any one topic.

Instead, the CCSS proposes that curriculum content be focused and coherent in order to foster students' attainment of knowledge and skills in accordance with expectations. The curriculum focus is critical for centering everyone's attention on topics that are essential to mathematical proficiency. Coherence between topics is key so that content is rendered in a logical way, aligned with the structure of mathematics as a discipline (Zimba 2014). The focus and coherence of mathematics content is comparable between the top-achieving countries in international assessments of mathematics, and the U.S. Common Core Standards have a striking similarity with those seen in these countries (Schmidt, Houang, and Cogan 2002; Schmidt and Houang 2012). The standards seek to harness the foundational content necessary for students to develop complex mathematical thinking focused on mastery of procedures, conceptual understanding, and the application of mathematics to real-world situations. While the entire curriculum cannot be covered here, the core mathematical ideas for primary school mathematics can be summarized as (1) number, and (2) geometry and measurement. Administrators evaluating educational software might consider whether the content being taught aligns with these foundational knowledge areas. Table 1.3 provides a list of content curriculum areas covered by the content standards for primary school mathematics.

TABLE 1.3
KEY PRIMARY SCHOOL MATHEMATICS CONTENT CURRICULUM AREAS

Counting and cardinality
Operations and algebraic thinking
Number and operations in base 10
Number and operations—fractions
Measurement and data
Geometry
Ratios and proportional relationships
The number system
Expressions and equations
Functions
Statistics and probability

Source: Prepared by the authors based on CCSS Initiative (2010).

1.4 The Development of Mathematical Thinking

This section describes in more detail the two categories of the mathematical skills presented earlier in Table 1.2: (1) number skills, and (2) geometric/

measurement/spatial skills. A wide spectrum of early mathematics software is available for these two skill categories.

1.4.1 Number Skills

At the heart of the mathematics curriculum during the preschool, elementary school, and middle school years is the concept of numbers (NRC 2001). Research on numerical development encompasses a wide gamut of mathematical skills, from rudimentary knowledge emerging in early infancy to complex mathematics in adulthood, examined from various developmental perspectives and disciplines, including cultural, linguistic, cognitive, and neurological ones, to name a few.

While one may be used to thinking about mathematics as a precise discipline centered on numbers, the example of numbers substituted with alphabetic symbols above shows that numbers are tools to help us think about deeper mathematical concepts relating to quantity, shape, and measurement. Before babies have ways of talking explicitly about numbers (e.g., with words, "one" "two," and later with symbols, e.g., "1, 2"), they have a system in their brains that helps them discriminate coarse magnitudes and shapes. For example, six-month-olds can distinguish between 2:1 ratios (e.g., seeing eight ducks versus four ducks), which is supported by neurological data. Infants do not count, but instead have an approximate representation that can discriminate eight ducks as more than four ducks. However, children do not begin to make the connection between the elementary, separate quantities 1–4 and symbols to describe those set sizes until they are 3 or 4 years of age, or later for children from impoverished backgrounds. This progression from nonsymbolic (size, quantity, amount) ways of thinking about numbers to symbolic ways (e.g., number words or agreed-upon conventions for shapes that stand for amounts such as "1" and "2") happens at approximately 3 or 4 years of age. As children grow, this correspondence between symbols and senses of quantity expands to account for a wider range of whole numbers. This progression is slow and stepwise, starting with 0–10 when they are around 4 or 5 years old, then 0–100, until they comprehend 0–1,000 when they are 8 or 9 years old. At the core of the developmental trajectory underlying numbers lie the mathematical concepts of ordinality, cardinality, and one-to-one correspondence that present a particular challenge for children and are described next.

Numbers: Ordinality, Cardinality, and One-to-One Correspondence

It is hard to think of a concept that can match the versatile nature of numbers. Numbers can be thought of as an infinitely long, ordered list of

distinct numerals, that is, ordinality (e.g., 1, 2, 3, 4, …, which we will refer to as the "number list" for ease of reading) and also as quantifying a set of things, that is, cardinality (e.g., "I have four apples"). As adults, we may think of the number list and cardinality as a single system of numbers, but children still need to make a connection between the two. These concepts are very familiar to adults, and to remind the reader of how difficult these connections can be when working with abstract symbols, we refer back to the example used in the beginning of the chapter (e.g., how many fingers is "H"?). To use numbers for mathematical thinking, people need a physical representation either in spoken or written form (NRC 2001). Number symbols are arbitrary: for example, the quantity of three could be represented by the symbol 3, but we could have adopted a different symbol (e.g., III, or D, as was the case in the example). The connection between the number list and cardinality underlies all mathematical thinking.

As children begin to understand the one-to-one correspondence between certain items and a list of numbers and cardinality (a set of items), they begin to realize that counting is a form of addition. If a student has 4 apples and the teacher gives him or her 1 more, the student does not need to count the 4 apples from the beginning to realize he or she has 5 apples, if the student realizes that 5 comes after 4 on the list. Yet, children who are taught to follow a procedure for addition will often revert back to counting their set from the beginning (i.e., count the set of 4 apples then count 1 more apple to make 5 apples) because they have been taught to follow such procedures for "addition." Thus, children need to be guided to observe that counting is a way of adding, whereas subtracting is counting backwards. Educators need to keep in mind, however, that these concepts are connected through symbols that children are still mastering (e.g., imagine if one had to count onward from "J"). Often children will be able to "add" by counting, without understanding the increase in magnitude, just as an adult would try to perform the procedure of counting onward (i.e., recite the letters) from "J" by knowing that after "J" comes K, L, M and so on, but may not be able to immediately think of the overall quantitative increase or the total quantity—denoted by the last symbol of the set. Much too often, teaching and technology reinforce the recitation of numbers, not understanding. Instead, symbols need to be accompanied by magnitude representations, such as number lines that increase/decrease in quantity in alignment with the list of symbols (Siegler and Ramani 2008). As children master the one-to-one correspondence, they need to be guided toward understanding our number system as a base 10 place-value system, leading them to become fluent in making 1-to-10, 1-to-100, 1-to-1000 etc. correspondences, since children think of each place value as a single unit.

Base 10 and Place Value

A key element of gaining number sense in the primary grades is developing an understanding of the base 10 pattern and place value and using these for arithmetic calculations of addition, subtraction, multiplication, and division with increasingly large numbers. Early place-value skills predict arithmetic skills in middle primary school (Moeller et al. 2011), and experimental data reveal that high-quality training in base 10 understanding leads to gains in overall number understanding (Mix et al. 2017). Interestingly, Mix et al. (2017) found that all students receiving directed instruction about how to decompose numbers into their base 10 structure using symbols (e.g., 112 = 100 + 10 + 1) and those who did so with both symbols and with base 10 blocks improved on number line magnitude skills, which are a key indicator of number skills. Also interestingly, they found that while all students gained, the concrete manipulatives worked best for students who started with lower understanding, while the symbol version was most effective for students who started with a higher understanding.

In arithmetic, children must first understand the mathematical concepts underlying their calculations. Understanding can be evidenced by their ability to group and decompose numbers flexibly (e.g., understanding easily that 4 + 3 = 7, which also equals 2 + 5 or 1 + 6 or 6 + 1, or the number of cookies held by a child with 4 cookies who gains 3 more). By grouping and regrouping in these ways, children can begin to learn what happens when the sum reaches above 9, leading to a second 10, which introduces place value. Regrouping numbers into tens and ones (e.g., 121 = 100 + 20 + 1), and building on that to add numbers by adding the ones, tens, and hundreds separately, can be used to gain a strong understanding of the role of place value.

An important consideration when teaching any mathematical rule or algorithm is that children seek efficiency. Therefore, if they are taught an algorithm, such as "carrying" or "borrowing" for multidigit addition or subtraction, they will very likely attempt to use that rule regardless of whether they truly understand it. The main problems stemming from this are that (1) children make errors in the rule execution, but their lack of understanding means that they do not recognize their answers as implausible; or (2) they do not understand the limits to applying a rule, leading to over- or under-application. Importantly, once students start using an algorithm, an instructor will have to work very hard to motivate them to pay attention to more conceptual discussions or activities that prove the rule. Thus, it is essential to ensure that students have a strong base for understanding a rule, such as "carrying" or "borrowing" for multidigit addition or subtraction, before it is introduced.

Fluency

Once this understanding of place value is strong, and only then, is fluent memorization of mathematical facts important. Children must become fluent in these calculations, meaning that they must practice speeded memorization of routine calculations of addition, subtraction, multiplication, and division of integers between 0 and 12. This enables students to free up conceptual resources for thinking through increasingly more complex problems. Thus, both conceptual understanding and memorization are important to developing strong number sense, but memorization that proceeds without understanding is unlikely to support full mathematical proficiency. Technology can provide an excellent tool for such memorization practice, with optimal efficiency produced by creating some spaced time between each repetition of a number fact to be memorized, but making shorter intervals between repetitions for items that were answered incorrectly, and longer intervals for items answered correctly (Kang 2016).

Fractions

Fractions are an additional core curriculum area in primary mathematics. However, fractions are highly challenging, in part because they involve a different understanding of numbers than emerges from experience performing arithmetic calculations with integers. For example, larger numbers in the denominator of a fraction signify an increasingly small quantity, which is counterintuitive unless children fully understand the role of fractions as partial quantities. Children's understanding of fraction representations seems to develop around second grade, but some adults never reach high levels of fraction proficiency (DeWolf et al. 2014). Foundational knowledge of fractions is thought to be critical for children to successfully advance to algebra. In fact, the evidence suggests that fraction knowledge at age 10 predicts algebra knowledge at age 16, after accounting for other types of mathematical knowledge (e.g., addition, multiplication), cognitive ability measures, and family income and education (Siegler et al. 2012).

There are several competing theories for how children advance from a rudimentary stage of distinguishing magnitudes to proficiency with fractions. A common thread among these theories suggests that the development of children's knowledge of fraction concepts is distinct from that of whole numbers. This implies that an understanding of whole numbers interferes with the later learning of fractions (Wynn 2002; Gelman and Williams 1998; Vosniadou, Vamvakoussi, and Skopeliti 2008; Geary 2007). These theories have provided fruitful ways for thinking about how children learn whole numbers, but have generally provided incomplete accounts

of children's developing thinking, suggesting that there are indeed strong relationships between whole number and fraction understanding.

More recent studies indicate that children who can more accurately put a number on a number line and more fully understand base 10 structure and number decomposition can more quickly build fraction understanding (Ischebeck, Schocke, and Delazer 2009; Bailey, Siegler, and Geary 2014; Meert, Grégoire, and Noël 2009; Siegler and Lortie-Forges 2014). This has important implications for educators and technology designers because it suggests that children should be taught to integrate their knowledge about whole number magnitudes with their understanding of fraction magnitudes. Similar to issues of one-to-one correspondence with whole numbers, children need to understand the correspondence regarding the relationship between two numbers in a fraction and the magnitude they represent. A successful intervention that could translate to educational technology is to teach children to represent fractions on a number line in order for them to understand the magnitude related to the fraction and connect that with their whole number knowledge (Fuchs et al. 2013, 2014). This is also supported by a recent theory of number development from Siegler and colleagues, known as the integrative theory of numerical development, which suggests that children's conceptual development of fraction knowledge lies on a continuous progression with their conceptual development of whole numbers (Siegler, Thompson, and Schneider 2011; Siegler and Lortie-Forges 2014). Based on this theory, children's number development is comprised of four developmental steps that build on each other:

1. Nonsymbolic representations of quantity
2. Moving from nonsymbolic to symbolic representations of quantity
3. Extending symbolic representations to larger quantities
4. Extending knowledge of whole numbers to rational numbers (fractions).

These stages are useful for discussing numerical development through primary school. There is research support for the idea that improving knowledge of whole numbers extends to fractions and, subsequently, fraction arithmetic (Fuchs et al. 2013, 2014). Thus, moving from nonsymbolic to symbolic representations between magnitudes and number symbols of increasingly large or fractional numbers is not a rote memorization process. Rather, it is the key developmental progression in children's understanding of numbers and should be considered in the design of primary school curricula. Also crucial is that at each stage the teacher guides students to recognize the relationships between each of these four steps, for example showing nonsymbolic quantities next to

symbolic representations. While this is more commonly enacted for small numbers in the early grades, it is also important for larger numbers and fractions. This could be accomplished with manipulatives, an abacus, or even by drawing and counting marks such as tallies.

1.4.2 Geometry, Measurement, and Spatial Thinking

Beyond numbers, primary school mathematics builds on children's fundamental spatial and geometric skills. From complex natural structures of flower petals to the intricate architecture of skyscrapers built to resist earthquakes, humans perceive objects in the world as shapes of various measurements existing in space. Geometry may be defined as the study of shapes and space, and measurement as a manner of specifying the size of objects. Together, these play a vital role in children's development of sophisticated skills needed in many modern undertakings, including science, engineering, architecture, and art. The purpose of geometric shapes (triangles, circles, cylinders, etc.) is analogous to the purpose of numbers. They are abstract objects that approximate objects in the real world and serve as thinking tools that help us represent, measure, and manipulate objects around us.

Abstract objects, such as the cube, provide us the freedom to focus on their varying attributes of two-dimensional (2-D) and three-dimensional (3-D) space. For example, children can attend to length and area in 2-D and volume in 3-D. Working with these attributes necessitates an understanding that various units can be used to measure a cube. One could use a one-meter stick to measure the length of the sides, one-meter-squared tiles to measure the area, or one-meter-cubed blocks to measure the volume. These measurements are extensions of a way to understand that the size of each unit always adds up such that the final number means the total number of units (e.g., 3 meters long, 3 square meters, or 3 cubed meters). This is the same notion of cardinality described earlier, where children must first understand, when counting, that the final in a count list of objects refers to the entirety of the set. Thus, measurement and geometry can provide a supporting context for learning cardinality, a key mathematical concept, as well as for understanding how to use cardinality to understand measurement.[1]

There are object properties that can be observed/discovered by composing/decomposing shapes and/or moving them through space. Analogous with the conceptual benefits of decomposing and combining

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[1]   For a fuller theory of children's learning trajectory for measurement skill, see Szilágyi, Clements, and Sarama (2013).

numbers into sets, smaller shapes can be combined to form one large shape, or a single large shape can be decomposed to form smaller shapes. Composing and decomposing are important geometric manipulations that will help children understand concepts of area in 2-D space and volume in 3-D space. Reasoning about mathematics in the number domain and in the geometry and measurement domain is deeply intertwined, such that understanding in one domain can be used to facilitate understanding in the other.

Spatial thinking skills include how an object is positioned in space, alignments between objects and the relationships among them (above, below, half, etc.), ways of representing ideas in relation to one another (e.g., 1/2 or 1:2), and the vocabulary itself used to describe spatial relationships ("above," "under," "behind;" see NRC 2006). Humans and animals use spatial thinking to navigate their environment—this is how we find our way home after a long walk. While navigation has important implications for finding our way through space, other aspects of spatial thinking are also theorized to be fundamental to mathematics education.

Spatial thinking skills afford the learner a way to conceptualize problems before solving them (Clements and Sarama 2007) and to categorize and represent shapes and objects and manipulate them through transformations (e.g., rotating objects, translating or moving objects, zooming in or out of objects, and folding; see NRC 2009). All species of animals that move through space use some form of spatial thinking, but only humans can extend their spatial knowledge through symbolic representational systems and figures such as numeric and geometric symbols, language, units of measurement, maps, diagrams, and graphs (NRC 2009). Thus, humans have the advantage to be able to learn and build on representations by reasoning. The sophistication of spatial thinking skills requires humans to use all three aspects of spatial thinking—space, representation, and spatial reasoning—in concert. While children have rudimentary spatial thinking skills from early infancy, their development of sophisticated spatial thinking seems to largely depend on their experiences with symbolic representations and figures, including puzzles, blocks, and digital environments. This suggests that spatial thinking abilities can and should be formally taught in schools with the use of strategically designed curricula and technology (NRC 2006).

The research evidence on spatial thinking suggests that infants can recognize and categorize shapes, objects, and distances on a coarse level before they can talk (NRC 2009). It is not until the second year of life that infants begin to make connections between their spatial abilities and use of spatial language. For example, 3-month-olds can differentiate spatial categories such as up versus down and left versus right (Quinn 2004). By

five months, infants can use geometric cues to learn the spatial location of objects (Newcombe, Huttenlocher, and Learmonth 1999). These abilities progress by 12–16 months to help them search for hidden objects (Hermer and Spelke 1994). By 4–5 years, children start showing abilities to perform transformations—moving objects in space and mental rotation (that is, turning an object on a vertical or horizontal axis by visualizing it in space). At this stage, children's skills are not always reliable, but the spatial thinking skills needed to transform objects in one's mind (e.g., imagining how a paper might look if it was folded), mental rotation, and visualizing objects from different perspectives continue to increase over time. Importantly, the development of spatial thinking depends on experiential skills.

Gender differences have played an important part of the literature on spatial skills and related mathematics. In the United States, clear gender differences emerge by 4½ years old, and differences based on parents' income and education emerge by the second grade (Levine et al. 2005). While some have raised the possibility that these differences are genetic, much work suggests that socialization of gender differences in key mathematical and spatial skills begins early (Levine et al. 2016). A longitudinal study examining how U.S. parents play with their children suggests that parents used more spatial words and games when playing with their boys than with their girls (Levine et al. 2012). The amount of parent use of spatial terms predicts children's own spatial language use (Pruden and Levine 2017), and parents' spatial talk predicts children's later spatial skills (Levine et al. 2012). Parents' use of spatial language and children's play experiences (e.g., playing with blocks and puzzles) also vary across different levels of socioeconomic status, which may lead to spatial thinking differences between high and low socioeconomic status children upon school entry, which in turn can lead to differences in school mathematics skills such as geometric thinking (Lourenco et al. 2011). This malleability is an opportunity to enhance the skills that underpin one aspect of primary school mathematics. Also important with regard to gender differences, it may be that identified gender differences across spatial and/or mathematical skill domains are driven by adult socialization, rather than any genetic or sex differences (Levine et al. 2016). While there are no comparable studies conducted in LAC, awareness of the potential for gender differences in educational experiences is important and could help to mitigate any tendencies to differentiate by sex.

Mental rotation and transformations represent the foundation for developing more sophisticated spatial thinking skills. These are simple skills that are highly correlated with broader mathematical achievement (Mix et al. 2016). Additionally, symbolic representations of space, such as spatial language, seem to play a key role in shaping children's geometry

development in later years. Language helps children retain spatial concepts (Gentner 2003). Teachers and educators can use more spatial language and measurement terms (units, cardinality statements) to increase children's knowledge and use. Children's understanding greatly depends on their experiences.

Early spatial skills together with environmental factors represent foundational knowledge for the later development of geometry and measurement. Measurement is important because it represents the intersection between geometry and numbers: that is, measurement can attach a "number to spatial dimensions" (NRC 2009). Children's ability to measure seems to arise from their ability to compare lengths of objects, which begins around 4–6 months old (Baillargeon and DeVos 1991). Though these length discriminations are coarse, they become more precise between 2 and 4 years old. Units, meanwhile, pose a challenge that children do not overcome without explicit instruction. This is particularly true for transformations between units (e.g., 1 meter = 1,000 millimeters). Despite the important role that spatial thinking plays in the development of measurement and geometry, spatial thinking has not been successfully integrated into educational curricula. Nevertheless, the research base supporting spatial thinking has drawn interest in educational communities because it is trainable and has been linked to achievements in science and engineering careers. In LAC, the teaching of spatial reasoning has already been piloted and found to increase early mathematics learning (Näslund-Hadley, Loera Varela, and Hepworth 2014).

In the last two decades, scientists have discovered promising interventions that could close the gap between students with high and low spatial thinking skills. A meta-analysis of over 200 studies revealed significant gains after explicit training (Uttal et al. 2013b), suggesting this is an important area of focus in primary school mathematics. This meta-analysis suggests that training works for both males and females equally and lower performers gain more spatial skills than do higher performers. Spatial thinking interventions were categorized into three categories: course training (e.g., engineering courses), video games, and spatial tasks. There were no significant differences in the results observed across these three methods—all led to improvements (Uttal et al. 2013b). There are two reasons why spatial training might be effective. First, interacting with spatial tasks makes people more comfortable attempting them in social situations, reducing performance anxiety and fear of gender stereotypes, and thus boosting confidence (Ramirez et al. 2012; Estes and Felker 2011; Campbell and Collaer 2009). Second, these tasks may improve the cognitive skills necessary for spatial thinking—for example, the working memory needed to master a video game might lead to improvements in

spatial thinking (Dye, Green, and Bavelier 2009). Spatial thinking is effective probably because of both cognitive skills and social factors. Teachers and educational technology have an important role in bridging these more intuitive, rudimentary skills with mathematical concepts through spatial representations, such as number lines, measurements, blocks, etc.

Spatial Thinking in Education and Careers

A series of studies provide evidence that not only might these early spatial skills impact mathematics learning in a short-term way, they may also have long-term effects on students' persistence in school and careers that are related to science and mathematics. Super and Bachrach (1957) examined the personal characteristics of scientists and engineers and found a strong relationship between people's spatial ability and their potential to move into careers in science, technology, engineering, and mathematics (STEM). In the decades since, many other studies have examined whether spatial abilities could predict future careers (Benbow and Stanley 1982; Shea, Lubinski, and Benbow 2001). The findings from a congregate of 50 years of research on data from more than 400,000 participants consistently show that spatial ability around middle- or high-school levels predicted career placement in STEM fields (Wai, Lubinski, and Benbow 2009). Importantly, spatial ability is predictive beyond students' general mathematics and verbal abilities.

Spatial skills may matter most when students are at a point of entry in STEM disciplines and are grappling with elementary content (Uttal and Cohen 2012). Spatial skills are necessary for rotating molecular structures and for understanding maps and graphs (Hegarty 2010), and those who feel uncomfortable with these initial practices may not persist in STEM career trajectories. Thus, from an early stage, students with weaker spatial skills may be deterred from STEM topics (Wai, Lubinski, and Benbow 2009; Uttal et al. 2013b).

Overall, children's development of mathematics through primary school requires them to apply specific mathematical reasoning skills that revolve around understanding units, composing/decomposing quantities and shapes, relationships and order, looking for patterns and structures, and organizing information throughout multiple domains and at every grade level. The content of mathematics learning standards should change appropriately with each grade level based on what children are able to learn at each respective age.[2] At each grade level and with all content, children

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[2] See, for example, the CCSS public website at http://www.corestandards.org/Math/.

TABLE 1.4
POINTS TO CONSIDER WHEN EVALUATING EDUCATIONAL TECHNOLOGY OR INSTRUCTION RELATED TO MATHEMATICAL LEARNING

Curricular Factor	Implication for Learning	What to Look For
Instruction should be clearly related to a goal for broad mathematics proficiency, rather than to adequate performance on individual practices or content areas.	Instruction will be more effective in creating proficient learners if instructors have a set of standards that they can aim toward, and particularly if these standards address both curriculum goals and goals for mathematical practice.	Materials that outline goals for curriculum attainment and student thinking, and evidence of these goals in the instruction.
Children are born with interest in quantity and small numbers.	Teaching number symbols and calculations should build on these skills and interests.	Activities that move back and forth between interacting with real quantities and symbols that represent them.
Children must understand the base 10 structure of numbers to fully comprehend place value.	Many primary school students learn rules for multidigit arithmetic without understanding place value.	Number pattern activities, i.e., counting by 2s, 3s, 4s, etc.; breaking numbers down into their ones, tens, hundreds; using manipulatives or technology to handle ones, tens, hundreds, etc.
Spatial skills are trainable and are foundational to measurement, geometry, and many future careers that build on math or science.	Giving students experience in building spatial skills while learning curriculum content related to numbers, measurement, or geometry, can build understanding of those topics as well as spatial skills themselves.	Experience using maps, mentally rotating objects, solving puzzles, or using spatial words to describe information.

Source: Prepared by the authors.

need to display mathematical reasoning skills outlined as mathematical proficiency in the practice standards. Table 1.4 describes the learning and classroom implications of different mathematical curriculum goals.

1.5 Challenges to the Learning Process

While instructional designers benefit from clear standards and aims, they also benefit from awareness of key challenges that their students will face when acquiring primary school mathematics proficiency. While a catalogue of all the challenges is beyond the scope of this chapter, it is important to note first that efforts to improve educational outcomes will be streamlined

by attention to, and awareness of, students' common misconceptions and areas of potential difficulty. This section first highlights some of the key misconceptions students develop during the primary school period and encourages educators to take explicit care that their students avoid or correct these misconceptions. It is a common belief that students who are not allowed to make errors will be more successful in the long term, though errors can actually enhance future learning through increased motivation and curiosity, as well as enhanced memory (Richland, Kornell, and Kao 2009; Butterfield and Metcalfe 2006). The section then highlights factors that exacerbate achievement gaps within regions: feelings of pressure and stereotypes that may affect girls and minorities in particular, and which systematically lower math performance and the willingness to continue working in this content area.

1.5.1 Common Misconceptions

As a complement to determining optimal curriculum and practice standards, research has also demonstrated the utility of identifying common areas where children are likely to have difficulties and develop persistent misconceptions. Educators who are aware of potential misconceptions in a given content area can assess their students to determine whether they exhibit the misconception, and can then guide them towards correcting it. This is a more effective strategy than teaching the correct information and not engaging with the misconception. In that case, the learner will often exhibit the correct procedures, but after a delay, or faced with a similar but distinct type of problem, the misconception will resurface. If it was never addressed in the first place, the students' mental representations will not have changed. While this chapter cannot cover all areas of misconception in primary school mathematics, it highlights two areas—fractions and fraction arithmetic, and equivalence and equations—to demonstrate the importance of documenting and reorienting misconceptions in support of student learning. It is important to be vigilant in identifying instruction (or educational technology) that creates or reinforces misconceptions versus instructional conditions that support students in refining their understanding to be more accurate.

Fractions and Fraction Arithmetic
As noted above, children often develop misconceptions about fractions and fraction arithmetic, failing to understand how quantity concepts of whole numbers cannot directly translate to quantity in fractions. For example, children will often add two fractions by adding both the numerator and

the denominator (e.g., 2/7 + 3/4 = 5/11) and/or believe that multiplying two fractions leads to a larger quantity and dividing leads to a smaller quantity. These misconceptions may stem from misunderstanding the operations of multiplication and division.

Division is largely taught as dividing a quantity, or sharing, and it is difficult to conceptualize that one can divide or share a quantity and the outcome will be a larger number (e.g., $2 \div \frac{1}{4} = 8$). In these cases, it may be more useful to conceptualize division as a measurement or as an investigation of how many x's fit into a y—namely, how many times a divisor (e.g., $\frac{1}{4}$) goes into the dividend (e.g., 2), where the result is a quotient (e.g., 8), from the Latin word *quot* which refers to "how many."

Similarly, multiplication is thought of as repeated addition, but this becomes difficult to mentally simulate, even for adults, when involving fractions (e.g., $2 \times \frac{1}{2} = 1$). On the other hand, misconceptions about fractions may also stem from a lack of understanding about the magnitude represented by symbols used to represent fractions, which is apparent when children are asked to compare fractions (and decimals), and think the larger number also represents the larger quantity (e.g., 1/12 is larger than 1/2 or 0.452 is larger than 0.51). These errors are a clear indicator that children think about mathematics as following procedures without understanding the meaning behind the symbols or operations. It is important to note that these errors are also common in students attending two-year colleges. Successful interventions have revolved around teaching students to draw relationships between fractions and broad differences in percentages (e.g., 50 percent is "half," 100 percent is "everything," 99 percent is "almost everything," 1 percent is "almost nothing;" see Moss and Case 1999) and emphasizing fraction magnitudes (Fuchs et al. 2013, 2014).

Equivalence and Equations

A second area where children in primary school regularly develop misconceptions pertains to understanding equivalence and equations. Children are regularly shown only equations with the calculations on the left of the equal sign, and a blank on the right in which the answer to the left calculation should be entered. This also coincides with everyday nonmathematical notions of the equal sign as an intermediary between cause and effect, or operation and result, which are not in fact equivalence relationships (e.g., "buy one = get one free," but not "get one free = buy one;" see Hofstadter and Sander 2013), which differs from equivalence. This leads to the belief that an equal sign means "put the answer here to the right" (e.g., for problem 2 + 5 = __). While this often leads students

to solve a problem correctly, if it is in the standard format, these children do not develop a more full and flexible understanding of equivalence. For example, they may enter a "7" for the blank in the following equation: $2 + 5 = \underline{\hspace{1em}} + 3$, not knowing what to do about the extra 3 in place. Also, they may have trouble with the same problem rewritten as $\underline{\hspace{1em}} = 2 + 5$. This misconception contributes to learners' particular challenges when doing algebra, where they must take the equivalence relationship as a starting point for many calculations. This challenge may be mitigated by providing students with experience solving equations in nonstandard forms.

1.5.2 Anxiety and Stereotypes

Another significant challenge for certain children is their fear and anxiety about math and their feelings of performance pressure. Anxiety about mathematics can be culturally based and has been found to be more common among women than men (Hembree 1990). Mathematics anxiety leads learners to perform below their actual math abilities. For example, children may be anxious when they perform math at a chalkboard and the rest of the class is watching, during a math exam, or even when figuring out whether they have enough money to buy a candy bar (Ashcraft and Kirk 2001). Teachers who themselves suffer from mathematics anxiety might pass it on to their students, particularly the girls in their classrooms (Beilock et al. 2010).

The fear that one will confirm others' stereotypes can also cause people to overload their working memory system with worried thoughts. For example, a female student might experience an inner discourse that goes something like: "Everyone will expect me to do poorly on this test since I'm a girl, but I really don't want to do poorly, I have to do well, oh no, I think I'm getting this question wrong and that's going to confirm their stereotypes..." (Steele and Aronson 1995). Engaging in this type of worry takes cognitive resources away from the actual mathematics problem, meaning that students are likely to impair their performance despite the goal of performing well. A body of literature has shown that simply reminding individuals about their family income, race, gender, or other stereotyped categorizations before a test can lead to differential performance based on whether their group is stereotyped as low-performing. Interestingly these effects can shift, such that Asian-American girls in the United States at an elite college (where the stereotype is that Asian-Americans are good in math) did better than their average when reminded of their race, but lower than their average when reminded of their gender (Shih, Pittinsky, and Ambady 1999). Together with gender differences in socialization of

spatial skills, gender differences in mathematics at the university level may begin to be understood as having multiple sources.

Educators and technology designers thus need to be mindful to use gender-sensitive material that excludes biases and engages both genders and all races equally. Similarly, in the design of both paper-based and online assessments, it is important to include any questions about students' personal characteristics at the end of the test, rather than the beginning, to ensure that students will not score differentially based on beliefs related to their personal identity. Unfortunately, when students begin to feel anxious or threatened because of their personal characteristics, they perform worse and this perpetuates stereotype-based beliefs. It is important to disrupt this insidious cycle. Table 1.5 outlines common challenges in mathematics instruction that need to be addressed to ensure effective use of education technologies for all students.

1.6 Tools for Supporting Students' Mathematical Development

Many mathematics interventions have successfully used tools such as manipulatives or visual representations to help with children's acquisition of mathematical concepts, often lowering students' anxiety by moving away from pure symbol manipulation and making classroom instruction more approachable to all.

1.6.1 Manipulatives

Manipulatives are typically concrete objects (such as blocks), but they can take any shape or form (such as interactive technology), with the intention of providing children a tactile quantitative experience to help them learn abstract symbolic ideas. The abacus is one example of a concrete, tactile manipulative that has recently received much attention (Figure 1.1), but one could even include educational technology in this category as providing virtual manipulatives.

Manipulatives provide a way for children to go beyond the abstract thinking about mathematics described in the introductory example. A recent meta-analysis examined the effects of providing manipulatives for learning across 55 studies and found evidence of its benefits (Carbonneau, Marley, and Selig 2013). However, details about how the manipulatives are used in the instructional context are essential. If manipulatives are used without strong instruction or clear tasks, children may be engaged during activities with manipulatives but may not connect this learning to their mathematical understanding.

TABLE 1.5

POTENTIAL CHALLENGES TO CONSIDER WHEN EVALUATING EDUCATIONAL TECHNOLOGY OR INSTRUCTION

Cognitive or Curricular Factor	Implication for Learning	What to Look For
Children's misconceptions should be clarified and addressed directly, rather than ignored, and a new strategy should be employed. At the same time, focusing on a misconception can lead to some students reinforcing that misconception.	Misconceptions will resurface if not addressed directly. But discussing them also runs the risk of directing the attention of children with low ability back to these misconceptions.	Undertake activities that elicit misconceptions by having children draw out their thought processes or express them in another nonstandard way, compare a misconception to a correct solution, or set up children to make errors so that they can be corrected and discussed. Using counterexamples to show why a misconception that does not always work can be helpful, and explicit demonstrations of why and how something is a misconception is essential.
Students and teachers may feel anxiety about math performance, either because doing math makes them anxious or because they are worried that others see them as poor in math.	Instructors must be vigilant not to activate feelings of anxiety or stereotype-based beliefs in their students.	Make sure instruction does not begin with reminders of children's race/ethnicity/cultural or language background, socioeconomic status, or gender. Materials should not contain clear biases toward one of these categories or against others. Instead, make sure to transmit a message that effort, rather than ability, is responsible for success in mathematics. Children suffering anxiety use executive function and working memory resources toward that end, so they are especially susceptible to the problems described above when these brain resources are overwhelmed.

Source: Prepared by the authors.

FIGURE 1.1

EXAMPLES OF ABSTRACT (LEFT) AND PERCEPTUALLY RICH (RIGHT) MANIPULATIVES
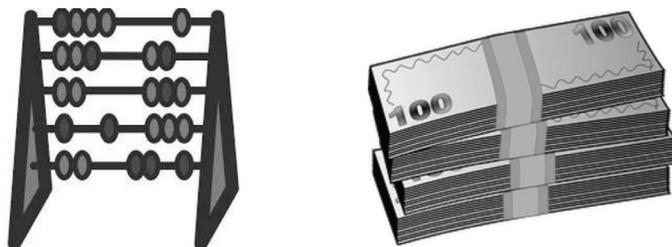
Several elements of the instruction turn out to be important. First, for manipulatives to be effective, they must be well understood by children (who sometimes need training in this). Teachers must highlight the key relationships being taught. The perceptual richness of manipulatives and adequate instructional time are also important to ensure their usefulness (Carbonneau, Marley, and Selig 2013).

In terms of development, younger children may encounter two hurdles to successfully working with manipulatives. First, they may think of a block as simply a toy, not a representation of a mathematics concept of quantity. Considering manipulatives as mathematical symbols may be conceptually challenging. This does not detract from their educational value, but teachers must be aware of the students' learning process and explicitly support them in learning how to use manipulatives to conceptualize mathematics.

Second, children may follow or be able to repeat demonstrated mathematical procedures with blocks but struggle to see the relationship between blocks and written numbers. Thus, young children may learn to correspond between quantity and manipulatives, or may provide the correct answer to a problem they have been shown how to solve with manipulatives. But they may not use what they have learned when solving calculation problems later—in other words, the manipulative activity was an interesting diversion for them, but not central to their conceptualization of mathematics.

The success of making the shift from concrete to abstract representations, at any age level, depends on the amount of teacher support. Teachers must be explicit and remind students to think about the manipulatives even when solving problems without them. Manipulatives represent an additional system of symbols, but in physical form, and it takes additional mental resources to process and determine their utility (Uttal et al. 2013a). Some research evidence suggests that higher levels of instructional guidance promote increased retention and problem-solving. On the other hand, strategically presenting manipulatives in a way that complements instruction over longer time periods seems to help students gain a deeper conceptual understanding when they are left to their own devices or with little instructional guidance (Carbonneau, Marley, and Selig 2013). This does not imply that providing manipulatives to children will, on its own, lead to better learning. On the contrary, even when students are meant to discover how to use manipulatives with limited or no guidance from teachers, it is imperative for educators to be thoughtful and to strategically choose when and how to integrate manipulatives into everyday instruction (Ball 1992). Manipulatives can take many shapes

or forms, and common intuition may favor manipulatives with the rich perceptual features of everyday objects (e.g., a pizza instead of a round one-color object, or money instead of blocks) as being more beneficial for children.

The research evidence suggests there are trade-offs. When manipulatives appear similar to the phenomenon that they are supposed to represent, they are easier for students to use successfully right away. If they are too abstract, manipulatives may require additional time for students to learn how to use them, though, in fact, abstract stimuli can be most helpful in the long run. Features that are unrelated to mathematical concepts can distract students from the main learning, whereas simple objects help children focus on the mathematical structure (Carbonneau, Marley, and Selig 2013). This suggests that abstract shapes such as plain squares, circles, and lines are more efficient than using toy animals, flowers, or food as manipulatives. Abstract shapes can help children generalize mathematical concepts to multiple contexts, whereas with toys, children seem to restrict their learning to the context of playing with the toy.

Many in the field of developmental psychology or education will consider research by Piaget (1977) when making decisions about the use of manipulatives. Piaget argued that there is a developmental trajectory such that younger children would benefit from early use of concrete tools for thinking about mathematics, moving to abstractions only once they have reached adolescence. In contrast, more recent research shows that all children can benefit from manipulatives, but that manipulatives are not helpful simply because they are concrete objects children can handle. Rather, the key to making physical manipulatives (or those in a technology resource) useful is to be sure that children can handle them appropriately and understand how they are related to the mathematical understanding they are supposed to support. Then they can transition back to using symbolic representations. If students enjoy and engage in a mathematical topic with a manipulative, but then do not recognize how to solve a mathematics problem on another day without the manipulative, this suggests the learning was not generalized.

One model for using manipulatives and slowly moving back to symbolic representations has been called "concreteness fading" (Bruner 1964; Goldstone and Son 2005). The idea is that students first use physical manipulatives—for example, small pebbles to count the number of cookies in a division problem such as the following: If a boy has 12 cookies and shares them fairly with his best friend, how many cookies would his friend get? Now what about sharing them between three friends? The teacher could first have students use pebbles to solve problems like this, dividing them

into piles on a piece of paper. Then, they could write the mathematical symbols on that paper, in order to show the same process but in a symbolic format, mirroring the half relationship of 1 whole (all 12 cookies) split into two equal groups, recording that this is the total group of cookies (1) divided into two parts (/2), so each pile gets the label 1/2. The same can be done with 1/3. Eventually, the teacher could ask students to solve problems like these using only the symbols, but, at the same time, could reference the pebbles to help the students connect their now-abstract learning to the manipulatives: "Remember how you can always just imagine pebbles to help you think about these problems." Thus, the teacher is moving from the more concrete use of a manipulative to a more abstract, symbolic form.[3]

1.6.2 Visual Representations

Beyond the use of concrete manipulatives, incorporating visual images and written versions of mathematics offers a powerful way to help students draw connections and reason mathematically (Begolli and Richland 2016). One way that visual depictions, or representations, of mathematics can be useful is to show one concept in multiple ways. For example, a teacher might demonstrate the concept of powers by first showing powers of two as a set of manipulatives, showing the size of two blocks, then four, then eight, and so on. Next the teacher could show powers of two as a two-dimensional graph, which often provides the same information but in a way that highlights different aspects of the concept—perhaps using concreteness fading as well. Showing these multiple representations of the same ideas is known to build broader and more generalizable understanding (Ainsworth 1999).

At the same time, a key to successfully using multiple representations is that instructional designers or teachers cannot simply provide the multiple representations, but must make the connections between them clear and evident to all students (Ainsworth 1999). There are many pedagogical decisions that teachers make about using multiple representations. One could present the manipulatives for powers during one class period, and, in another period, show the same patterns but using graphs. Or, a teacher could show the two ways of explaining powers one at a time, and then move to the formulas. The teacher will understand that these are all showing the same information, and some students may too. Many other

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[3] For a fuller review and discussion of how concreteness fading could be accomplished using technology, see Fyfe et al. (2014).

students, however, will need the teacher to be very explicit about how these representations are showing the same information. Teachers can use strategies such as showing the representations together on the board or in front of the whole class, or having students see or use them both together in a technology platform. Also, teachers can use hand gestures and explicit statements that show how these representations are similar and related (Alibali et al. 2014). The key is to ensure that students notice similarities or differences between these representations, and make use of those comparisons to build broader understanding (Gentner 2010; Richland and Simms 2015).

Another role for visual representations is to provide a visible record of the instruction that students have just accomplished, whether it be in a classroom or in a technology-enhanced setting. Mathematics teachers around the world often lead their students through a series of problems that build in a certain way, perhaps demonstrating that a common procedure can be used for multiple problems, or that certain problems may appear similar but are in fact different (Hiebert et al. 2003). These sequences are meaningful and important, but students often do not notice the progression if teachers do not make it explicit (Gick and Holyoak 1980, 1983).

A technique used in high-achieving Asian countries is to leave problems or key solution strategies on the board in order to create a visual record of the lesson (Hiebert et al. 2003). That way, if students miss a step or need support for their executive function processes in recognizing the key elements of a lesson, they can look back at the visual information provided on the board. At the same time, the visual record of key mathematical information should not be overwhelmingly distracting or difficult to parse, or it will have the potential to overload students' inhibitory control resources (Fisher, Godwin, and Seltman 2014). Table 1.6 highlights tools that have been found to be particularly effective for learning mathematics.

1.7 Conclusion

This chapter has aimed to provide educators and administrators in Latin America and the Caribbean with an overview of key aspects of children's mathematical development that will be useful for improving educational outcomes. The hope is that readers have gained an appreciation for the importance of attending to student thinking in planning instructional experiences, including developmental lines in maturation and mathematical skills, and avoiding misconceptions and overloading cognitive processing skills. Further, the aim has been to convey the power of using a strong standards-based curriculum that can be generalized across student

TABLE 1.6
TOOLS FOR SUPPORTING THE LEARNING OF MATHEMATICS

Cognitive or Curricular Factor	Implication for Learning	What to Look For
Manipulatives can be highly effective.	Allowing children to handle manipulatives or technology that show concepts in different ways builds more flexible, generalizable understanding.	Introduce concepts in concrete ways, and gradually fade to more symbolic ways of showing the ideas (like an equation). Children must fully understand the manipulatives involved and have enough instructional time to learn about them before using them to learn math ideas.
Children learn from visual representations, interacting with multiple representations of the same information.	Provides a broader understanding of the concepts, and more avenues for remembering the content. However, it can overload children's attention, so care must be taken to organize representations so that students know what they are looking at.	Undertake activities that enable children to grapple with the same ideas in different representations (e.g., in a story and an equation, in a physical set of blocks or a quantity of dried beans to connect to an equation, or in a game and symbolic notation) or different versions of concepts (e.g., multiple polygons of unusual configurations to teach about angles, sides, and area). Care must be taken to ensure that children see the relationship between the more and less concrete representations.
Compare and contrast.	Learners understand the underlying mathematical concepts, developing a deeper understanding that can be generalized across types of problems.	Employ question-and-answer activities in which instructors have a clear goal for comparing problems, solution strategies, representations, or mathematical objects to real-world phenomena.

Source: Prepared by the authors.

populations within a country, and using a refined definition of mathematical proficiency.

This theoretical overview has been provided to drive home a central point. Educators and administrators involved in improving educational outcomes need to understand how children think, what they know and need to know, how age impacts learning, and misconceptions commonly observed throughout development. This knowledge allows for the design of more effective curricula and technologies, and provides a lens through which to evaluate educational technologies. In this vein, it is also important to understand what specific aspect of children's mathematical knowledge

an intervention addresses. Clear standards help separate what has been addressed from what still needs to be addressed. The hope is that current and future educators can use this knowledge to make informed decisions regarding the appropriateness of interventions.

Table 1.7 presents the chapter's key conclusions and policy take-aways for policymakers.

TABLE 1.7
KEY CONCLUSIONS AND RECOMMENDATIONS

Conclusion		Policy Implication or Recommendation
Access to education technologies is not enough; teachers need knowledge of how technologies can be used to evaluate and support children's mathematical development.	→	Teacher training in the use of math education technologies must focus on their application in the classroom to reflect the knowledge that children already have so as to appropriately build their skills, respond to their anxieties, and foster their thinking.
The learning goals that are typical in Latin America and the Caribbean are often content- rather than skills-based, and lack teaching goals.	→	To evaluate the use of technologies in mathematics instruction, education systems must define learning goals that go beyond content to measure skills (e.g., specific number or spatial skills) as well as theory-based practice standards with goals for student behaviors and approaches to mathematics.
Mathematics is often taught and learned as a set of rules to be memorized, sometimes leading to technologies that teach children the correct answers in the short term (devoid of understanding), resulting in student confusion and alienation in the long term.	→	Technology that is promoted must build on children's abilities by making explicit connections between mathematical concepts and children's intuitions (e.g., spatial representations), avoiding unnecessary distractions, and facilitating concept comparisons.
Girls and students from minority backgrounds often have stereotypes about being poor at mathematics. This may lead them to perform below their ability. A related problem is that boys may monopolize the use of new technologies.	→	School systems must train teachers in the use of measures to ensure equal and fair participation by all students. In a class that uses education technologies, tasks must be structured to ensure equal access to tools such as computers, software, and robots.

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A Learning Path Framework for Balancing Mathematics Education: Teaching and Learning for Understanding and Fluency

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In an urban school in South America, a young elementary school teacher, Catalina, is teaching her second grade students how to add two-digit numbers. Catalina was never a good math student herself, and she feels awkward and uncomfortable teaching the subject. As she tries to go over the procedure, her students ask:

¿Por qué funciona de esta manera? (Why does it work this way?)

Catalina feels lost, not knowing how to explain. However, she notices some students trying to help each other, using blocks and drawings, to show the meaning of the regrouping conceptually. As Catalina stands back and listens for a while, she feels a new sense of excitement that she never felt before as a teacher. She feels proud of her students for trying, and she wants to help them. She joins the student discussion of different methods and, at certain moments, can actually provide explanations that clarify students' confusion. Catalina feels empowered for the first time as a mathematics teacher. She remembers that in a recent professional development session, her colleagues had discussed a different way of teaching called "balanced teaching," which starts with students sharing ideas. The teacher then facilitates learning by helping students make connections between their ideas and math concepts using math drawings. Catalina decides that she will talk to her colleagues during the planning period that week to learn more about it.

2.1 A New Way Forward

Countries around the world are trying to help as many classrooms as possible prepare students for the math needs of the 21st century. The old focus on memorizing and copying what the teacher shows is no longer good enough. Students must make sense of and understand what they are doing because the math needs of the workplace are changing. Students need to be prepared for future changes, too. In recent years, three U.S. National Research Council committees have studied research from around the world and summarized the findings: NRC (2009), Donovan and Bransford (2005), and Kilpatrick, Swafford, and Findell (2001).[1] This chapter provides a framework that summarizes these international research results, focusing especially on how students think about math ideas. The summary also draws on decades of experience with a similar framework in Japan and on two decades of classroom-based research in English- and Spanish-speaking classrooms in the United States (Fuson and Murata 2007; Fuson, Murata, and Abrahamson 2014; Murata and Fuson 2006, 2016; Murata 2008). The research for the framework comes from all over the world, including Latin America and the Caribbean (LAC). Much of the research is about student learning and teaching of specific math topics, and it emphasizes sense-making and explaining as crucial for 21st century goals that balance understanding and fluency. This "balanced teaching" approach is related to several models of teaching that emphasize learning trajectories (Clements and Sarama 2004, 2014), task sequences (Simon, Placa, and Avitzur 2016), "math talk" in the classroom (Yackel and Cobb 1996; Hufferd-Ackles, Fuson, and Sherin 2004, 2015; Murata et al. 2017), linking ideas (Alibali et al. 2013), embodied thinking (Abrahamson 2014), visual learning (Mason 1989), productive failure (Kapur 2014), and discovery-based learning (Abrahamson and Kapur 2018). This framework can guide national decisions about teaching and learning, including choices regarding what kinds of technology to use, and how.

This chapter first outlines the balanced teaching framework, then explains its central aspects in more depth and gives examples using fractions. The chapter next describes and illustrates the importance of drawing, using the examples of problem-solving with key math concepts, before briefly discussing issues of supporting students and teachers with

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[1]   This chapter is based on the research summarized in the National Research Council reports, but the examples given are relevant to Latin American and Caribbean classrooms and uses of technology.

balanced teaching/learning. The chapter concludes by discussing some uses of technology drawn from the framework.

2.2 The Balanced Teaching Framework: Proficiency for All

Box 2.1 outlines the balanced teaching framework. At the top of the table is the high-level 21st century goal for all students presented and discussed in the National Research Council report, *Adding It Up: Helping Children Learn Mathematics*. (Kilpatrick, Swafford, and Findell 2001). This goal can focus changes in teaching because it broadens typical goals for teaching. Its aim is to nurture resourceful and self-regulating problem-solvers using five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These five strands guided subsequent reform efforts in the United States and Canada, for example in the writing of the 2010 U.S. Common Core State Standards (CCSS Initiative 2010). The strands of conceptual understanding and procedural fluency are balanced in the framework, as the teacher helps students develop and then move from conceptual understanding to fluency. Strategic competence and adaptive reasoning are two vital aspects of problem-solving and reasoning. The fifth strand, productive disposition, involves a positive self-image as a problem-solver, characterized by Dweck as a "growth mindset" (Dweck 2010; Blackwell, Trzesniewski, and Dweck 2007). All of these strands require students to develop their self-regulating capacity as they become more aware of and can take more control of their math thinking and problem-solving.

This high-level goal involves Principle 3 (The importance of self-monitoring) from the National Research Council Report *How Students Learn* (Donovan and Bransford 2005). The report organizes its research summaries around three principles that are used in the three-phase, balanced teaching model summarized in Box 2.1. These principles are stated where they appear in Box 2.1 so that readers who wish to find research about them in the report can do so. Box 2.1 also draws on research-based recommendations about teaching and learning made by the main organization of researchers and teachers in the United States, the National Council of Teachers of Mathematics (NCTM), in its report *Principles and Standards for School Mathematics* (NCTM 2000). This report identifies five process standards for teaching: problem-solving, reasoning and proof, communication, connections, and representations. These process standards are identified in Box 2.1 where they are relevant. Later research related to these process standards is discussed in NCTM (2014).

BOX 2.1 THE THREE-PHASE BALANCED TEACHING MODEL

The high-level goal for the balanced teaching model is to build resourceful, self-regulating problem-solvers (Principle 3 from *How Students Learn*: The importance of self-monitoring) by continually intertwining the five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, and Findell 2001*).*

How? Create a Year-long, Nurturing, Math-Talk Community
- The teacher orchestrates collaborative instructional conversations focused on the mathematical thinking of classroom members (Principle 1 from *How Students Learn:* Engaging prior understandings; and the *Process Standards on Problem Solving, Reasoning and Proof, and Communication* of the National Council of Teachers of Mathematics (NCTM).
- Students and teachers use responsive means of assistance that facilitate meaningful learning and teaching by all; the teachers seek to engage, involve, manage, and coach (model, clarify, instruct/explain, question, and give feedback).

Use Three Balanced Teaching Phases for Each Math Topic
The teacher and students use and relate ("interform") coherent mathematical situations, pedagogical forms, and cultural mathematical forms (the NCTM's *Process Standards on Connections, Representations, and Communication*) as they move through these phases.

Phase 1—Guided Introducing
- Supported by the coherent pedagogical forms, the teacher elicits and the class briefly works with the understanding that students bring to a topic (Principle 1 from *How Students Learn*: Engaging prior understanding).
- Teacher and students value and discuss student ideas and methods (which allows teachers to know how students approach the topic).
- Teacher identifies different levels of solution methods used by students and typical errors and ensures that these are seen and discussed by the class.
- *Student methods may be basic and slow, contain errors, or be Phase 2 methods (see below).*

Phase 2—Learning Unfolding (In-depth Meaning-making Phase)
- The teacher helps students form emergent networks of forms-in-action (Principle 2 from *How Students Learn:* The essential role of factual knowledge and conceptual frameworks in understanding).
- Explanations of methods and of mathematical issues continue to use math drawings and other pedagogical supports (external forms) to stimulate correct relating (interforming) of the forms.

(continued on next page)

- Teacher focuses on or introduces mathematically desirable and accessible methods.
- Erroneous methods are analyzed and corrected with explanations.
- Advantages and disadvantages of various methods, including the current common method, are discussed so that central mathematical aspects of the topic become explicit.
- Student methods become predominantly mathematically desirable and accessible methods. Errors decrease. Students may also use mathematically desirable and not accessible methods.

Phase 3—Kneading Knowledge (Fluency)
- The teacher helps students gain fluency with desired methods.
- Students may choose a method. Fluency includes being able to explain the method.
- Some reflection and explaining will still continue (kneading the individual internal forms).
- Students stop making math drawings when they do not need them.
- *Each student fast-forms one mathematically desirable method; many students interform more than one method.*

Source: Prepared by the authors.

The second part of the balanced teaching framework (how to meet the goal) describes how a teacher can increase the understanding levels of all students in the classroom by creating a year-long, nurturing "math talk" community focused on how students make and discuss mathematical meanings.[2] To do this, the teacher orchestrates collaborative instructional conversations (math talk) focused on the math thinking of students. Visual models (e.g., math drawings) are introduced to support the thinking of students and the teacher. The teacher can explain concepts, but the students are also encouraged to explain their thinking and discuss it with other students. Explanations that use visual models (such as drawings) are vital to the sense-making and understanding of all participants. The math talk also helps students use math language and notation.

The third part of the balanced teaching framework is a three-phase model of how a teacher supports student understanding and fluency in

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[2] As explained by the National Council of Teachers of Mathematics, "math talk" is an instructional conversation directed by the teacher, but with as much student engagement as possible. The idea behind it is that if students take time to explain their mathematical thinking, this will increase their understanding.

each new math topic. The bottom part of Box 2.1 describes central aspects of each phase. Phase 1, "Guided Introducing," is emphasized in reform approaches that focus on eliciting and discussing children's invented methods. Traditional curricula emphasize Phase 3, "Kneading Knowledge (fluency)," and focus on fluency development. The new, important part of the model is Phase 2, "Learning Unfolding." This in-depth meaning-making phase connects Phases 1 and 3 and provides opportunities for deep and ambitious learning. As students compare, contrast, and analyze different methods in Phase 2, core math concepts can be lifted up from the problem contexts or specific methods and connected together. Coherent facilitation of mathematically desirable and accessible (MD&A) methods via math drawings in classrooms helps students express their ideas and ultimately fosters individual learning. The general word "form" is used in several places in the box (pedagogical forms, cultural mathematical forms, interform, internal forms-in-action using external forms, individual internal forms) instead of the range of other terms (teaching materials or drawings, mathematical symbols or words, relate, mental representations using external materials or actions or words or written symbols, mental representations or actions) to emphasize the relatedness of all of these different internal and external structures and how mathematical thinking involves developing and using increasingly complex and accurate forms.

Phase 2 is the heart of this process, as the class focuses on and discusses MD&A methods with the help of visual models.[3] MD&As are general and can be abstract and conceptually nontransparent. If a teacher only presents and explains information without any visual models, it can be very confusing for students. In second grade classrooms, students may approach a two-digit addition problem (e.g., 68 + 76) by using drawings or "tens and ones" blocks. Research in many countries underlines the importance of math drawings (visual models, diagrams) as pedagogical forms to support individual thinking, problem-solving, and instructional conversations (math talk). Math drawings facilitate problem-solving because students can relate steps in the math drawing to steps with math symbols and can label the drawing to relate to the problem situation or to math concepts (e.g., hundreds and tens). These drawings can help bridge problem situations with mathematical solutions through mathematizing

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[3] Visual models are the range of displays—including images with figurative, diagrammatic, and symbolical elements—that teachers, students, math programs, or math technologies create to show the meaning of a mathematical concept. Visual models can be physical things, but this chapter emphasizes the importance of diagrams and math drawings as visual supports for teaching and learning. All of these terms for visual models are used in this chapter because they all have slightly different meanings in context.

(focusing on the math structure). Math drawings assist math talk because they can be put on the board or be projected on a screen for all to see and they leave a trace of all steps in the thought process, so each step can be explained later. Math drawings are inexpensive, easy to manage, and remain after the problem is solved to support reflection and further explanation. Teachers can collect pages containing them and reflect on these as windows into the minds of students outside of class time. Many East Asian elementary math programs have a history of using diagrams, as do some other countries around the world. Math drawings initially can show all of the objects and later be simplified into diagrams with numbers in them. The drawings of concrete objects may be helpful for very young children or for some special-needs children, but for many math topics, students need only use simplified diagrams and numbers.

It is essential for any math teaching to involve all three phases. Depending on the level of complexity of the mathematics concept, these three phases may occur over several lessons, or they may occur in one lesson. Throughout the phases, it is important for the teacher to maintain high expectations of students, accept the different ideas and varied learning paths students may take, and understand that every student will come to use a mathematically desirable method in time with varied degrees of fluency. These three phases and their relationships are summarized in Figure 2.1. The double arrows connecting and showing how students

FIGURE 2.1
MATH TALK COMMUNITY: EVERYONE FOCUSES ON MAKING SENSE OF MATH STRUCTURES USING DRAWINGS TO SUPPORT EXPLANATIONS
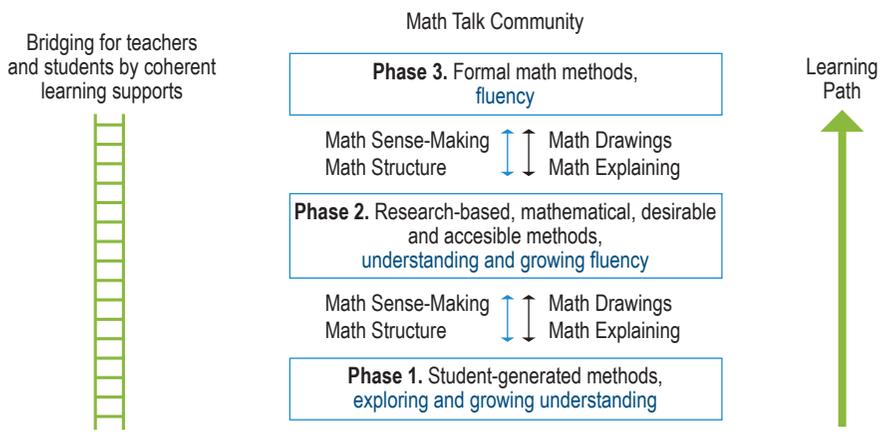
Source: Prepared by the authors.

proceed between phases summarize what teachers need to emphasize in the classroom—making sense of math structures using math drawings to support math explaining. Research concerning teaching aspects of the crucial Phases 1 and 2 are briefly summarized here.[4] Even when practicing solution methods in Phase 3, students can fall back to Phase 2 and to drawings to help them remember a method or fix an error.

   After all three phases are completed for a given topic, it is important to maintain fluency by engaging in practice distributed over time and also to relate the concepts to new concepts that will be learned later. The teacher may help students remember concepts they have already learned by giving them related problems on occasion. For new, related topics, the teacher initiates and orchestrates discussions to assist re-forming students' individual internal forms to support and stimulate the building of an extended individual internal network of related topics. With this, each student remembers and maintains Phase 3 performance. Practicing and discussing over an extended period helps students revisit the previously learned concepts and reinforce their understanding by creating new connections with other math topics they are learning. This supports the development of a broader and deeper conceptual network for their understanding of mathematics, and it adds new meanings to the previously learned concepts. This final phase is referred to here as the Review Phase, and it can extend for the rest of the year following the initial teaching of Phases 1, 2, and 3.

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[4] One "gold standard" study, using random assignment to groups, found that students who passed through Phase 2 outperformed those who had a long Phase 1 or who moved rapidly to Phase 3 (Agodini et al. 2010). A related study found that the effects of two curricula that balance the phases are largely robust across variations in student achievement and teachers' mathematical knowledge (Agodini and Harris 2016). A meta-analysis that included studies using different instructional process strategies suggests vital parts of the math talk community shown in Figure 2.1. This meta-analysis found that changing the way that teachers and students interact in the classroom increases achievement, especially if children are given opportunities and incentives to help one another learn and are kept productively engaged and interested in mathematics (Slavin and Lake 2008). Another meta-analysis supports the importance of the move from Phase 1, focused on eliciting student methods, to Phase 2, where mathematically important methods are discussed and explained. This study found that unassisted discovery (Phase 1 in Box 2.1) does not benefit learners as much as instruction that includes feedback, worked examples, scaffolding, and elicited explanations, which are all important activities in Phase 2 (Alfieri et al. 2011). A free program available online carefully uses the approaches in both of these meta-analyses and has had a high degree of success across all levels of learners (Mighton 2013).

2.3 Explanation of the Phases with Visual Supports (Math Drawings)

This section uses examples to describe how different phases might look in instruction on the topic of adding fractions. The section will highlight connections between different student thought processes and visual representations to illustrate a learning process that moves through the three phases and uses math talk to support the connections.

Figure 2.2 outlines four possible student methods for solving the example problem: "We had 4/7 liters of milk in the bottle. We added 2/7 liters of milk. How many liters of milk do we have now?" As students attempt

FIGURE 2.2
STUDENT SOLUTION METHODS FOR A FRACTION PROBLEM: 4/7 + 2/7

Problem: "We had 4/7 liters of milk in the bottle. We added 2/7 liters of milk. How many liters of milk do we have now?"

Problem Representation	Student Explanation
Method 1. 4/7 = 6/14 2/7	[Student 1 does not add fractions by adding the unit fractions 4 of the sevenths and 2 of the sevenths and so gets the wrong answer.] Student 1: "First, I drew a bar to show one liter, divided it into 7 parts, and shaded 4 of them to show 4/7 liters. I drew another bar and showed 2/7 liters in the same way. I then counted how many parts there are for the whole, and how many parts are shaded, to find the answer. 6/14 liters."
Method 2. Step 1 Step 2 4/7 + 2/7 = 6/7	Student 2: "First, I drew a bar to show one liter, and I divided it into 7 equal parts to show the one sevenths. Then I shaded 4 of them to show 4/7 liters. Then I shaded 2 more sevenths after the 4 sevenths to add on 2 sevenths. I shaded them differently so I could see the 4 sevenths and the 2 sevenths. So I can see that the answer is 6 sevenths, and I wrote it below my drawing."
Method 3. 4/7 = 1/7 + 1/7 + 1/7 + 1/7 2/7 = 1/7 + 1/7 4/7 + 2/7 = (1/7 + 1/7 + 1/7 + 1/7) + (1/7 + 1/7) = 6/7	Student 3: "I thought that 4/7 is four of the 1/7ths and 2/7 is two of the 1/7ths. So, I added all the 1/7s, all 6 of them, and found the answer as 6/7 liters."
Method 4. 4/7 + 2/7 = (4 + 2)/7 = 6/7	[Student 4 thinks math is following a rule] Student 4: "I followed the rule. When adding fractions, if the bottom numbers are the same, you leave them alone, and only add the top ones. So, 4 + 2 = 6, and the answer is 6/7."
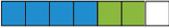
Source: Prepared by the authors.

to find the answer, they typically approach this problem in various ways, based on their prior math experiences and fraction concepts. All the solution methods shown in Figure 2.2 are possible in any upper-elementary mathematics classroom.

For Method 1 in the figure, the student first uses bars to show the fractions 4/7 and 2/7. In attempting to combine the two fractions, the student counts the parts shown in both of the representations altogether, and ends up with the answer of 14 parts, with 6 of them shaded to find 6/14 of a liter as the answer. The student does not see a fraction as the total of the unit fraction 4 of the sevenths and 2 of the sevenths to give 6 of the sevenths, but just counts all of what he or she sees, and gives the wrong answer, 6/14.

The student using Method 2 shows how each seventh is a part of the whole made by dividing the whole into 7 equal parts. The student draws the whole bar and divides it into 7 equal parts. Then the student shades 4 parts and then 2 more parts with a different kind of shading. He or she writes the fractions below the drawing to connect the math notation to the drawing.

Using Method 3, the student shows his or her thinking numerically. The student thinks of each fraction (2/7 and 4/7) as collections of a symbolic unit fraction (1/7), and thus adds 4 of the 1/7ths and 2 of the 1/7ths to find the total of 6/7 liter.

Regarding Method 4, although the student arrives at the correct answer, the student's explanation is solely procedural, and he or she only seems to think of problem-solving as following a rule. This explanation is likely if the student had received prior instruction in adding fractions based only on procedures.

When anticipating all these different student methods, it may feel overwhelming for teachers to facilitate meaningful math talk. If the teacher wants to value all student ideas but is not sure how to focus student attention on the core mathematical ideas, the teacher can elicit as many student solutions as possible but then not help students further to relate to these methods. This kind of teaching focuses on Phase 1 of the balanced teaching model. The students and the teacher may feel good about celebrating multiple solution methods, but students may come out of this experience without meaningful conceptual connections among the mathematics concepts and methods discussed. This type of teaching can invite a lot of criticism that learning outcomes are not clear, and it does not correct errors or allow all students to understand the problem.

On the other hand, in a traditional mathematics classroom, teachers do not invite students to share their thinking about the solution process to begin with. In these classrooms, it is more likely that the teacher presents the problem and shows the steps of the desired solution (most often as in

Method 4). The teacher then asks students to solve similar problems using the same method by following the steps carefully. There is no discussion of why and how these steps work in the solution process. In these traditional classrooms, students are likely to experience math as a set of rules and procedures to follow, and these students are likely to explain their thinking as described in Method 4 in the figure. This type of teaching can be considered as Phase 3 teaching. While Phase 3 is important as a part of balanced teaching, if done alone it does not help students understand how their ideas are related to mathematically valued rules and procedures or why these rules and procedures work. Understanding why each procedure works can help students remember them better or figure them out later, and it reduces interference among different procedures later on, when they may become all jumbled.

What is proposed in the research-based model of balanced teaching is to bridge Phases 1 and 3 with a crucial Phase 2. In balanced teaching (Figure 2.1), after students explore and generate their own methods and share them, the teacher facilitates student math talk by focusing their attention on aspects of the core math concept. In the example presented in Figure 2.2, after all four methods are shared, the teacher may ask students what similarities and differences they notice across these methods, often for two methods at a time. By questioning, listening to, and connecting different student ideas, the teacher will be able to help students see how similar Methods 1 and 2 are, but why their answers are different because they consider the "whole" differently. At this point, the teacher may want to bring a 1-liter bottle to demonstrate what adding 4/7 and 2/7 of a liter of liquid looks like, and whether the resulting amount of liquid is 6/7 liter or 6/14 of a liter. This is an ideal place to discuss how to determine the "whole" in the problem's context with fractions.

After students understand Method 2 well, comparing it with Method 3 will lead students to see how to represent the solution method using fraction notation. Method 3 also shows very clearly that each written 1/7 corresponds to a shaded section drawn in Method 2. By making reference to the relationship, the students who may be thinking about fractions only at the most concrete level will be able to see how the written unit fraction 1/7 relates to the concrete model in Method 2.

At this point, the teacher may want to help students describe the rule for adding fractions of like denominators as the student explained in Method 4. Once the different solution methods using different representations have been discussed, the rule can emerge as something described by students, as it came out of their math talk. This leaves the ownership of the process to the students.

In the balanced teaching framework, it is emphasized that all three phases are necessary for students to learn mathematics coherently and meaningfully. In many cultures, traditional math teaching focusing on Phase 3 has been the major way of teaching. In an effort to help students learn math more constructively in recent years, some teachers have attempted to shift their teaching completely to focus solely on Phase 1. This can leave the students feeling lost and frustrated, as it can be difficult for them to grasp mathematical structures and patterns when they are left alone to explore without a summary highlighting and discussing important math concepts. Phase 2 brings these two seemingly very different teaching approaches together by creating a space and time for the teacher and students to make sense of the mathematics. Connection-building is the key in this phase, and visual supports and drawings play a critical role in the process. Use of similar representations to bridge different student methods can focus student attention on an important mathematics concept, because students can focus on mathematical structure rather than on superficial differences in representations. The visual supports must be carefully chosen to highlight the concepts students learn in the problem context. In the example shown in Figure 2.2, the bar representation and written math notation help clarify fraction relationships and support students' understanding.

In Phase 2, teachers might introduce more advanced visual supports or solution methods, as informed by the expectations of a given school or country. For example, number lines are an important math tool in many countries. They are more difficult than the fraction bars the students used in Phase 1 methods in Figure 2.2. Helping students relate simpler fraction bars to fraction number lines and to fraction notation extends and deepens their understanding of various methods. Figure 2.3 shows how this might be done with drawings shown to students that they or the teacher would then explain, as shown on the right side of the figure. Those who explain would point to parts of the drawings and make other gestures to help their listeners make the connections they are emphasizing. Having such carefully designed drawings available in student books or in a technology program could ensure that students see and explain important mathematical representations and engage and extend their sense-making.

2.4 The Lengths of the Phases Can Vary: An Example from Japan

The length of each phase in an instructional unit may vary according to the complexity and nature of the topic. In Japan, where the three-phase balanced teaching framework has been used for decades, the curriculum is organized according to the framework. Table 2.1 summarizes the numbers of

FIGURE 2.3

PHASE 2: RELATING A FRACTION BAR, A FRACTION NUMBER LINE, AND FRACTION NOTATION

Problem Representation	Student Explanation
a. [empty bar]	"First, I draw the one whole. Every fraction is some number of equal parts of one whole."
b. 1/7 + 1/7 + 1/7 + 1/7 + 1/7 + 1/7 + 1/7 [bar divided into 7 parts]	"Now I divide the one whole into 7 equal parts to make 7 sevenths and I label each part with a unit fraction 1/7."
[number line] 0/7 1/7 2/7 3/7 4/7 5/7 6/7 7/7	"Now I draw a line segment as long as the one whole, and I divide it into 7 equal small lengths to make the unit fraction lengths. Fractions are shown on fraction number lines as the number of lengths from 0. So I label the start as 0/7 because I have no length yet, and then I label the end point of each length to tell the number of lengths it is from the 0. See 1/7 [sliding a finger along that length], 2/7 [sliding a finger along 2 lengths], etc. up to 7/7."
c. 1/7 + 1/7 + 1/7 + 1/7 + 1/7 + 1/7 + 1/7 [bar divided into 7 parts] 4/7 + 2/7 = 6/7 [number line] 0/7 1/7 2/7 3/7 4/7 5/7 6/7 7/7	"Now we'll show our problem 4/7 + 2/7 and discuss how the fraction bar and fraction number line are alike and different in how they show the problem." Maria: "They both show 4 of the sevenths and then 2 of the sevenths to make 6 of the sevenths" Jose: But the unit fractions are small parts in the fraction bar and small lengths in the number line. Eusebio: "The number line labels tell the total as they go. We have to count the two sevenths added on to make six sevenths." Rosa: "And with the fraction bar we have to count all of the unit fractions, the four and the two, to find the total six."
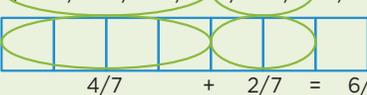
Source: Prepared by the authors.

lessons (each lesson is typically 45 minutes) recommended for each phase for all the units in Grades 2 and 5, using the curriculum "Study Mathematics with Your Friends: Mathematics for Elementary School" (Gakkotosho 2010).

TABLE 2.1

NUMBERS OF LESSONS DEVOTED TO BALANCED TEACHING PHASES IN A JAPANESE CURRICULUM FOR GRADES 2 AND 5

Grade 2

	Unit Name	Phase 1	Phase 2	Phase 3	Review	Total
1	Charts and graphs	0.5	1.5	2	1	5
2	Numbers up to 1,000	0.5	2.5	4	3	10
3	Addition algorithm	1.5	3	5.5	2	12
4	Subtraction algorithm	1.5	3	4.5	2	11
5	Shapes	3	1	2	1	7
6	Clocks	0.5	0.5	0.5	0.5	2
7	Addition and subtraction (1. Solving problems using tape diagrams and other representations)	1	1	1	1	4
8	Length (1. Understanding the concept of length, measure, and estimated length using standard units)	2.5	1	4.5	2	10
9	Multiplication (1. Understanding the concept of multiplicative relationship and representations)	0.5	2	2.5	2	7
10	Multiplication (2. Multiplication of 2s, 5s, 3s, and 4s)	2	2.5	7.5	1	13
11	Multiplication (3. Multiplication of 6s, 7s, 8s, and 9s)	0.5	3.5	7	3	14
12	Multiplication (4. Investigating patterns and problem-solving with multiplicative relationships)	1	1	3	2	7
13	Length (2. Solving addition and subtraction problems with standard units of measure)	1	0	3	3	7
14	Numbers larger than 1,000	1	1.5	5.5	2	10
15	Triangles and quadrilaterals	2	1	4	2	9
16	Addition and subtraction (2. Solving problems using the reverse relationship between addition and subtraction)	0.5	1	2.5	0	4
17	Review	0	0	0	5	5
	Total	19.5 (14%)	26 (19%)	59 (43%)	32.5 (24%)	137

(continued on next page)

TABLE 2.1 *(continued)*

NUMBERS OF LESSONS DEVOTED TO BALANCED TEACHING PHASES IN A JAPANESE CURRICULUM FOR GRADES 2 AND 5

Grade 5

	Unit Name	Phase 1	Phase 2	Phase 3	Review	Total
1	Decimal numbers and integers	1.5	1.5	5.5	2.5	11
2	Estimation and approximation	1	0.5	0	0.5	2
3	Multiplication with decimal numbers	1.5	3.5	4.5	3.5	13
4	Perpendicular and parallel lines	0.5	1	3.5	3	8
5	Quadrilaterals	1	2	7	3	13
6	Division with decimal numbers	3.5	4	5.5	3	16
7	Angles	1	1.5	2.5	1	6
8	Areas	1	3	7	3	14
9	Fractions	2	2	7	3	14
10	Circles	2	2	3	3	10
11	Percentages and graphing	0.5	1	8.5	4	14
12	Review	0	0	0	8	8
	Total	15.5 (12%)	22 (17%)	54 (42%)	37.5 (29%)	129

Source: Prepared by the authors.

Overall, across two grade levels, the curriculum devotes approximately 13 percent of the entire instructional time to Phase 1, 18 percent to Phase 2, a little less than 50 percent to Phase 3, and the rest to reviews at the end of each unit (about 26 percent). The review of particular content can happen at the end of the unit, as well as later on in other unit reviews, and reviews may thus contain more than one topic. While the frequencies provide a general sense of balanced teaching, it is important to note how this can vary across different content topics. For example, for the shapes unit in second grade, students spend three lessons out of seven in Phase 1 sharing their observations of different shapes, one lesson in Phase 2, two lessons in Phase 3, and the remaining lesson in review. In fifth grade, for the unit of estimation and approximation, 1.5 lessons are spent on sharing student ideas (Phase 1), followed by 0.5 lessons in Phase 2, and 0.5 lessons of review (there was no Phase 3). In thinking about how to balance different parts of the instructional unit, educators and curriculum designers need to be careful not to generalize the relationships among phases too much in their communication with teachers. It also needs to be emphasized that,

although Phase 3 focuses on fluency, students who still lack a full conceptual understanding can also receive support in Phase 3.

2.5 The Importance of Using Math Drawings to Support Math Talk

There is a lot of international research about representing and solving word problems as the bases for understanding operations (+ – x ÷) and for building algebraic thinking. Figure 2.4 shows diagrams that can represent the three addition/subtraction and three multiplication/division situations that have been identified in worldwide research. Seeing the diagrams together shows their coherence: for example, equal group situations in multiplication arise from addition situations (add to/take from, or put together/take apart) when an addend is a group that is added repeatedly. Additive comparisons likewise add a group repeatedly to become multiplicative

FIGURE 2.4
WORD PROBLEM SITUATIONS AND DIAGRAMS FOR ADDITION (TOP ROW OF PANELS) AND MULTIPLICATION (BOTTOM ROW OF PANELS)
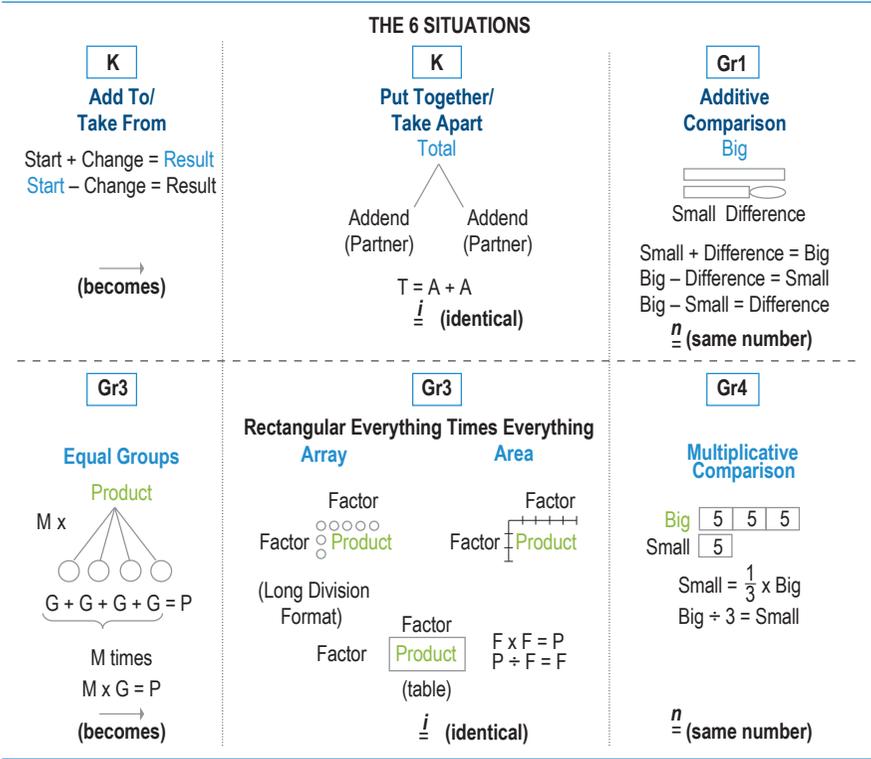
Source: Prepared by the authors.

comparison situations. Both of these multiplication situations involve one factor, as the multiplier telling how many groups there are, and the other factor as telling how many in a group. The rectangular "everything times everything" situation involving arrays or area does not have factors with these different roles, although either the row or the columns can be regarded as a repeated group. Figure 2.4 shows how different situations actually involve different meanings of the equals sign, indicated at the bottom of each problem type. This single set of diagrams can be used for all of the quantities students experience from grades 1 through 6 (from single-digit numbers through fractions and decimals) and for many multistep problems.

An example of a second grade student's use of diagrams to support solving and explaining a difficult word problem is shown in Figure 2.5. This problem is a "take from" problem because Eddie took some balls from the box. It is also a "start unknown" because the first number in the problem, the number the student is starting with, is not known. Neither of the first two students really makes sense of the whole problem. Student 1 in Figure 2.5 just draws the numbers in the problem without thinking about how they relate to the actions in the situation. Student 2 focuses on the fact that Eddie took some, but does not really make sense of the whole problem and so has no problem representation of the situation. If the student had been asked to make a drawing such as the third student made, this error might have been avoided. Student 3's solution might have been a student-generated method made in Phase 1 of balanced teaching for these difficult types of problems. This student makes sense of the whole situation and shows it in the drawing. Student 4's solution uses the Math Mountain (number bond) diagram at the top middle panel of Figure 2.4 to show the relationship between the numbers 9, 4, and ? in the problem. This diagram shows how the quantities in the situation relate to one another. The last student's solution is a situation equation that shows the balls that belong to Yolanda as an unknown quantity (students can use a small rectangle to show the unknown), the 9 balls that belong to Eddie that are taken away, and the 4 balls left. Once they have represented the problem situation, students can find the total of 9 and 4, possibly in different ways.

If students did not use the last three methods in Phase 1 of teaching this topic, the teacher could introduce them as effective approaches for the students to try. These methods then could continue to be discussed and compared in Phase 2. Seeing and explaining the different problem representations help, students think about problems in different ways and relate these different representations. Labeling the quantities to relate

FIGURE 2.5
START UNKNOWN SOLUTION APPROACHES FOR SINGLE-DIGIT NUMBERS

Yolanda had a box of balls. Eddie took 9. Now Yolanda has 4 left. How many balls did Yolanda have in the beginning?

Problem Representation	Student Explanation
OOOOOOOOO OOOO	Student 1: "I drew 9 circles for the balls Eddie took. I then drew 4 more circles for what Yolanda had left. I then counted all the circles to find 13."
5	Student 2: "The problem says that Eddie took some. So it is a take away problem. So 9 take away 4 is 5. The answer is 5."
—OOOOO—OOOO|— OOO— Took Left	Student 3: "I drew the 9 balls Eddie took away and then I drew a line through them to show they were taken away. I drew this small stick to show the end of the balls Eddie took. Then I drew the 4 balls that were left. Then I labeled the problem. Now all of the balls are shown, and I can see 13 because 4 gives one to the 9 to make 10."
in box ☐ 9 4 Eddie Yolanda now	Student 4: "I take apart the balls that belong to Yolanda to show the 9 that Eddie took and the 4 that Yolanda has now at the end. My total that I don't know is at the top. I put the 9 and 4 back together and that makes 13: 9, 10, 11, 12, 13."
Yolanda Eddie end ☐ – 9 = 4	Student 5: "I made an equation to show the problem situation. We don't know how many balls Yolanda had to start with. Then Eddie took 9. And there are still 4 at the end. I see that 9 and 4 are the addends in the equation, and I know that 9 and 4 make 13."
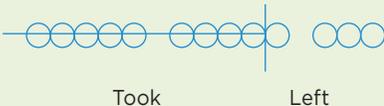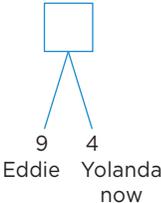
Source: Prepared by the authors.

them to the problem situation is important for supporting the understanding of the solver and the classmates listening to his or her explanation.

In Figure 2.6 we see problems with the same "start unknown structure" but with multidigit numbers and fractions. Now the situation equations and the Math Mountain (number bond) diagram are very useful because the numbers are bigger and/or more complex. Students may also understand the situation and just add the addends to make the unknown total,

FIGURE 2.6
START UNKNOWN SOLUTION APPROACHES FOR MULTIDIGIT NUMBERS AND FRACTIONS

Yolanda had a huge box of balls. Eddie took 157. Now Yolanda has 189 left.
How many balls did Yolanda have in the beginning?
Yolanda had part of a sandwich. Eddie took 4/7 of it. Now Yolanda has 2/7 left.
How much of the sandwich did Yolanda have in the beginning?

| Solution computation | $\begin{array}{r} 157 \\ + 189 \\ \hline 200 \\ 130 \\ 16 \\ \hline 346 \end{array}$ Eddie Yolanda in all | $\dfrac{4}{7} + \dfrac{2}{7} = \dfrac{6}{7}$ in all E   Y   B |

Numerical relationships in Math Mountain

Y beginning

346

157   189

$\begin{array}{r} 157 \\ + 189 \\ \hline 346 \end{array}$

Student Explanation

Y   E   end

$346 - 157 = 189$

$\begin{array}{r} 157 \\ + 189 \\ \hline 346 \end{array}$

Y at first

$\dfrac{6}{7}$

$\dfrac{4}{7}$ $\dfrac{2}{7}$

Y   E   left

$\dfrac{6}{7} - \dfrac{4}{7} = \dfrac{2}{7}$

total

$\dfrac{1}{7} + \dfrac{1}{7} + \dfrac{1}{7} + \dfrac{1}{7} + \dfrac{1}{7} + \dfrac{1}{7} = \dfrac{6}{7}$

or

$\dfrac{4}{7} + \dfrac{2}{7} = \dfrac{6}{7}$

as in the solutions at the top. So we see that the diagrams and equations with which teachers build understanding in the lower grades with small numbers can be used with larger numbers and fractions (and decimals) in the upper grades.

In Figure 2.7 we see diagrams and equations for an additive comparison problem. Students initially can make matching drawings (as at the top left) to make sense of comparing quantities. But with larger numbers, as at the bottom, the comparison bars are useful for showing the problem situation, and numbers can be written in them. Students represent comparison problems in various ways.

FIGURE 2.7
SOLUTION APPROACHES TO AN ADDITIVE COMPARISON PROBLEM, GRADES 2 AND 3

Grade 2 Solutions

Jana read 15 books. Lisa read 8 books. How many fewer books did Lisa read than Jana?

Matching drawing of quantities

Situation equation

$$8 + \boxed{7} = 15 \qquad \boxed{7} \text{ books}$$

Lisa   more   Jana

Numerical relationships shown in Math Mountain

Solution equation

$$15 - 8 = \boxed{7} \qquad \boxed{7} \text{ books}$$

Jana   Lisa   fewer

Grade 3 Solutions

Jasmin made 346 tortillas. Luisa made 189 tortillas. How many fewer tortillas did Luisa make than Jasmin?

Comparison bar drawing of quantities

Situation equation

$$
\begin{array}{c}
L \qquad d \qquad J \\
189 + \boxed{157} = 346 \\
189 + 11 = 200 \\
200 + 146 = 346 \\
157 \qquad \boxed{157} \text{ tortillas}
\end{array}
$$

Numerical relationships shown in Math Mountain

Solution equation

Jana   Lisa   fewer

$$346 - 189 = \boxed{157}$$

$$
\begin{array}{r}
\overset{13}{} \\
2\ 14\ 16 \\
3\ 4\ 6 \\
-1\ 8\ 9 \\
\hline
1\ 5\ 7
\end{array}
\qquad \boxed{157} \text{ tortillas}
$$

Source: Prepared by the authors.

For all problem situations, one can see how helpful it is for students to use the diagrams to represent the problem situation. Using a diagram or a situation equation helps students read and make sense of the whole problem situation. This is the key to problem-solving. It takes very little time to make a diagram or write a situation equation, so these visual supports can be used to support fluency in Phase 3 when students are solving problems that are routine for them.

Learning paths of problem subtypes, ranging from easy to difficult, have been identified in a great deal of research. These depend on the problem situation and the particular unknown within that situation. Algebraic problems are those where the situation equation, such as $\square + 6 = 9$, is not the same as the solution equation, $9 - 6 = \square$, that shows the solution operation. The equation $\square - 9 = 4$ in Figure 2.5 was a situation equation, because it showed the action of the quantities in the situation. Students can also work in kindergarten with forms of equations with one number on the left (e.g., $5 = 2 + 3$ and $5 = 4 + 1$), as they decompose a given number (here, 5), and record each decomposition by a drawing or equation. Experience with these various forms of equations can eliminate the typical difficulty many students have in algebra, where their limited experience with one form of equation leads them to expect equations with only one number (answer) on the right.

The diagrams support a student in algebraic problem-solving: the student can represent the situation by making a diagram or writing an equation and then use the numerical relationships in that diagram or equation to find the solution. The diagrams are used in the Phase 2 MD&A methods in the middle (Figure 2.1) for algebraic problem-solving. The diagrams have moved beyond students' basic math drawings that show all of the objects, and they are not yet algebra, which uses only an equation to represent the situation. These diagrams bridge these two levels and give students extensive experience with writing, understanding, solving, and explaining/discussing situation equations like $\square - 538 = 286$ or $5/7 = \square + 2/7$ that show the situation.

2.6 How to Support Students and Teachers in Balanced Learning

2.6.1 Motivating Teachers

Many teachers around the world may feel unexcited about teaching mathematics because of their own math experiences as students. They might have learned mathematics as a set of rules and procedures to follow and felt no personal connection to the subject. They might have considered themselves not good at math at one time or another because they could not find the right answers as quickly as their peers who were considered

smart. They might have tried to make sense of mathematics, only to end up feeling lost and confused because they could not find the support and resources they needed. These teachers may feel unmotivated to learn to teach math in a new way out of fear of repeating their bad experiences as well as the possibility of exposing what they do not understand. They may prefer to give their students worksheets and drills and minimize the time they have to discuss mathematics concepts in lessons.

To address such issues, first, these teachers need to be heard and accepted for all the difficult experiences they have had with math and then be encouraged to be agents of change in their students' lives and help avoid producing another generation of adults who dislike mathematics. They need to be empowered to understand that they have the position and the ability to break the cycle and contribute to a better future for their students.

While it is possible that teachers might hesitate to follow the balanced teaching model initially, in reality it will create a new space for them to address their own negative experiences with mathematics. The teachers may lack some content knowledge of mathematics, but the balanced teaching model creates a space for them to relearn mathematics along with their students. By placing their students at the center of mathematics learning (e.g., in Phase 1, when students share their ideas and the teacher takes the role of clarifier and questioner), the teacher can temporarily feel a reduced level of responsibility in terms of feeling the need to always know the right answer. At the same time, as the teacher listens to the students more, he or she can come to realize that the students actually have a lot of great ideas and see how excited and empowered they are when their ideas are valued in math classrooms. This will produce a new sense of success among teachers and gradually help shift the way they see their role in the classroom. Teachers may further investigate certain mathematics concepts they did not understand conceptually in order to develop a new mathematics understanding based on their students' learning. The perception of a good teacher shifts from a person who knows the right answer and shows how to get it to a person who asks good questions, helps students represent a problem and then find a solution to it, as well as helping students explain their thinking. Mathematics over time becomes something they think about along with their students, and they will come to experience the inquiry-based learning process as exciting and fun. Many teachers who attempt the balanced teaching model report that it was the first time that mathematics made sense to them, and some even call it a "math therapy for teachers" for this reason.

In order to help teachers be successful with the process, it is important to provide them with a safe social structure where they can share their

experiences, voice their concerns, ask questions and receive answers, and find resources as needed. Teacher communities or lesson study groups are ideal places where teachers can safely discuss their practices and receive support. Instructional leaders, curriculum supporters, or knowledgeable others can join the groups as needed to provide necessary resources and information, as the teachers will need to expand their existing knowledge base to continue to grow professionally. A good conceptual curriculum to teach from can also be very helpful.

2.6.2 Motivating Students

Motivating students to learn and undertake necessary practice, including doing homework, can be especially difficult in modes of traditional teaching where students just memorize what a book or a teacher says. When the focus is initially on understanding and on explaining their own math thinking, students can become invested in what they are learning and in helping their classmates learn. They can feel competent as learners and come to understand that learning increases their ability to learn in the future—the "growth mindset" mentioned earlier (Dweck 2010).

Today's classrooms include many students from different cultural, linguistic, and ethnic communities. When students' personal lives and values differ significantly from those of their school, they are likely to find themselves struggling to make sense of the differences. These students often feel alienated and have a hard time picturing themselves as possible learners whose ideas are valued. When math is taught in a top-down manner (solely Phase 3 traditional teaching), without incorporating student ideas as a part of the learning process, it further distances the students, and they remain peripheral to the learning community. For any student to learn, he or she has to maintain a level of social intention to be part of the classroom community and to share in the learning process. The greater the distance between the classroom (often operated by the values of the dominant cultural group) and the student's community, the harder it is for the student to bridge the gap. The balanced teaching model helps the process by inviting student ideas from the beginning and gradually connecting the ideas with the formal and mathematically desired methods. The teachers also maintain reasonable expectations that every student comes with different ideas and interests and that it takes time for students to develop appropriate levels of fluency with any concept. Gradually, the students will communicate and share their ideas with peers and come to feel included in the classroom community. This will increase the sense of membership in that community. In these ways, the

balanced teaching model helps motivate students in diverse classrooms and supports the productive learning disposition in the high-level goal for all (top of Box 2.1).

The concept of "means of assistance" was developed by Tharp and Gallimore (1988) as they described aspects of responsive reading instruction in elementary school classrooms with students from diverse backgrounds in Hawaii. Murata and Fuson (2006, 2016) and Fuson and Murata (2007) identified responsive means of assistance used by students and teachers to facilitate meaningful mathematics learning and balanced teaching. The means of assistance include three larger elements:

1. The teacher engages and involves students in meaningful mathematics learning activities.
2. The teacher manages participation so everyone is included.
3. The teacher coaches productive math talk by modeling, clarifying, explaining, questioning, and giving feedback.

With these means of assistance, the teacher will orchestrate collaborative instructional conversations focusing on the math thinking of students, and visual models will allow everyone to focus on making sense of the math structure. The teacher also helps students use all of these means of assistance to help their classmates so that the classroom becomes a place where everyone is a learner and a teacher.

2.7 Using Technology to Support Balanced Teaching

Technology comes in many forms and can offer different kinds of solutions to educational problems. Table 2.2 summarizes five uses of technology that can be helpful in various phases of balanced teaching; more detailed information on the five uses are then presented in the main text that follows. It is useful to consider the whole range of teaching needs when making decisions about purchasing or supporting technology use, so that there is a balance between the needs for teacher learning and professional development, student understanding of math topics, and student fluency with math topics. This section suggests ways in which technology can help teachers understand both the mathematics at hand and their students' thinking and lead their class productively through learning paths from building understanding to building fluency for all students. It should be emphasized, however, that while technology can and should help teachers teach better, it cannot and should not be used to replace teachers completely or unnecessarily complicate their teaching lives.

TABLE 2.2

USES OF TECHNOLOGY RELATED TO PHASES OF BALANCED TEACHING

Use 1. Teachers confer on teaching methods.
This is helpful in all phases of teaching, from preparing to teach through Phases 1, 2, 3 to the Review Phase.

Use 2. Teachers and students develop an understanding of math content by seeing visualizations and hearing explanations of problem representations and solutions.
This is helpful in preparing to teach in Phase 1 (Guided Introduction) and in Phase 2 (Learning Unfolding).

Use 3. Students practice solving problems to develop fluency. The technology gives feedback on answers and keeps problems in individual student practice zones (where students need practice).
This is helpful in Phase 2 (Learning Unfolding) for doing homework, in Phase 3 (Kneading Knowledge for Fluency) practice, and in practice for the rest of the year to maintain fluency (the Review Phase).

Use 4. Students confer on representing problems and problem-solving.
This is helpful for all of the phases and for problem-solving in Use 3 above.

Use 5. Teachers manage classroom discourse about problem representations and problem-solving with student visual models.
This can be helpful in Phase 2 (Learning Unfolding).

Source: Prepared by the authors.

Use 1. Teachers confer on teaching methods. Technology can enable teachers to interact with colleagues in many ways and locations. Conferring on teaching methods is useful in all phases of teaching from preparation through Phases 1, 2, and 3. It can be as simple as using email to ask questions or sharing prepared teaching materials. It can be as complex as sharing student work with a small or large working group or sharing ideas via conferencing technology, much of which is now or soon will be free (e.g., Skype, Google Hangouts, GoToMeeting, etc.). Teacher conferring can be done from a teacher classroom or involve a dedicated videoconference room in locations accessible to teachers. Such conferring can have different formats and organizational structures. For example, one teacher can take the lead for a given math topic at a given grade level and provide overviews to initiate a conversation with colleagues. Such interactions can provide teachers a community rather than an authority, and teachers can contribute rather than just receive. They can grow to become local or regional experts. Teachers who develop personal identities as reform-oriented mathematics teachers are more apt to perceive mathematical content in pedagogically productive ways and vice versa (Ma and Singer-Gabella 2011).

Use 2. Teachers and students develop an understanding of math content by seeing visualizations and hearing explanations of problem

representations and solutions. Developing teacher understanding is helpful in preparing to teach in Phase 1 (Guided Introducing) and Phase 2 (Learning Unfolding). Computers with Internet access and video functions can help teachers see familiar mathematical representations through the eyes of their students and thus be better prepared to explain the content or understand student errors. Specifically, computers enable teachers to access instructional resources dedicated to demonstrating students' point of view as well as to videoconference with colleagues who lecture and discuss these matters.

Mathematical conversations between students or between the teacher and students should be based on an agreement over what the intention of the conversation is. But sometimes two or more people can have a difficult time communicating about a mathematical representation because, unknown to them, they are attending to it differently, and so they do not understand why their inferences are different. Computers can prepare teachers to foresee and manage these situations so that they and the students share common ground in their mathematical conversations, no matter where the children begin (Figure 2.8).

In Jastrow's ambiguous "Duck-Rabbit" image, teachers need to see both the duck and the rabbit in order to understand why children want to feed this animal fish or carrots. Teachers also need to understand that some students may see the image above on the right (the *b* image) as 2/5 instead of 2/7 because they see 2 dark parts and 5 light parts and do not see the total 7 parts. A teacher can help students understand such drawings by first making all 7 parts to see each 1/7 as in image *c* then shading the number of parts in the fraction (here, 2 of the 1/7 make 2/7) in a new drawing so that students can still see all 7 of the 1/7ths in the whole. Computers can help teachers as well as students see mathematical representations in different ways: just as highlighting the rabbit's mouth "brings out" the rabbit, so highlighting 2 blocks as part of 7 blocks shown visually "brings out" 2/7.

An important aspect of the teaching practice is helping children build relationships among visualizations and mental links between mathematical signs, such as between a diagram of 2/7 and the symbol "2/7." It is known that teachers use hand gestures as well as speech to communicate such links (Alibali et al. 2013). Furthermore, these communication efforts are largely centered on orienting the children's visual attention in new ways toward these mathematical signs (Ingram 2014; Stevens and Hall 1998). In so doing, teachers show children not only what to look at but also how to look at it and what to do with it, for example, to show how Figure 2.8b is a representation of 2/7 by relating it to Figure 2.8c. Technology can effectively simulate and complement teachers' natural ways of using gestures to draw

FIGURE 2.8
VISUALIZATION AND PROBLEM-SOLVING
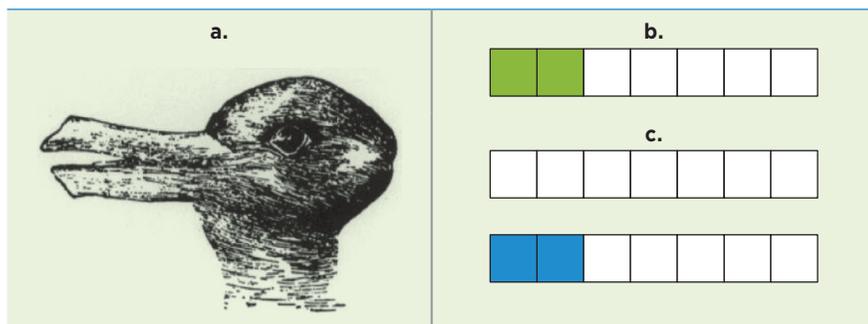
Source: Prepared by the authors.

attention to and relate to features. For example, objects on a screen can be highlighted by blinking, changing color, or jittering to show that they are the focus of a demonstration. So, for example, the boxes in panel *b* of Figure 2.8 can blink, one after the other, to accompany a counting of 2 or of 7 boxes. Labels can be overlaid on objects to link between symbol and referent, for example, overlaying "1/7" on or above each of the seven boxes in panels *b* or *c* of Figure 2.8. Logical or arithmetic relations between objects can be shown by choices of color and special effects, such as by alternating between showing all 7 boxes and showing just the 2 boxes to demonstrate containment. These technological features can also help teachers become aware of how they explain ideas and emphasize effective communication methods, such as gestures. In Figure 2.7 shown earlier, one can imagine different ways in which important connections could be made using the above features of technology, along with a voiced explanation, as students discuss and gesture at the drawings of additive comparison problems to make connections for classmates.

Students, too, should be able to shift between different visualizations of mathematical representations. This will enable them to reflect on their own reasoning, make the link between conceptually compatible visualizations, and understand the teacher and their classmates. More generally, visualization—how we are looking at and thinking about a representation—is an important aspect of mathematics and science activity and discourse. But it is rarely made explicit in classroom conversations. When children do speak explicitly about how they are seeing a mathematical representation, they draw directly on their experiences and create opportunities for their teachers to help them (Abrahamson, Gutiérrez, and Baddorf 2012). Teachers should create safe spaces for students to share how they are

seeing the mathematical objects, even if these ways of seeing are unfamiliar or surprising to other students (Feucht 2010).

An important benefit of recording visually supported explanations is that people watching the video can stop it at any moment, repeat it, or watch it at a slower or faster pace. Therefore, students working individually either in the classroom or at home can review a teacher's or a student's explanations at their own pace. Moreover, students can watch multiple explanations—just as many as they need to understand a new solution method. Finally, students or teachers can watch different solution methods, which can both expand their solution repertoire and increase the chances that a particular solution method will better connect with their own approach developed during Phase 1.

For classrooms in which students have never explained their thinking, a video of a student explaining a given topic can be shown to the class to start student discussion. Teachers who are not confident that they can explain why a given solution method works can show a video of a teacher explaining it to a class or an individual student in a tutoring situation. Such interactive explanations are more powerful than just a video of a teacher explaining (though this also can be useful) because the interaction is more natural and more methodical (Chi, Kang, and Yaghmourian 2016).

Thus, there is immense potential for policymakers and technology experts to invest in video technology and content. They can build professional-development video archives in collaboration with curricular designers or from freeware on the web. Teachers and students can make additional videos that extend the topics available. The video and animation functionalities described above are already available either as free downloads or in generic software packages native to personal and laptop computers, such as the QuickTime™ on Macs and VLC on PCs.[5]

Finally, a related, unique attribute of powerful educational technology is enabling learners to access ideas that are difficult for a teacher or a math program to present in the classroom. This need increases as

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[5] The authors of this chapter have websites where helpful free resources are available for teachers. See karenfusonmath.com for classroom videos that show students from impoverished backgrounds learning conceptually and explaining their thinking, and that show teaching progressions for math topics in elementary school that demonstrate visual models and how they can support student thinking. See lessonstudynetwork.com for lesson study resources and support as teachers learn together. See edrl.berkeley.edu for videos demonstrating productive ways of discussing mathematical ideas with students. Matif.com also has useful interactive games that support conceptual learning and practice. Dor Abrahamson serves on the Advisory Board for these games.

the mathematical ideas get more complex, and addressing it becomes important for many concepts in grades 6 to 8 and above. For example, the mathematical ideas involved in proportion (the equivalence of two ratios, e.g., 2:3 = 4:6) are difficult to "phenomenalize" (Pratt and Noss 2010), that is, to make into a situation in which students can experience the core notion. When students are not given interactive situations to experience a new concept, they cannot access the basic meanings of what the concept is about. Consequently, the students only learn to execute procedures, but without understanding them (see Abrahamson, 2014, on the embodied-design framework).

Use 3. Students practice solving problems to develop fluency. The technology gives feedback on answers and keeps problems in individual student practice zones (in which a student needs practice). This is helpful in Phase 2 (Learning Unfolding) for doing homework, in Phase 3 (Kneading Knowledge for Fluency) practice, and in later practice to maintain fluency. The problem sets practiced by students change across the phases, from those focused on limited types for a given lesson in Phase 2, to broader ones across more types of problem sets in Phase 3 within a unit, and across many types of problem sets and many math topics in review for the rest of the year. For example, a solution to the problem $4/7 + 2/7$ needs to be expanded in several ways:

1. To $2/7 + 4/7$, which can focus students on mathematical properties
2. To $4/7 + 5/7$, which involves an answer that goes beyond one whole, $7/7$
3. To $4/9 + 2/9$, which uses a new number of unit fractions
4. To $4/7 + 2/5$ in which both fractions must be changed to equivalent fractions in order to add them.

Expansion to (1), (2), and (3) might be done in class and then practiced in mixed problem sets during Phase 2 homework for that lesson. Some math programs or teachers might delay discussing (2) until after they have practiced expressing the answer as 1 1/7 (as a whole number and a fraction), while others might want students to solve the case right away to emphasize that adding is always done by finding the total number of unit fractions. But (4) is a new, large topic that needs several days of development and discussion and might even be taken up in a later grade. The problem sets would likely include subtracting examples, like the addition examples in the unit, and would mix addition and subtraction examples, for the unit fluency work. Adding and subtracting mixed numbers might be included in this unit or in a later grade, and practice would eventually be

done across these types. Similarly, fluency practice on all four operations would eventually be done in Phase 3's unit practice or in the mixed review for the rest of the year.

Over the rest of the year, students can review a mix of older topics, even as they focus on the more targeted Phase 2 and Phase 3 kinds of practice. This allows them to achieve fluency in new topics while maintaining fluency in older topics.

Computer practice systems can have more or less of a management system that gives feedback about student performance to teachers or the family. Such feedback can be useful for teachers or family members to help with motivation or conceptual help if needed. Or, such practice can be related to Use 2 (technology) to provide a user-adaptive, cognitive tutor based on individual student performance.

Many computer games designed to provide fluency practice fail in how they classify practice problems. They have far too many easy problems, wasting student time. This is especially true for single-digit addition, subtraction, multiplication, and division games. Then a game may jump to much harder problems, missing problems of intermediate difficulty. Some games contain visual information, such as moving images, that is peripheral to the core problem and only taxes students' mental resources and distracts them away from the mathematical ideas (Hirsh-Pasek et al. 2015; Rosen, Palatnik, and Abrahamson 2018). So such programs need to be chosen carefully so that they give problems to a student that fits the student's practice needs.

The classroom and technological learning environment should support the students in sustaining their motivation (see Chapter 6). Some advantages of computer-based practice software are (1) providing immediate feedback, (2) diagnosing inappropriate beliefs or suboptimal strategies, and (3) providing teachers with current statistics on both individual and aggregate achievement and learning paths. The common game features of practice technology are fine and can be motivational, as long as the game aspect does not take too much time or distract too much from the math.

Use 4. Students confer on representing and solving problems. This is helpful for all of the phases and problems in Use 3 and also in solving problems and representing them in class in Phase 2. This can, of course, be done without technology, but increasingly students themselves may be using available technology to communicate with classmates about school work.

Use 5. Teachers manage classroom discourse about representing and solving problems using student visual supports. This can be helpful in

Phase 2 (Learning Unfolding). When students are explaining their thinking, their math drawings and any written notation need to be made visible to their listeners. To achieve this, students can do their problem-solving with math drawings in low-tech formats such as on paper, chalk or dry-erase boards, or big reusable plastic write-on sheets, and then explain their problem to their classmates who are able to see their work. However, electronic tablets increasingly have writing/drawing functions that support student or teacher problem-solving with math drawings, and these can be projected for everyone to see.

2.8 Summary

Educating students for the mathematical needs of the 21st century is a challenging task. Doing this well during the elementary school years is an important element of a country's success, because that is when many students fall behind, and those who do are rarely able to catch up in later grades. Table 2.3 summarizes the major conclusions drawn from research and reviewed in this chapter and provides an overview of recommendations along with their policy implications.

Focusing on the entire learning progression for a given math topic in the elementary grades can help educators think deeply about priorities in learning the topic. Educators can then provide the support needed to help students build understanding, master topics more coherently, and develop fluency with those topics. This chapter provided examples of fraction addition and algebraic problem-solving because these are centrally important math topics in these grades. The examples illustrate how important visual models are for understanding and explaining mathematical concepts, and for eliciting students' different ways of making sense of the situations. Providing an overview of classroom teaching and learning within the balanced teaching framework allows for presenting results from a large international body of research in a coherent way. This framework brings together different views of teaching to focus on both student ideas and mathematically desirable methods. In a sensible progression, teaching begins with the ideas that students bring into classrooms, then rapidly builds to mathematically desirable and accessible methods that are explained by the teachers and students, and finally moves on to fluency practice. Table 2.2 showed some important roles that technology can play at various points of this teaching/learning progression. Altogether, the framework can help teachers and students in Latin American and Caribbean countries and the rest of the world learn and use mathematics to participate productively in the global world of the 21st century.

TABLE 2.3
SUMMARY OF CONCLUSIONS AND IMPLICATIONS

Conclusions	Recommendations and Policy Implications
1. Technology changes the goals of math teaching. In the 21st century, teachers need to help students understand a math topic. Practicing solution methods for that topic follows and builds from understanding.	• Educational policymakers need to share this message with all educators and parents and implement policies to help teachers undertake this approach.
2. Students think in different ways, and their ideas can be anticipated and connected through teacher assistance. All ways can be valued, and students can be helped to move to mathematically desirable, accessible and formal math methods.	• Instructional materials and standards documents should clearly outline how students think mathematically and how teachers can facilitate student learning by making connections between key mathematical methods though classroom activities, lessons, and discussions.
3. Teachers can learn to teach differently, moving away from traditional rote teaching to more conceptual teaching by focusing on how students think mathematically, how their ideas support certain solution methods, and why. The three-phase balanced teaching model (Box 2.1) can help teachers shift their thinking. Teachers can bring forward key math concepts in student "math talk" and help students make connections among methods.	• Professional development opportunities should be provided for teachers as they make sense of the curricular materials and standards documents and learn to teach differently. Local leaders need to be involved in such efforts.
4. Visual models are essential to helping students learn mathematics and teachers learn about student learning of mathematics. Effective teaching involves the purposeful use of visual models as students share their solution methods in math talk.	• Examples of visual models should be consistently presented in instructional materials so that teachers can learn to use them effectively to support student learning.
5. Technology can help with these policy recommendations (see Table 2.2 and its discussion). It is particularly important to provide examples of visual models for teaching and for teachers to confer with each other.	• Technology dollars for education need to be focused on the goals in this table, and free resources need to be identified and shared by teachers and experts.

Source: Prepared by the authors.

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Mathematics Learning in Latin America and the Caribbean

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In the preceding chapter, a large body of international research on mathematics teaching and learning was reviewed using the balanced teaching framework. The goal of the framework is to encourage learning progressions using student ideas, connecting these ideas to significant mathematics concepts, and applying other strategies to generate meaningful opportunities to learn in mathematics classrooms.

This chapter reviews the evidence from research in Latin America and the Caribbean (LAC) to understand what mathematics is learned in the region's primary schools today. It considers how students' attainment in mathematics in LAC is likely connected to the opportunities afforded them to learn specific mathematics topics, skills, dispositions, and mental routines, and how current curriculum policy is related to those opportunities. The chapter begins by examining students' performance on international tests, contrasting those attainments with the types of goals that their national educational systems set out for them, and then further considering specific areas of mathematics and specific populations of students that show disparate weaknesses and strengths in mathematics attainment.

In 2007, fourth grade students from more than 60 countries participated in the Trends in Mathematics and Science Study (TIMSS), a test measuring what they had learned in mathematics (Mullis, Martin, and Foy 2008). Two Latin American countries participated: Colombia and El Salvador. Students from both countries performed significantly below the midpoint of the test scale among the 10 countries with the lowest average achievement on the test. Indeed, every time a country from LAC has

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[1]   M.I. Khan and E. Villalobos assisted in the preparation of this chapter.

FIGURE 3.1
TIMSS 2007 TEST QUESTION, SAMPLE 1
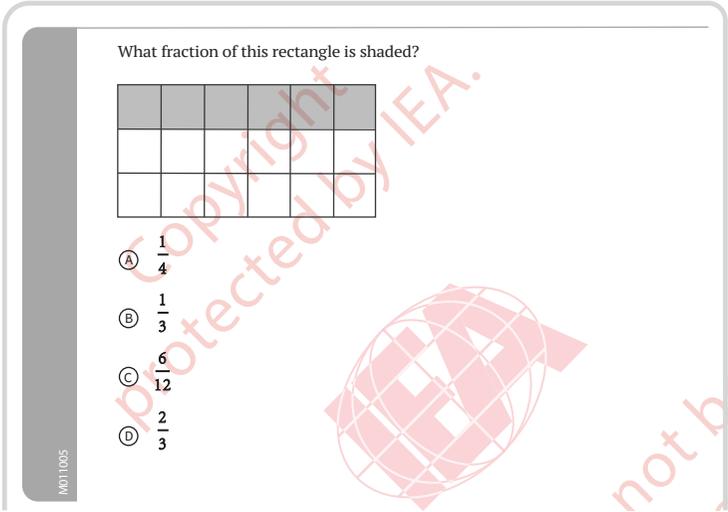
Source: TIMSS 2007 Assessment. Copyright © 2009 International Association for the Evaluation of Educational Achievement (IEA). Publisher: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.
Note: TIMSS: Trends in Mathematics and Science Study.

participated in TIMSS, it invariably has performed among the countries with the lowest average achievement levels.[2] An example of individual test questions intended to measure key areas of mathematics learning is represented in Figure 3.1. In this question, one of the few released to the public, students were asked to determine which fraction of the rectangle is represented by the shaded area.

This question proved difficult for Colombian and Salvadoran students, as can be observed in Figure 3.2. A large majority of students chose the incorrect answer, C, presumably because the sum of the shaded segments is six. Over 40 percent of students who took the TIMSS test across the globe were able to identify the correct answer, whereas in Colombia and El Salvador only about 10 percent of students were able to do so. This result is striking because in all LAC educational systems—and in Colombia and El Salvador in particular—national curriculum policy prescribes work

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[2] LAC participation in TIMSS has been low: Chile participated in 1999, 2003, 2011, and 2019, Argentina in 1995 and 2003, Colombia in 1995 and 2007, Mexico in 1995, and El Salvador in 2007.

FIGURE 3.2
PERFORMANCE OF COLOMBIAN AND SALVADORAN FOURTH GRADERS ON TEST QUESTION IN FIGURE 3.1 (PERCENT)
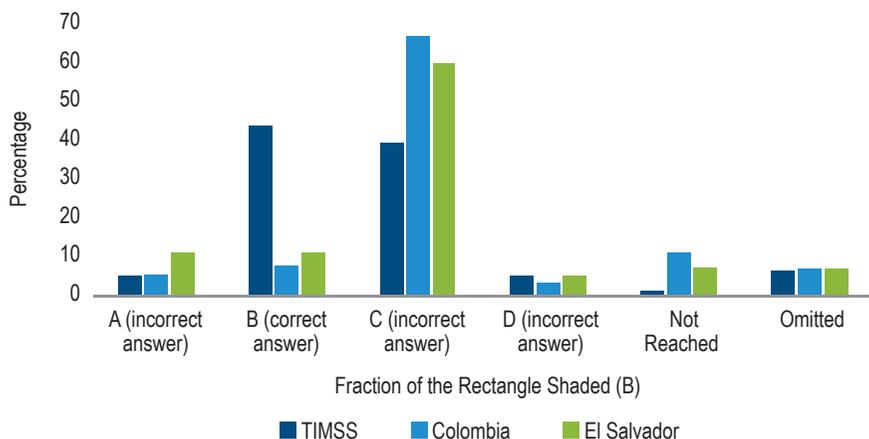
Source: Trends in Mathematics and Science Study (TIMSS), 2007.

with common fractions as a focal learning goal starting in about the third grade. The use of models, such as the one in the test question in Figure 3.1, is also promoted extensively, and third and fourth grade textbooks include examples and exercises that are very similar to this TIMSS test question.[3]

Although the findings of low average achievement are striking, there are, of course, some test questions that were easier for Colombian and Salvadoran fourth graders to solve. Yet, even on test questions that were relatively easy, these students scored lower than their international peers. For example, from among the TIMSS 2007 questions that have been released, one of the least difficult proved to be the question in Figure 3.3, which asks students to choose the set of numbers arranged from largest to smallest.

As can be seen in Figure 3.4, even on this question, which 40 percent or more of students in both Colombia and El Salvador were able to answer correctly, they performed worse than their international peers. On average, 20 percent more students were able to answer this question correctly in other parts of the world.

Once again, the question tests mathematics knowledge that is a required focus in early primary grades in LAC. For example, in the Salvadoran National

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[3] For example, the Colombian National Standards give modeling a prominent place among the five general processes in mathematical activities promoted for learning in primary grades (Colombia Ministerio de Educación Nacional 2006).

FIGURE 3.3
TIMSS 2007 TEST QUESTION, SAMPLE 2
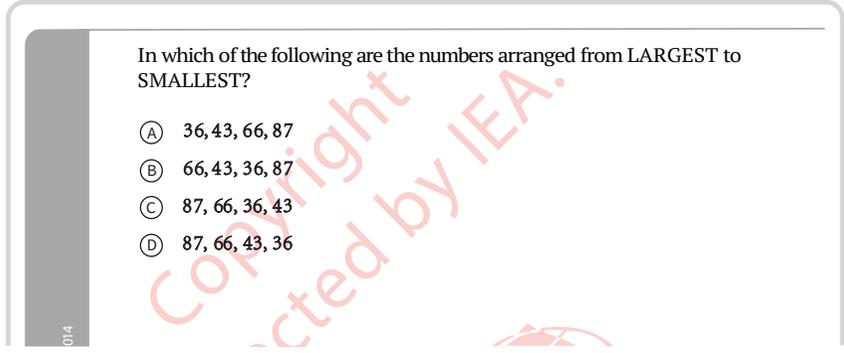
In which of the following are the numbers arranged from LARGEST to SMALLEST?

(A) 36, 43, 66, 87

(B) 66, 43, 36, 87

(C) 87, 66, 36, 43

(D) 87, 66, 43, 36

Source: TIMSS 2007 Assessment. Copyright © 2009 International Association for the Evaluation of Educational Achievement (IEA). Publisher: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.
Note: TIMSS: Trends in Mathematics and Science Study.

Program of Study for the Third Grade (in force since 2008), school children are expected to master counting and ordering numbers up to 9,999. Indeed, this is the first set of learning objectives mentioned (El Salvador Ministerio de Educación 2008, 56).

These illustrations give a glimpse of the scope of the educational challenges in primary school mathematics in the region. Most evidence

FIGURE 3.4
PERFORMANCE OF COLOMBIAN AND SALVADORAN FOURTH GRADERS ON TEST QUESTION IN FIGURE 3.3 (PERCENT)
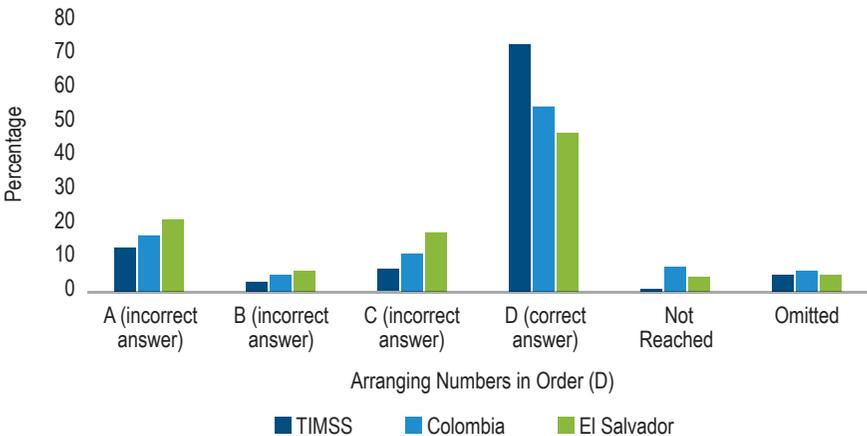
Source: Trends in Mathematics and Science Study (TIMSS), 2007.

indicates that LAC faces important and immediate challenges in creating opportunities for school children to successfully learn mathematics at levels comparable to their peers in other parts of the world. As will be shown in this chapter, some of these challenges may be related to differences in the curricular intentions of LAC countries relative to global peers. Yet there are other issues that can be attributed to significant difficulties in classroom implementation, inequities in the distribution of educational opportunities to different segments of the school population, and other structural factors.

Evidence of the challenges faced in the region does not come only from large-scale global tests in which LAC countries participate infrequently. However, this evidence is quite valuable because it offers the only available way to contrast the attainment of the region's students with students outside the region. Meanwhile, evidence restricted to the LAC region also suggests serious and pervasive challenges in average levels of achievement in mathematics.

The Third Regional Comparative and Explanatory Study (*Tercer Estudio Regional Comparativo y Explicativo* – TERCE) is a test designed to be aligned with curriculum expectations in the region. The blueprint for the test was developed after an analysis of the national curricula in each participating country (ICFES 2013) and thus was planned to closely line up with current curricular policies. In other words, the test was aimed at asking students mathematics questions regarding content that most participating nations intended for them to learn. Students are measured against a standard of performance designed to represent regional learning goals fairly. The TERCE assessed the third and sixth grades.

The TERCE found critically low average levels of mathematics performance among third grade students (UNESCO-OREALC 2015), even involving content that their educational systems emphasized in the curriculum. The test distinguishes four performance levels in mathematics, with Level I at the low end and Level IV at the high. A scant 8.3 percent of school children in the region performed at Level IV. Indeed, less than a quarter of the region's primary school students performed at either of the two top performance levels. Slightly over half the students in the region performed at the lowest performance level or below. A description of the results of the TERCE, including results at the national level, will be discussed later in this chapter.[4] It is important to note that even in the LAC

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[4] See also Figure 3.7 for descriptions of what the TERCE results mean in terms of the mathematics that school children are capable of doing.

countries with the highest average scores on the TERCE, approximately a quarter of third grade students are performing at the lower levels.

The TERCE offers the latest and most comprehensive evidence of the important weaknesses in mathematics attainment of primary school children in LAC. The evidence of low achievement in this critical domain is pervasive, long-standing, and documented in various studies. There is evidence of substantial differences associated with whether a child attends a public or a private school. This achievement gap is even recognized formally in some countries. For example, in Brazil, the Mathematics Olympiads hold public school students to completely different (and lower) standards than their peers in private schools (Biondi, Vasconcellos, and Menezes-Filho 2012). There are also countries that show substantial gender-related achievement gaps favoring boys. This is the case in Chile, for example (Zambrano Jurado 2013), where the gender differences may in part be related to gender stereotypes regarding interest and ability in mathematics, stereotypes that are possibly already in place when boys and girls are only 5 years old (del Río and Strasser 2013).

There is also a growing body of information regarding structural inequities in the distribution of opportunities to learn. The quality of school experiences in mathematics afforded to school children in rural areas differs significantly from that in urban areas. Such inequity also exists between ethnic and linguistic groups, as well as between students from affluent or poor families—mirroring structural inequalities in the distribution of wealth, services, and other opportunities that pervade the societies in the region (Cueto, Ramirez, and Leon 2006; Ramirez 2006; Valverde and Näslund-Hadley 2010).

The use of computers in mathematics classrooms has received attention as a promising way to overcome some of these challenges, and this book arises from a concern that its promise needs to be better understood. However, existing evidence on the effectiveness of computers in primary school mathematics in LAC is mixed. Analysis of data from the second regional test in LAC (*Segundo Estudio Regional Comparativo y Explicativo* – SERCE) suggests that whereas students with computers at home perform better in mathematics, when they use those computers to do their homework, they actually perform worse on the test (Carrasco and Torrecilla 2012). A study in Brazil found that the students of primary school teachers who use computers and the Internet as pedagogical tools perform a bit better in mathematics than their peers, but also that students in schools with computer labs have significantly lower average achievement levels in mathematics than their peers in schools without labs (Sprietsma 2012).

3.1 What Mathematics Are Primary School Students in Latin America and the Caribbean Expected to Learn?

Curriculum policy in LAC merits special attention. In curriculum studies, the official expectations regarding mathematics learning promoted by ministries, secretariats, or other official national, state, or provincial educational agencies are termed the "intended curriculum." The intended curriculum is embodied in national curricula or programs of study, standards, frameworks, or other similar documents, and has a primary role in defining learning goals for the educational system. Educational policymakers put forth documents to guide the experiences of students in classrooms. These curriculum policies promote, constrain, and guide the opportunities to learn that take place in mathematics classrooms. This is the "implemented curriculum." They have a demonstrated, measurable impact on the final category, which is the "attained curriculum," that is, the mathematics that school children effectively master (Schmidt et al. 2001; Valverde et al. 2002).

To explore the question of what LAC educational systems intend students to learn, this chapter takes advantage of the rich data set available at the International Curriculum and Textbook Archive (ICATA) at the University at Albany, State University of New York. The archive specializes in documents pertaining to the intended curriculum in primary school mathematics (and reading) in developing countries. It includes national curricula, syllabi, programs of study, teacher's guides, and officially sanctioned textbooks from developing countries around the world, and it is especially strong in its representation of the LAC region.[5] Given the scope of this chapter, the focus of the analysis here is on official curriculum policy documents such as national curricula, national scope and sequence policies, and so on. Each of these documents was coded, page by page, by trained coders focusing on the mathematics content intended for instruction and the expectations regarding what students should be able to do with that content. The coding procedure follows the one developed for a large-scale international study of curricula, with an extended version of the coding frameworks used for that study (Survey of Mathematics and Science Opportunities 1992; Schmidt et al. 1997b; Valverde et al. 2002).

In previous international comparative studies of the intended curriculum, important insights were derived through comparisons of the

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[5]   For more information on ICATA and its continuously expanding archive of primary school curriculum documents and the dataset compiled by its coding, contact the lead author of this chapter at gvalverde@albany.edu.

number of mathematics topics intended for students to learn in each grade (Schmidt et al. 1997a, 1997b). Comparing the number of intended topics provides a first measure of what might be termed the "breadth versus depth" problem. Those previous studies found that higher-achieving countries predominantly intended for the coverage of fewer mathematics topics to be covered at greater depth than countries with mediocre or poor average performance. LAC stands out as a uniquely different case. The problem was first observed in a previous replication of the TIMSS curriculum analysis in Chile (Valverde 2004): Chile's intended curriculum at that time had even fewer topics than high-achieving countries with "focused" curricula. However, rather than "focused," the Chilean curriculum looked "shallow," to use the term employed in the literature.

An examination of recent curricula across many LAC countries shows further evidence of this troubling phenomenon. For example, Table 3.1 compares aspects of the intended curriculum for grades 5 and 6 across a selection of LAC countries. It also compares them against a prominent reform-oriented curriculum in the United States, the Common Core State Standards (CCSS Initiative 2013), that attempts to increase the level of depth and focus in the U.S. intended curriculum.[6]

Upper-primary-school students in LAC are expected to learn as many topics in number as are students following the U.S. Common Core State Standards. Indeed, many of these topics represent foundational aspects of arithmetic that many countries continue to stress through the upper primary grades. More challenging content, such as integers and rational and real numbers, is commonly omitted in LAC at this grade level. These are intended in the Common Core Standards, and certainly large-scale studies of mathematics curricular intentions indicate that these contents are typically intended at this grade level in countries with higher average performance on international tests (Valverde 2000; Schmidt et al. 1997b).

Further evidence of important differences in the regional curricular intentions of LAC and its international peers can be seen if attention is shifted to key areas of mathematics that distinguish high-performing countries.

Table 3.2 shows data on curricular intentions in the mathematics areas of proportionality and functions, relations, and equations. Few countries intend students at this grade level to learn about proportionality problems. A more striking difference—and one with a more likely impact on levels

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[6] Using this document as a point of comparison has the additional advantage that the Common Core Standards are based on research findings from large-scale cross-national mathematics tests.

TABLE 3.1
TOPICS IN THE MATHEMATICS AREA OF NUMBERS INTENDED IN FOURTH AND FIFTH GRADE CURRICULA IN SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES

	ARG	CHL	COL	CRI	DOM	MEX	PRY	PER	BHS	JAM	United States (Common Core)
Whole Numbers											
Meaning	X	X	X	X	X	X	X		X	X	X
Operations	X	X	X	X	X	X	X		X	X	X
Properties of operations	X	X	X	X	X		X	X	X	X	X
Fractions and Decimals											
Common fractions	X	X	X	X	X	X	X	X	X	X	X
Decimal fractions	X	X	X	X	X	X	X	X	X	X	X
Relationships of common and decimal fractions	X		X	X	X	X	X	X	X	X	X
Percentages	X		X	X	X	X	X		X	X	
Properties of common and decimal fractions	X								X		X
Integer, Rational, and Real Numbers											
Negative numbers, integers, and their properties				X						X	X
Rational numbers and their properties			X								X
Real numbers, their subsets and properties											X

Source: International Curriculum and Textbook Archive.

TABLE 3.2
INTENDED TOPICS IN SELECT MATHEMATICS SUBAREAS IN FOURTH AND FIFTH GRADE CURRICULA IN SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES

	ARG	CHL	COL	CRI	DOM	MEX	PRY	PER	BHS	JAM	United States (Common Core)
Proportionality											
Proportionality concepts	X		X	X	X	X	X	X	X	X	X
Proportionality problems	X					X		X	X	X	X
Properties of operations											
Slope and trigonometry											
Linear interpolation and extrapolation											
Functions, Relations, and Equations											
Patterns, relations and functions			X						X		X
Equations and formulas									X	X	X

Source: International Curriculum and Textbook Archive.

of achievement in mathematics—is the general absence of intentions to cover functions, relations, and equations content. This indicates the omission from primary grades of foundational content that is preparatory for further learning of algebra. Countries with higher levels of average student achievement begin such introductory work in algebra much earlier and with more sustained focus. As seen in the table, the intended curriculum in LAC rarely promotes such educational opportunities.

This is one of the most striking aspects that set the curricular intentions of Latin American (but not the two Anglophone Caribbean countries in the sample) primary education policy apart from international peers, and one that may help explain differences in mathematics achievement. The U.S. Common Core Standards aim to prepare students from kindergarten onward for algebraic thinking. It may seem strange to intend for students that young to learn algebra. What is meant, however, is not the rigorous formal algebra common in secondary school, but rather foundational aspects of algebraic reasoning that are essential to a variety of important quantitative skills throughout subsequent years of schooling. Recognition and use of patterns and relations are two of the foundational components of mathematical reasoning, key to progressing toward the formal use of equations, formulas, and functions and other numeracy skills in later grades. LAC countries rarely have curriculum policies that call for learning progressions that begin with such content and lead to increasingly more challenging skills in algebraic thinking. A few countries do intend for the lower primary grades to include this content, and it is instructive to consider how curriculum policy in those nations structures these intentions.

In grades one through three in Colombia, one strand of the national standards, Variational Thinking and Algebraic and Analytical Systems (*Pensamiento variacional y sistemas algebraicos y analíticos*), has four goals, followed by five goals in grades four and five (Table 3.3).

Table 3.3. shows that the Colombian intended curriculum includes a progression of learning goals that build up to increasingly more challenging material.

In The Bahamas (Ministry of Education 2010), curricular intentions in primary school mathematics are specified for each individual grade, rather than by groups of grades, as in the case of Colombia. The national Scope and Sequence for Primary School Mathematics puts forth objectives for "patterns, functions, and algebra" into two subgoals, one of which is shown in Table 3.4 as an example.

These examples show approaches to curriculum policy that are uncommon in the region, where, as has been shown, most countries do not focus

TABLE 3.3

CURRICULAR INTENTIONS IN THE AREA OF PATTERNS, RELATIONS, FUNCTIONS, AND EQUATIONS IN COLOMBIA'S NATIONAL STANDARDS

Grades 1 through 3	Grades 4 and 5
• Recognize and describe regularities and patterns in different contexts	• Describe and interpret variations represented in graphs
• Qualitatively describe situations of change and variation	• Predict patterns of variation in numerical, geometric, or graphical sequences
• Recognize and generate equivalencies between numeric expressions	• Analyze and explain relationships of dependency between quantities that vary with certain regularity over time in economic, social, and natural science contexts
• Build numerical and geometrical sequences using properties of numbers and geometric figures	• Represent and relate numeric patterns with tables and verbal rules.
	• Build numerical equalities and inequalities as representation of relationships between different data

Source: Colombia Ministerio de Educación Nacional (2006).
Note: Material translated, summarized, and organized in tabular format by the authors.

on this content area in mathematics. The examples from Colombia and The Bahamas suggest possible approaches.

Recognizing and using patterns is a key component of mathematical reasoning. Returning to Table 3.2, one can observe another area of mathematical reasoning where regional curricular intentions for primary school mathematics differ from intentions in other parts of the world. The ability to recognize proportional situations (proportional reasoning), and understand the multiplicative relationship between quantities in these situations is central to the eventual understanding in later grades of algebraic expressions, coordinate graphs, trigonometry, and so on. Yet, although it is common in the region to promote the learning of some proportionality concepts, it is much less common to intend that students be able to understand ratio concepts and use ratio reasoning to solve problems. This is one of the goals for upper primary school common in better-achieving countries, and it is also a goal present in the U.S. Common Core Standards.

This chapter has identified a few of the most striking contrasts between curricular intentions in LAC and countries outside the region that have been more successful in promoting higher levels of attainment in primary school mathematics, as measured in large-scale cross-national tests. "Shallow" areas—including important numbers concepts, foundational aspects of algebraic thinking, and meaningful content in proportionality—are key

TABLE 3.4
CURRICULAR INTENTIONS IN THE AREA OF PATTERNS, RELATIONS, FUNCTIONS, AND EQUATIONS IN THE BAHAMAS
Sub-goal 2: Use algebraic and analytical methods to identify and describe patters and relationships in data, solve problems and predict results.

Objectives	Preschool	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7
1. Sort, classify, and order objects by size, amount, and other properties.	I	D	A	A	M	D	D	A
2. Identify, describe, and extend various patterns such as sequences of sounds, shapes, or simple numeric patterns, and analyze how both repeating and growing patterns are generated.	I	A	A	A	M	D	D	A
3. Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.				I	D	D	M	A
4. Model situations that involve addition and subtraction of whole numbers, using objects, pictures, and symbols.				I	D	D	M	A
5. Identify and construct rectangular, triangular, oblong, and L-shaped numbers.					I	D	D	M

(continued on next page)

TABLE 3.4 *(continued)*

CURRICULAR INTENTIONS IN THE AREA OF PATTERNS, RELATIONS, FUNCTIONS, AND EQUATIONS IN THE BAHAMAS
Sub-goal 2: Use algebraic and analytical methods to identify and describe patters and relationships in data, solve problems and predict results.

Objectives	Preschool	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7
6. Describe qualitative change using various attributes.	I	D	D	M	A	A	D	D
7. Describe, extend, and generalize about geometric and numeric patterns.		I	D	M	A	A	D	D
8. Represent and analyze patterns and functions using words, tables, and graphs.			I	D	M	A	A	D
9. Identify and illustrate general principles and properties as commutative, associative and distributive, and use them to compute with whole numbers.				I	D	D	M	M

Source: Ministry of Education (2010).
Key: I= Introduce, D- Develop, M- Maintain, A- Advance.

to mathematical reasoning. The deficit in expectations is further confirmed when one looks at curriculum policy for how students are expected to reason with and apply the mathematics content they learn.

Students are expected to not only gain mathematical content knowledge but also to think and utilize that knowledge. These performance expectations are central to countries' curricular visions and intentions.

Table 3.5 lists the intentions of several LAC countries in the least-demanding areas of performance—knowing and using routine procedures—and compares these against the expectations of the TIMSS test (Mullis et al. 2005, 2009).[7]

As can be observed, expectations in the region align well with the expectations assessed by the fourth grade TIMSS tests. The use of equipment, including computational devices such as calculators and computers, is widely intended across the region. There are, however, variations across LAC: Jamaica and The Bahamas intend for all the skills assessed by the TIMSS to be learned, whereas some countries do not intend for students to perform routine procedures in graphing (Colombia, the Dominican Republic, Mexico) or using instruments (Colombia, Paraguay) at this grade level. Evidently, this would affect the likelihood that such content is taught, and, consequently, achievement levels in this area.

From Table 3.6 it is also apparent that in the important area of problem-solving, countries in the region have similar expectations as do the TIMSS fourth grade tests, with some noteworthy variations. Here, curiously, The Bahamas is the exceptional case, with fewer performance expectations in the area than the other countries analyzed. The presence of expectations for problem-solving represents an important and positive change. In the late 1990s, such expectations were largely absent from the curriculum policies in the region (Schmidt et al. 1997b).

Expectations in LAC also differ from those in other parts of the world for mathematical reasoning, as shown in Table 3.7. However, there are important regional variations. Argentina, Chile, Jamaica, and, to some extent, Colombia, have greater expectations for student performance in this area. Other countries show a curricular shallowness in this area, with important aspects of mathematical reasoning omitted, suggesting a lesser likelihood that children will have the opportunity to learn them.

Another important contrast between LAC and other parts of the world has to do with the grades when the introduction and development of learning goals in key curricular areas is intended. Here one can observe another

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[7] A list of analyzed curriculum documents is available from the corresponding author (gvalverde@albany.edu). All curriculum documents were officially in force in 2015.

TABLE 3.5
PERFORMANCE EXPECTATIONS FOR ROUTINE PROCEDURES FOR FOURTH AND FIFTH GRADES IN SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES

	ARG	CHL	COL	CRI	DOM	MEX	PRY	PER	BHS	JAM	TIMSS
Knowing											
Representing	X	X	X	X	X	X	X			X	X
Recognizing equivalents	X	X	X	X	X	X	X	X	X	X	X
Recalling mathematical objects and properties	X	X	X	X	X	X	X	X	X	X	X
Using Routine Procedures											
Using equipment										X	
Using instruments (e.g., measuring instruments)	X	X		X	X	X		X	X	X	
Using computational devices	X	X	X	X		X		X	X	X	
Performing Routine Procedures											
Counting		X		X	X			X	X	X	X
Computing	X	X	X	X	X	X	X	X	X	X	X
Graphing	X	X		X			X	X	X	X	X
Transforming	X	X	X					X	X	X	X
Measuring	X	X	X	X	X	X	X	X	X	X	X
Using More Complex Procedures											
Estimating	X	X	X	X	X			X	X	X	X
Using data	X	X	X	X	X	X	X		X	X	X
Comparing	X	X	X	X		X		X	X	X	X
Classifying	X	X	X	X	X	X	X		X	X	X

Source: International Curriculum and Textbook Archive.
Note: Trends in Mathematics and Science Study (TIMSS) performance expectations pertain to the fourth grade tests.

important element of the story. One difficulty when studying curriculum policies in LAC is that some countries, such as Colombia, do not specify curriculum by grade, but rather by group of grade levels (in the Colombian case, grades 1 to 3, 4 and 5, 6 and 7, etc.). Thus, it is unclear when content is intended for introduction. However, examining the countries that do specify expectations by grade, one finds that in fractions, expectations in Bermuda are representative of the region. Figure 3.5 shows how the sequence of expectations regarding the instruction of common fractions differs between Bermuda and those countries that were high performers on the TIMSS tests. Students in Bermuda are intended to encounter common fractions in the third grade, while in other countries outside LAC that introduction typically takes place at the preprimary level. Thus, by the third grade, students in high-performing countries outside the region would have already had two to three years of instruction in this topic.

The examination here of the intended curriculum in LAC has uncovered mathematics areas for which expectations in the region are very similar to expectations in other parts of the world—for example, in whole number topics, mastering routine procedures, and problem-solving. However, the analysis also finds indications of shallowness in expectations for students in important elements of mathematical reasoning. Children in LAC are likely to be introduced to topics such as fractions later than their

TABLE 3.6
PERFORMANCE EXPECTATIONS FOR INVESTIGATING AND PROBLEM-SOLVING FOR FOURTH AND FIFTH GRADES IN SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES

	ARG	CHL	COL	CRI	DOM	MEX	PRY	PER	BHS	JAM	TIMSS
Investigating and Problem-solving											
Formulating and clarifying problems and situations	X	X	X	X	X	X	X	X		X	
Developing strategy	X	X		X	X		X	X		X	X
Solving	X	X	X	X	X	X	X	X	X	X	X
Predicting		X	X	X		X		X		X	X
Verifying	X	X			X						

Source: International Curriculum and Textbook Archive.
Note: Trends in Mathematics and Science Study (TIMSS) performance expectations pertain to the fourth grade tests.

TABLE 3.7

PERFORMANCE EXPECTATIONS FOR MATHEMATICAL REASONING FOR FOURTH AND FIFTH GRADES IN SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES

	ARG	CHL	COL	CRI	DOM	MEX	PRY	PER	BHS	JAM	TIMSS
Mathematical Reasoning											
Developing notation and vocabulary		X				X				X	X
Developing algorithms	X	X								X	X
Generalizing	X		X							X	X
Conjecturing	X	X	X			X		X	X	X	X
Justifying and proving	X	X	X							X	X
Axiomatizing	X	X									

Source: International Curriculum and Textbook Archive.
Note: Trends in Mathematics and Science Study (TIMSS) performance expectations pertain to the fourth grade tests.

FIGURE 3.5

GRADES WHEN COMMON FRACTIONS ARE TO BE INTRODUCED: BERMUDA VERSUS THE TOP 70 PERCENT OF INTERNATIONAL PEERS THAT APPLY THE TIMSS
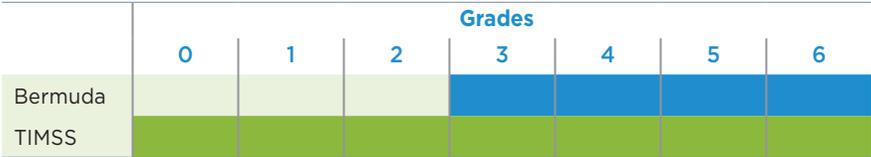
		Grades						
	0	1	2	3	4	5	6	
Bermuda				■	■	■	■	
TIMSS	■	■	■	■	■	■	■	

Source: International Curriculum and Textbook Archive (ICATA) at the University at Albany, State University of New York.
Note: TIMSS: Trends in Mathematics and Science Study.

peers in other parts of the world. These elements help gauge the scope of the challenges mathematics education policy faces in the region.

To further explore possible relationships between a shallow intended curriculum and low levels of mathematics achievement, one can return to the example of the two TIMSS test questions discussed at the beginning of this chapter. As observed, both questions belong to content areas that receive some attention in educational policy in LAC. However, they are notably more difficult for students to solve in LAC than outside the region.

Some reasons for this difference may be found in the pattern of omissions in mathematical reasoning. Yet this cannot be the full explanation.

The initial discussion of the fractions question in Figure 3.1 mentioned that common fractions are widely intended to be learned by the fourth grade across LAC, and thus poor achievement on such questions is difficult to understand. However, an examination of the question suggests an explanation worth further probing. To answer this question, students must not only have experience with common fractions (1/3), but also prior exposure to equivalent fractions and their representation in models of area and/or mathematical reasoning. Perhaps children know how to recognize fractions represented in models of area—and therefore can count the number of shaded squares, relate them to the full number of squares, and come up with the figure of 6/18. Not finding that option, they perhaps choose the closest available option: 6/12 (which is option C, by far the preferred incorrect option). This is likely because they cannot recognize that 6/18 is an equivalent fraction to 1/3. Yet this is not the only possible way to arrive at the correct answer, as an alternative would be to observe that the model could also be interpreted as being made up of three rows, and one row out of the three is shaded—thus 1/3. A fundamental misunderstanding of the way in which the model works may also be at play: students who may simply be adding the number of the shaded and the unshaded squares separately and coming up with the sums of 6 and 12, respectively, choose 6/12. A lack of opportunities to develop proficiencies of this type in mathematical reasoning might explain the difficulty of the question.

The second test item, in Figure 3.3, also shows a likely relationship to the diagnosis of weaknesses in curricular policy. Even though the intended curriculum in most LAC countries states that students should be able to count and order numbers up to 9,999, it is possible that most teachers instruct students to order numbers by having them compare pairs of numbers and determine which is greater (or lesser) than the other. The test question gives a sequence of four numbers and requires the student to recognize this as a pattern, rather than a simple comparison between two numbers. As shown in Table 3.2, one notes the common absence of work with patterns in the intended curricula in LAC. It is likely that a student who has not done work with patterns will look at a sequence of numbers and connect it with the idea of counting. Since counting is most often done in order from small to large numbers, the students will choose the option that shows numbers going from smallest to largest, which was indeed one of the incorrect answers preferred by students (option A).

There is then yet another part of the curriculum in which "shallowness" may be detected: the attained curriculum. When low average levels

of achievement are noted in areas that are not intended for instruction in LAC, there is one straightforward explanation. However, when one observes low levels of attainment in mathematics content that is intended for instruction, and present in a substantial portion of textbooks as well as in important areas of teacher pre- and in-service training, shallowness in attainment must be related in part to shallowness in implementation. Having examined the potential relationship between curriculum intentions and performance on mathematics test items, the next section turns to a deeper examination of mathematics achievement—the attained curriculum. The paucity of contemporary research on mathematics classroom practices in LAC leaves the key question of how intention is translated into attainment largely unexplored. This is a critical research priority for the future.

3.2 Further Exploration of Mathematics Achievement in Latin America and the Caribbean

The central argument in the first half of this chapter is that the attained curriculum—defined as the actual learning that takes place among students—is related to the curricular goals of the education system itself. Students should perform better in areas that are emphasized in the intended curriculum, but will likely struggle in areas that receive little or no attention in official textbooks and teacher lesson plans or are introduced in later grades. Using data from international studies like the TIMSS, it seems clear that at least part of the explanation for why LAC countries have generally performed poorly vis-à-vis other regions is related to differences in official curriculum goals and priorities.

Differences in the intended curriculum go beyond simple yes-no dichotomies, as in "do they teach this or not?" The concept of curriculum shallowness looms large, since student achievement levels can be low even in content areas that are officially part of the intended curriculum for that grade. Compared with instances of curricular exclusion—meaning students simply do not see particular content—shallowness is a more complex phenomenon. As the name suggests, shallowness can be related to the amount of time spent on a given topic, but it also refers to implementation and a potentially large number of "conditioning factors," such as the quality and relevance of learning materials and the ability of teachers to adequately instruct their students (this topic is covered in more detail in Chapter 4).

Here we return to the TERCE data from 15 countries in LAC for an overview of student achievement levels in sixth grade mathematics. The earlier extra-regional comparisons, using the TIMSS, allowed for considering

student performance in curricular areas that are not commonly intended in LAC countries. With the TERCE, one can instead examine how students perform on a test designed to measure what *is* commonly intended in the region's mathematics curricula.

The empirical review has three core objectives. The first is to summarize overall student performance in sixth grade mathematics in the region based on intercountry comparisons of different measures of achievement levels. The second is to examine learning gaps between boys and girls, ethnic groups, public and private schools, and different socioeconomic groups. Taken together, these first two objectives provide the global overview of mathematics achievement that complements the preceding survey of the intended curriculum. This chapter provides the current backdrop of mathematics education in the region—that is, the current "state-of-play" in which all new initiatives are taking place.

The third objective is to go beyond the global summaries of mathematics achievement and understand more about the potential linkages between curriculum and student learning in LAC. The TERCE tests are designed to cover common curriculum areas in participating countries. This essentially rules out cross-country comparisons of achievement on the basis of intended goals. Nevertheless, differences in student performance in various content areas (i.e., geometry, numbers, etc.) and cognitive areas (i.e., identification, complex problem-solving) that make up the TERCE sixth grade mathematics exam do make it possible to consider curricular-driven explanations for student performance. Once again, the concepts of curriculum shallowness—and implementation quality—come into play, although the analysis is regionwide.

3.2.1 Global Summaries of Overall Sixth Grade Mathematics Achievement

Figure 3.6 begins with the overall averages for sixth grade mathematics in the 15 countries that participated in the 2013 TERCE assessment. The summary is based on the scaled score that was created by the TERCE, which is set at a global (across all countries) mean of 700 points, with a standard deviation of 100. Averages are presented for both the national (overall) and urban school samples (see UNESCO-OREALC, 2015, for more detailed summaries). The results show that Chile, Mexico, and Uruguay have the highest national averages in mathematics (above 750 points). This group is followed by Costa Rica, Argentina, and Peru, which have averages above 720 points. Brazil, Colombia, and Ecuador have averages very near the overall mean of 700 points (see dotted line). There is a significant drop-off

after Ecuador, as Guatemala and Honduras come next with averages of 660–680 points. The final group of countries includes Panama, Nicaragua, Paraguay, and the Dominican Republic, all of which have averages below 650 points.

The intercountry learning gaps are considerable, since the national averages in Chile, Mexico, and Uruguay are more than one standard deviation higher than the averages for the four lowest-scoring countries. These kinds of gaps can potentially be explained by any number of factors, including differences in socioeconomic background and access to learning materials (see Chapter 4). But the large gaps also provide the first glimpse of likely differences in curriculum implementation—or, put differently, the low scores strongly suggest different degrees of curriculum shallowness across LAC classrooms. Again, it must be restated that the TERCE tests—unlike larger international assessments like the TIMSS—are based on a study of commonalities in curriculum that is carefully validated beforehand across all participating countries. So, there should be very little variation among participants in terms of the intended curriculum and its representation on the tests. This topic is returned to below in the analysis of TERCE mathematics test content and cognitive skill subdomains.

Are some countries doing better or worse than expected? This question is not easily answered, but Table 3A1.1 in Annex 3.1 compares TERCE performance against a measure of national wealth (gross national income [GNI] per capita). Based on this comparison, a handful of countries are doing better than expected: for example, Peru ranks ninth in GNI, but has the fifth-highest average on the TERCE. At the other extreme, Panama and the Dominican Republic perform at lower levels than what their national wealth would predict. This is a simplistic method for assessing national performance, but it does highlight one of the fundamental realities of comparative education analysis: countries are not locked into a given performance level based on their national wealth or development level. Specific policies and reforms can make a difference (Carnoy, Gove, and Marshall 2007), and this, of course, includes efforts to improve curriculum implementation.

Given the focus of this book on urban schools, Figure 3.6 also presents the national averages for urban samples only. The ordering of countries from top to bottom is virtually identical. In most cases the difference between urban and rural areas (calculated separately) is about 25–35 points, or 0.25–0.35 standard deviations, with relatively larger differences in Peru and Guatemala.

The discussion so far has focused on the TERCE scale score (average 700 points), which is a common way of presenting results of international

FIGURE 3.6

**SIXTH GRADE MATHEMATICS ACHIEVEMENT AVERAGES IN 15
TERCE COUNTRIES, OVERALL AND URBAN SAMPLES (MEAN = 700,
STANDARD DEVIATION = 100)**
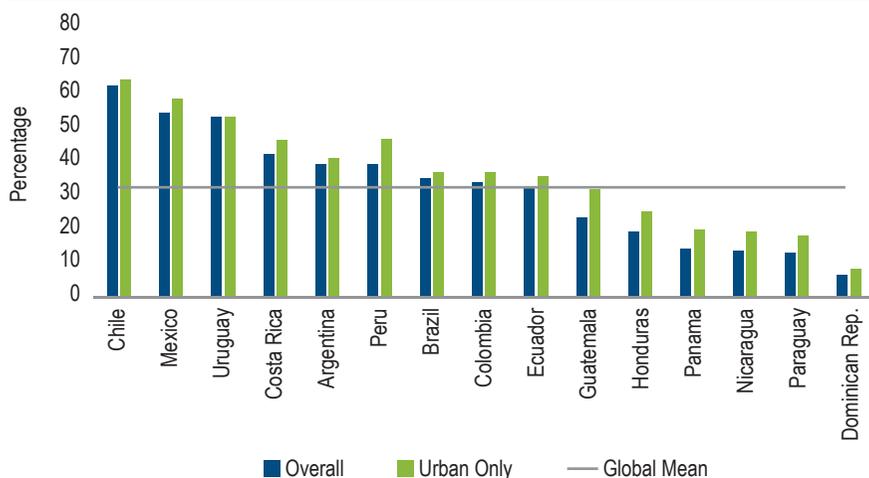
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

assessments of student achievement. Scale scores are useful for making quick comparisons of averages across countries (as in Figure 3.6) and examining differences between subgroups (boys-girls, urban-rural, etc.). But the scale score is not informative regarding what students can do, which is ultimately the most important piece of information generated by assessments. Therefore, most large-scale assessments also provide proficiency-scale summaries.

Figure 3.7 summarizes the TERCE proficiency-scale results for sixth grade urban students. TERCE curriculum experts defined four levels of proficiency in sixth grade mathematics (see UNESCO-OREALC, 2015, for more details on this process). Level I students have limited proficiency and are at best capable of answering items that require basic problem-solving skills, like reading data from a table or graph. Level IV students, on the other hand, have demonstrated proficiency in the most challenging aspects of the TERCE exam, including complex problem-solving related to conversions of units, fractions, and interpreting data from a table or graph. Levels II and III refer to subsets of demonstrated skills (based on correct answers on the TERCE) that are in between Levels I and IV.

The results in Figure 3.7 are a sober reminder of the challenges facing education systems in LAC—including high-scoring countries. They provide

FIGURE 3.7

PERCENTAGE OF SIXTH GRADE URBAN STUDENTS BY MATHEMATICS PROFICIENCY LEVELS

Country	Level I (lowest)	Level II	Level III	Level IV (highest)
Chile	15.1	39.4	26.3	19.3
Mexico	19.4	39.9	25.1	15.6
Costa Rica	24.6	49.8	19.3	6.2
Uruguay	26.7	36.1	24.0	13.2
Perú	28.5	43.5	18.6	9.4
Argentina	34.8	43.4	16.7	5.1
Colombia	37.4	44.0	14.6	4.0
LAC Average	37.8	38.8	15.6	7.8
Brazil	37.9	44.7	13.1	4.3
Ecuador	41.0	43.9	11.7	3.4
Guatemala	45.7	40.0	11.5	2.9
Honduras	55.0	35.5	7.8	1.8
Panama	58.7	35.0	5.0	1.2
Paraguay	62.9	30.0	5.9	1.2
Nicaragua	63.2	33.2	4.1	0.5
Dominican Rep.	77.6	20.6	1.7	0.2
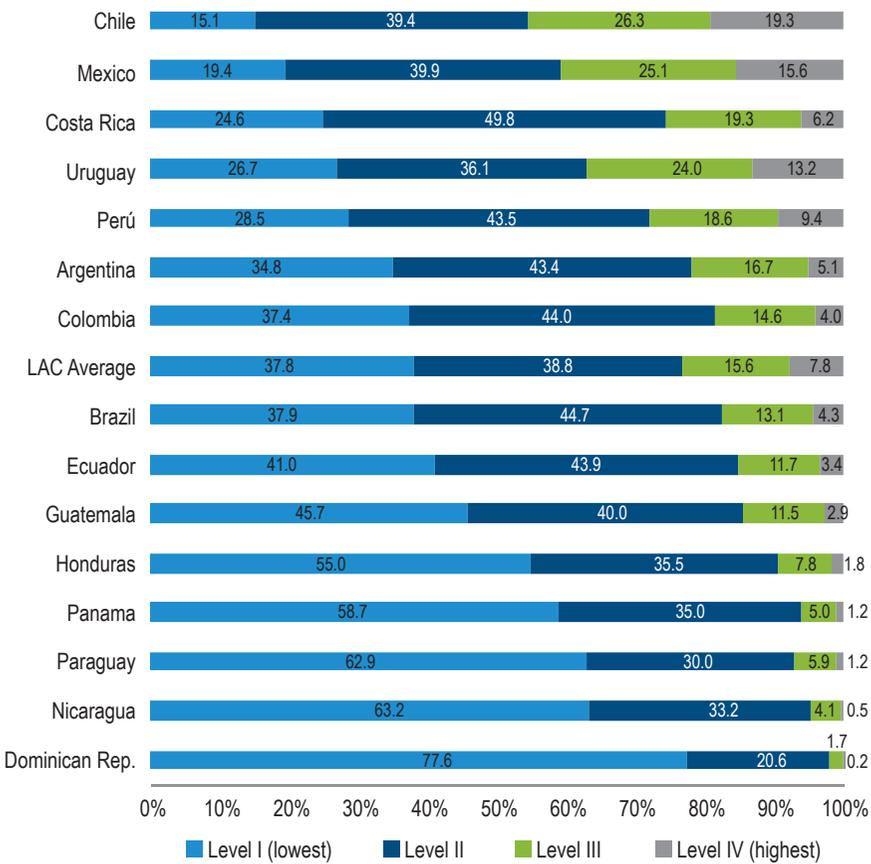
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

a useful counterpart to the results from large-scale assessments conducted across regions (i.e., the TIMSS) that were presented earlier. Even when restricting the analysis to urban areas, most sixth grade students are performing at the lowest levels. Across the entire region, roughly 75 percent of students are in Level I or II, compared with only 15.6 percent in Level III and 7.8 percent in Level IV.

The differences across countries are, not surprisingly, very large. Only 15.1 percent of Chilean urban sixth grade students scored in the lowest proficiency level (I), compared with nearly 80 percent of Dominican Republic students. For high-scoring students, the gaps are also quite large, as roughly 10–20 percent of students in high-scoring countries attained Level

IV status, compared with less than 5 percent of all urban students in the eight countries with averages below the LAC average.

The large number of students in Level I in most TERCE participant countries strongly suggests that students are far behind the expected achievement level for the end of the primary school cycle. This result, in turn, has three larger implications. First, it highlights the inherent challenges in maintaining quality (i.e., learning) while expanding access, as several of the low-scoring countries have made important progress in recent years in increasing primary school completion rates. Second, the concentration of students in the lower proficiency levels raises concerns about the preparation level of students entering lower-secondary (middle) school, especially given the high transition rates between primary and secondary education in urban settings. And, finally, the low scores point to shortcomings in the implemented curriculum in the LAC region, which, in turn, requires digging deeper to see what kinds of mathematics tasks students are struggling with the most, a question that will be addressed later in this chapter.

Despite the troubling state of student achievement levels in most countries in the region, the review of overall mathematics achievement can be concluded with a bit of positive news: student achievement levels do appear to be improving. The tests that were used in the TERCE are comparable with the earlier assessments (SERCE) administered by the Latin American Laboratory for Assessment of the Quality of Education (LLECE) through a process known as test equating (some common items were included on both the SERCE and TERCE). As described by the LLECE in its 2014 report (UNESCO-OREALC 2015), the comparisons show that student achievement levels in mathematics in grades 3 and 6 have improved in most countries. For sixth grade, the overall (or average) improvement between the SERCE and TERCE is about 20 points, or 0.20 standard deviations; for third grade, the difference is just over 30 points, or 0.30 standard deviations. Also, the proportions of students in the lower levels (I and II) have declined in most countries. These are encouraging findings that are hopefully indicative of a positive trend in the region that will continue.

3.2.2 Mathematics Learning Gaps

The review of mathematics achievement in LAC continues with a summary of achievement gaps by student gender, ethnicity, and socioeconomic status, as well as type of school administration (public/private). The focus is still on sixth grade mathematics in urban settings based on the TERCE

FIGURE 3.8

DIFFERENCES IN SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS BY GENDER, ETHNICITY, SCHOOL TYPE, AND SOCIOECONOMIC STATUS

a. Male (versus female) mathematics advantage in standard deviations

b. Nonethnic Group (versus Ethnic group) mathematics advantage in standard deviations
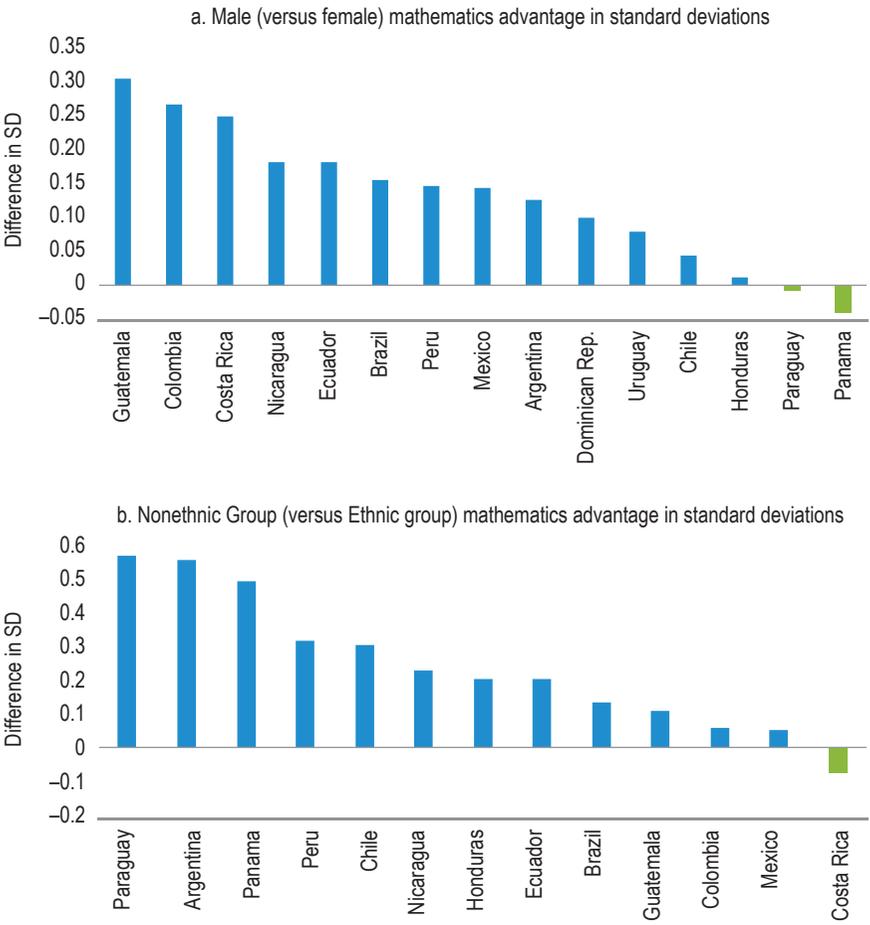
tests. Detailing these gaps—and comparing them across the region—adds to evidence about differences in opportunities to learn.

Figure 3.8 summarizes the results for four sets of comparisons, restricted in all cases to urban schools: boys versus girls, students who describe themselves as members of an ethnic group versus those who do not, public versus private administrations, and poorest versus wealthiest. In each comparison the advantage for one group over the other is measured in country-specific standard deviations. For gender, the comparisons indicate a consistent advantage for boys over girls at this grade level, although

FIGURE 3.8 (*continued*)

DIFFERENCES IN SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS BY GENDER, ETHNICITY, SCHOOL TYPE, AND SOCIOECONOMIC STATUS

c. Private (versus public) mathematics advantage in standard deviations

d. Wealthiest (versus poorest) quintile mathematics advantage in standard deviations
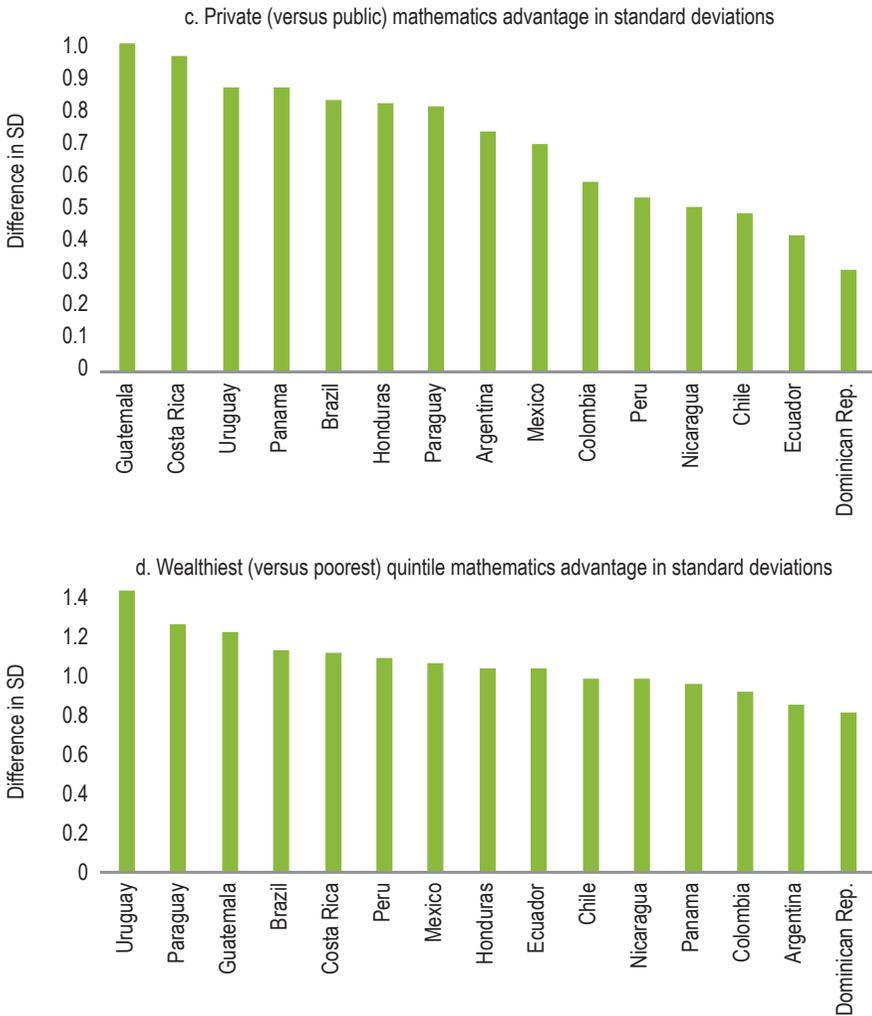
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: SD: standard deviation.

in only three countries is the difference greater than 0.20 standard deviations, and in two countries girls perform a little better than boys (Paraguay and Panama). The consistently modest advantage for boys in mathematics is not unusual, especially at lower grade levels (the gaps tend to be larger for higher grades).

The results for ethnicity show that students who report belonging to some kind of ethnic group have lower scores than their nonethnic counterparts in most countries. Ethnicity is not defined here based on language but is instead self-reported in general terms ("do you belong to an ethnic group?"). This general category is used because relatively few sixth grade students report speaking an indigenous language at home, especially in urban areas. The gaps in Figure 3.8 are large enough to confirm that ethnicity is a relevant factor, and in some cases may point to specific problems with incorporating minority groups into schooling.

The bottom half of Figure 3.8 confirms the very large achievement differences that are expected in a region with high levels of economic inequality, even when restricting the comparisons to urban areas. Private school students, as well as students from the wealthiest quintiles, score significantly higher than their counterparts in public schools and from poor households in all 15 countries. The private school advantage is greater than 0.50 standard deviations in all but three countries, whereas the Quintile 5 (wealthiest) advantage is never below 0.80 standard deviations.

3.2.3 Content and Cognitive Area Subdomains

This section breaks down the sixth grade mathematics results in the TERCE into subdomains of knowledge, which include five content areas (geometry, measures, numbers, statistics, and variation), and three cognitive areas (identification/recognition of objects and elements, simple problem-solving, and complex problem-solving). Along with adding to the descriptive overview of mathematics knowledge in LAC (including learning gaps), the subdomain information also provides an "indirect" method for assessing curriculum implementation and the degree of curricular shallowness across different areas of the intended curriculum.

The subdomain averages are presented using the percentage of correct answers for the subset of test questions that belong to each domain; this is the only option available, since the proficiency scales available in the TERCE are for the entire test, not individual subdomains.

Assessment projects, as a rule, tend to avoid using the percentage of correct answers, even though a percentage between 0 and 100 (or 0–1.0) is probably the most commonly used metric for communicating test scores around the world. One problem with percentages is that most people have their own opinion about what constitutes mastery: for example, 90 percent or higher is often an "A," and below 60 percent is an "F." Standardized tests tend to use very different scales for defining mastery. Proficiency scales are especially useful (Figure 3.7), since they translate the results from the test

into specific sets of skills and steer readers away from simplistic "pass/fail" dichotomies or from defining their own scales. The second related problem is that one does not know how difficult the test questions are within each subdomain. Only a handful of items have been released, and the TERCE did not create proficiency-scale summaries by domain. This lack of information about what the subdomains measure—and the lack of subdomain-specific proficiency scales—to some degree compromises the ability to compare achievement levels across the different domains.

Figure 3.9 summarizes sixth grade mathematics scores across the different content and cognitive skill subdomains.[8] Among the content areas, student scores are the highest in variation and lowest in geometry and measures. However, the spread from highest to lowest is not very large: the average for variation is 47.3 percent correct (for the entire sample), compared with 40.8 percent for measures. The overall average of correct answers is 42.3 percent for the whole sample, and 45.4 percent for urban students.

The uniform results across content domains are somewhat surprising. They suggest that TERCE test designers intended for the content areas to be relatively similar in terms of difficulty. More specifically, those who developed the test included similar numbers of relatively easy and difficult questions for each content area. Previous LLECE test reports (for the *Primer Estudio Regional Comparativo y Explicativo* [PERCE] and the SERCE) also did not include comparisons of content area scores, so this design feature appears to be consistent with previous assessments. The lack of variation across content areas in the TERCE significantly handicaps the task of understanding curriculum shallowness in the region based on strengths and weaknesses in content area performance. This means that it cannot be conclusively stated that LAC students are performing better in some mathematics content areas than in others, which in turn rules out even a basic assessment of curriculum shallowness. There are still some differences in results across countries, but the meaningful variation by content area has essentially been removed from the analysis by test designers.

Figure 3.9 also presents the averages for the cognitive subdomains. In contrast to the results for the content areas, the results for cognitive areas provide a more orderly ranking of difficulty. Students, on average, correctly answered more than half of the mathematics test questions that

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[8] The TIMSS and TERCE tests are developed by different teams in different countries, and used to measure the mathematics skills of different populations of students. Therefore, the way in which mathematics subareas are defined is different for each test. This is why the list of specific mathematics subareas differs. There is little public technical documentation as of this writing on how the subareas in the TERCE are defined and scaled.

FIGURE 3.9
SIXTH GRADE MATHEMATICS ACHIEVEMENT BY CONTENT AND COGNITIVE SUBDOMAINS (PERCENT)
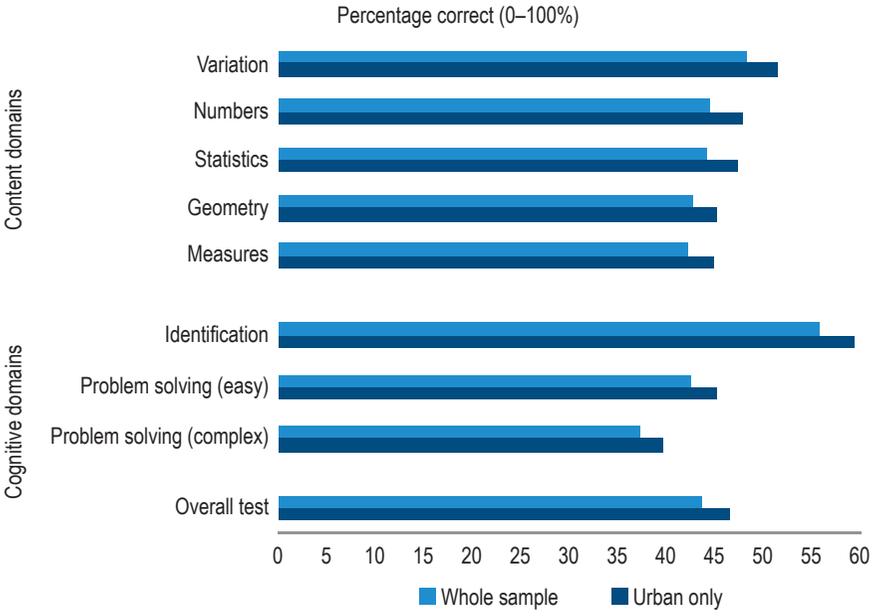
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

rely on basic identification of objects and elements. For problem-solving, the results show that 41 percent (for the whole sample) of the easy problem-solving activities were correctly answered, compared with roughly 35 percent of the harder (complex) problems.

Despite the aforementioned limitations of comparing content areas, the results in Figure 3.9 provide two important pieces of evidence of curricular shallowness in the LAC region. First, the scores across the five content domains are never above 50 percent correct (for the whole sample), which means that students cannot, on average, answer most questions in any of the content domains. It bears restating that these questions are taken from an intended curriculum that students are supposed to be familiar with. As noted above, the question of what constitutes mastery on a test like the TERCE is quite complicated, since standardized tests need to include items with a range of difficulty levels. It is not realistic to expect all students to answer all the questions correctly just because they are taken from official curriculum intentions; even very-high-scoring countries (i.e., South Korea, Singapore, and Cuba) fall far short of achieving this goal. However, global averages below 50 percent clearly indicate a sort of general curriculum

shallowness in these five content areas across the region, since it is not the case that student scores are substantially better in one or two areas, or that they are struggling in only a couple of content areas.

The results for the cognitive subdomain comparisons provide further evidence of curricular shallowness in LAC. Simply stated, the reason why student averages are never above 50 percent in the main content areas is that students are struggling to answer the most cognitively challenging test questions. This is not to say that students in LAC have mastered the basic elements of the sixth grade mathematics curriculum, such as place value identification, classification of angles according to their measure, simple numerical pattern recognition, or reading data from a table or graph. But they score lower on test questions that require them to solve problems that involve area and perimeter of polygons, conversion of measurement units, proportional reasoning, or statistical interpretation of data. This cuts to the heart of concerns about student mathematics achievement in the region. There is evidently an urgent need to provide teachers with effective tools to help students make these kinds of connections (see Chapters 2 and 4).[9]

What do the subdomain comparisons look like in high- and low-scoring countries? Figure 3.10 summarizes the percentage of correct answers by cognitive subdomain for seven countries, focusing only on urban schools. For the Mexico-Central American region and the South American region, one low-scoring country is included for each (Guatemala, Paraguay), as well as one high-scoring country (Costa Rica, Chile), as are the largest countries in the respective regions (Mexico, Brazil). The Dominican Republic is included as the sole representative of the Caribbean in the TERCE. The results show an identical pattern for proficiency across all seven countries, with relatively high scores in identification, followed by problem-solving (easy) and problem-solving (complex). The gaps between higher- and lower-scoring countries are also similar by cognitive subdomain.

The finding that stands out in Figure 3.10 is that the only three averages that are above 60 percent correspond to the highest-scoring countries (Chile, Costa Rica, and Mexico) in the area of identification. In other words, it is not the case that the overall averages in Figure 3.9 are being driven down by some very low-scoring countries. Even in samples restricted to urban students, the averages on the three cognitive skills subdomains are generally below 50 percent.

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[9] The challenges in this regard are daunting, and the difficulty of implementing changes in the intended curriculum in classrooms—although not a subject of this chapter—cannot be overstated. A recent videotape study supported by the Inter-American Development Bank documents teaching practices in the Dominican Republic, Paraguay, and the Mexican state of Nuevo Leon (Näslund-Hadley, Loera Varela, and Hepworth 2014).

FIGURE 3.10
SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS IN SEVEN LATIN AMERICAN COUNTRIES BY COGNITIVE SUBDOMAIN (PERCENT)
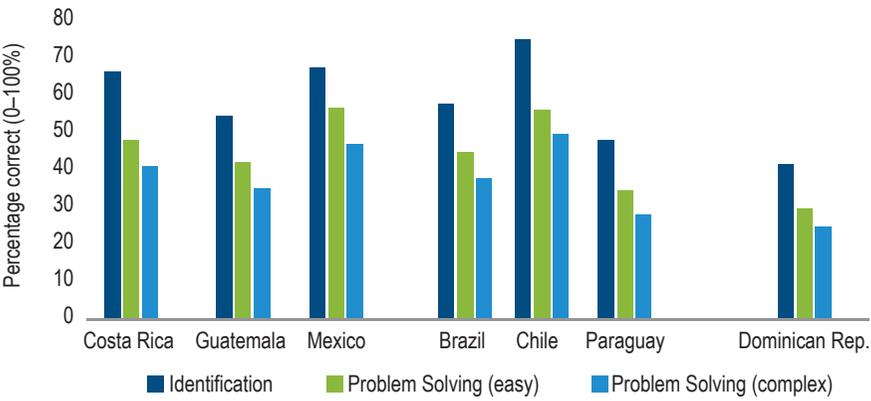
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

What kinds of gaps exist in the region by cognitive subdomain? Figure 3A1.1 in Annex 3.1 confirms that urban students in the highest socioeconomic status quintiles consistently perform much better than students in the lowest socioeconomic status quintile, and there is no real pattern across the three cognitive subdomains. But the results for gender (Figure 3.11) are quite different. On the easiest test questions related to identification, girls score higher than boys in four of the seven countries included in Figure 3.11. The results for identification are not entirely surprising, since it is not unusual to find that boys score higher than girls in more demanding cognitive subdomains (Leder 1992). But the results in Figure 3.11 show that the largest gaps between boys and girls are not in complex problem-solving, but rather in easy problem-solving. In fact, the gaps between urban boys and girls in these seven countries in complex problem-solving are never above 0.15 standard deviations, and in three countries they are below 0.10 standard deviations.

Figure 3A1.2 in Annex 3.1 provides a supplementary summary to the intercountry comparison of cognitive skills in Figure 3.10, using the TERCE test content areas (numbers, variation, etc.). Consistent with the Figure 3.9 global summary of content domains, the results show relatively small gaps between the content areas within countries, as the difference between the highest and lowest average is generally between 5 and 10 percentage points (compared with 20+ points in Figure 3.10). The cross-country variation is more substantial than the variation between contents, and

FIGURE 3.11

MALE ADVANTAGE IN SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS IN SEVEN LATIN AMERICAN COUNTRIES BY COGNITIVE SUBDOMAIN (IN STANDARD DEVIATIONS)
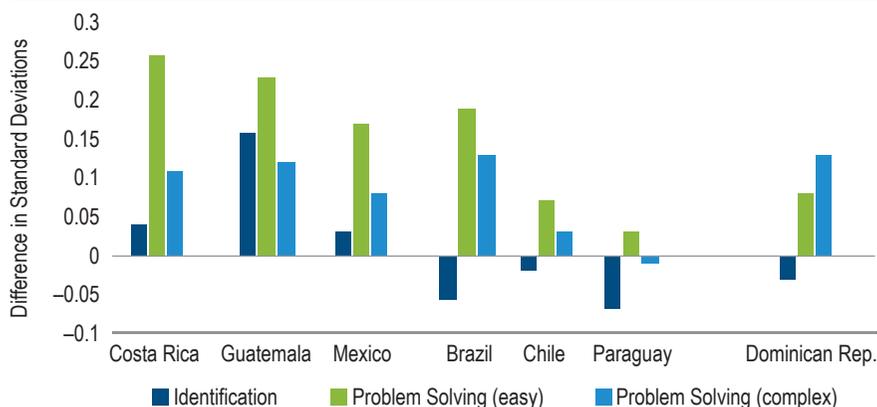
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

these differences, in turn, point to differences in curriculum implementation across countries. However, the reliance on the percentages of correct answers to subsets of test questions does not allow for inferences about what these skill limitations look like. As noted above, we do not have access to proficiency scales by content area. But the TERCE does publicly release some test questions, which makes it possible to see specific examples of relatively easy and difficult questions. Two examples are provided in Figure 3.12, one corresponding to a fairly easy question from the numbers content domain, and another more difficult item that is part of variation. The items also differ by the cognitive skill they require, since item 1 (easy) calls for the simple identification of place value up to millions in natural numbers, whereas item 3 (more difficult) is based on complex problem-solving skills that include proportional reasoning, identifying proper computations, and providing a solution to a problem.

Table 3.8 summarizes the results for these two items by country. Averages are provided for the whole sample, urban students only, by gender, and by socioeconomic status quintile. The countries are ranked from highest score = 1 to lowest score = 15, and the difference in the ranking between the two items is provided in panel b of Table 3.8.

The results in Table 3.8 provide a more specific glimpse of what low achievement looks like in the LAC region, albeit with a very small sample of test questions. Most students can answer the relatively easy test question correctly (Table 3.8, panel a), which gives some indication of

FIGURE 3.12
SIXTH GRADE MATHEMATICS TEST QUESTIONS 1 AND 3 PUBLICLY RELEASED BY THE TERCE

Matilde's identification card number has a 7 in the millions place, a 7 in the thousands place, and a 7 in the tens place.

Figure 1

67 071 007

Figure 2

77 377 070

Figure 3

57 470 271

Figure 4

81 187 037

Which figure corresponds to Matilde's identification card?

A) 1
B) 2
C) 3
D) 4

TERCE sixth grade public item 1

Content subdomain: **Numbers**
Cognitive skill subdomain:
Identification of objects and elements
Test proficiency level:
Level I
Global mean percentage correct: **72%**
Correct answer: **B**

When making juice, we produce on average 55 liters of juice per 100 kilograms of apples. How many kilograms of apples are needed to produce 220 liters of juice?

A) 121
B) 250
C) 375
D) 400

TERCE sixth grade public item 3

Content area: **Variation**
Cognitive skill subdomain:
Complex problem – solving
Test proficiency level: **Level III**
Global mean percentage
correct: **30%**
Correct answer: **D**

Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: Original document in Spanish. Translation by the authors of this chapter.

the kinds of basic skills that sixth grade students have obtained in urban primary schools. However, even within the easy item, the results clearly show pockets of low performance, both within high-scoring countries (see quintile 5 versus quintile 1 comparisons) and between the countries (comparing country averages).

The result that stands out in Table 3.8 is the very low overall average on the relatively difficult item (panel b). The regionwide average is just over 30 percent correct, and it should be noted that Item 3 is a multiple-choice question, with four options, so even if students guess at the answer they have a 25 percent chance of getting the correct answer. Furthermore, as difficult as this item is for most students, it is only from Level III of the

proficiency score scale, meaning that there are even more difficult items that students are expected to answer on the test.

The basic story of curriculum shallowness is apparent, to some degree, in these two example test questions. Students are exposed to the basic aspects of the curriculum, and, as a result, a majority of students in all countries can answer questions that cover fairly easy aspects of sixth grade mathematics. But when you move past the basics, the story is very different: test questions that require reasoning, thinking, and interpretation are answered correctly by a very small percentage of students. One piece of good news is that students seem to be exposed to all areas of the curriculum, including some that were traditionally absent in the past, such as statistics and pre-algebraic reasoning (numerical pattern recognition). Clearly what is missing is the implementation of

TABLE 3.8
TERCE COUNTRY PERFORMANCE ON SIXTH GRADE MATHEMATICS

a. Public Item 1

Country	Ranking	Overall Percentage Correct:		By Gender:		By Wealth Quintile (Q):	
		Whole Sample	Urban	Girls	Boys	Q5	Q1
Argentina	7	73.0	73.4	73.1	73.0	79.4	64.8*
Brazil	6	74.4	75.8	72.8	76.1	81.5	62.4*
Chile	1	84.5	84.8	84.4	84.6	90.7	82.0*
Colombia	8	71.6	74.6	70.9	72.1	83.2	65.7*
Costa Rica	3	77.4	78.9	77.7	77.0	83.8	72.8*
Dominican Republic	15	55.2	58.1	58.0	52.6	69.4	50.4*
Ecuador	9	70.2	72.6	70.7	69.6	78.5	58.9*
Guatemala	10	68.5	73.9	65.9	71.0	76.8	60.7*
Honduras	12	64.8	67.1	67.7	62.1	72.8	57.3*
Mexico	2	80.0	82.1	81.1	79.0	86.5	73.6*
Nicaragua	11	67.7	67.6	65.9	69.9	70.6	63.2
Panama	14	59.7	62.2	59.9	59.4	67.0	55.7*
Paraguay	13	60.2	66.7	57.4	62.6	76.2	51.1*
Peru	5	77.0	81.4	75.9	78.1	85.2	62.0*
Uruguay	4	77.1	77.0	73.4	80.3	83.0	63.3*
LAC average	—	71.5	74.7	71.0	72.1	79.7	63.6*

(continued on next page)

TABLE 3.8 (*continued*)
TERCE COUNTRY PERFORMANCE ON SIXTH GRADE MATHEMATICS

b. Public Item 3

Country	Ranking		Overall Percentage Correct:		By Gender:		By Wealth Quintile (Q):	
	On this item	Difference from Item 1	Whole Sample	Urban	Girls	Boys	Q5	Q1
Argentina	12	−5	26.4	27.6	26.4	26.3	29.0	19.9*
Brazil	13	−7	25.9	25.2	23.5	28.8	28.6	25.8
Chile	2	−1	35.5	36.3	32.4	38.7*	47.8	29.9*
Colombia	5	+3	31.2	31.9	27.3	34.1	38.9	27.3
Costa Rica	7	−4	29.4	31.6	27.5	31.3	35.3	27.4
Dominican Republic	15	0	24.6	26.3	25.1	24.0	23.2	23.9
Ecuador	11	−2	26.6	26.6	26.6	26.6	34.2	20.9*
Guatemala	3	+7	32.1	34.7	29.4	34.7	40.8	26.9*
Honduras	9	+3	29.0	25.8	26.7	31.1	30.0	31.4
Mexico	1	+1	38.6	40.1	38.1	39.1	43.3	35.9*
Nicaragua	6	+5	29.6	31.3	29.3	29.9	32.9	27.9
Panama	10	+4	27.1	24.7	28.2	26.0	27.7	26.6
Paraguay	14	−1	24.7	23.8	24.3	25.0	27.1	21.6
Peru	8	−3	29.1	31.3	29.0	29.2	34.7	23.5*
Uruguay	4	0	31.6	31.3	25.9	37.9*	44.1	15.6*
LAC average	—	—	30.2	31.1	28.7	31.6*	35.9	26.1*

Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: Ranking refers to numeric order (from 1 to 15) of country average. Wealth quintiles refer to wealthiest (Q5) and poorest (Q1) children in samples. Asterisks (*) in urban, boys, and Q1 columns mean that those averages are statistically different (p ≤ 0.05) from whole sample, boys, and Q5, respectively.

mathematical practices that enable students to perform at the highest level.

3.3 Conclusion

This chapter has set out to provide a synoptic view of the current status of mathematics education in Latin America and the Caribbean. It has surveyed and documented the scope and severity of the challenges countries in the region face, and it has identified the challenges in delivering the

opportunities to learn mathematics that would promote levels of achieve-ment commensurate with national aspirations and the skill requirements of 21st century citizens, workers, and lifelong learners. From the analysis emerges the theme of curriculum shallowness in both curricular inten-tion and student achievement. The chapter also surveyed variations in student achievement associated with gender, socioeconomic status, and geographic location, and found evidence of persistent structural factors leading to lower achievement levels for rural students, for girls (in some cases), and for the poor. Such structural challenges complement the ones identified in intended curriculum policy.

Curricular intentions are surveyed because of their influence on the types and qualities of actual learning opportunities in classrooms. National curricula, programs of study, standards, and the like are intended to identify learning goals. Together with tools such as textbooks and other pedagogical resources, and as facilitated by teachers, they are intended to shape educational experiences.

This look at curricular policy shows that intended curricula in LAC share some important priorities with other parts of the world, particularly in the areas of arithmetic and routine procedures. There is also evidence of ongoing and important innovation in curricular intentions: for example, the inclusion of learning goals in mathematics problem-solving. However, there are indications as well of important challenges facing mathematics education in the region, particularly regarding learning opportunities that the national curricula are not aiming to provide. The following are the most important challenges and policy implications:

1. All evidence of student achievement in mathematics in the region indi-cates that it not only is inferior to achievement in other regions, but is also inferior to countries' own national aspirations as put forward in their curricula. This may be addressed by prioritizing curriculum policy to promote better opportunities to learn and higher student achievement.
2. The intended curricula in mathematics in LAC are shallow compared to fundamental content commonly intended in higher-performing coun-tries. An effort could be made to include content in national curricula that has proven fundamental in promoting higher achievement in other countries. Priority content areas are identified in this chapter.
3. Shallowness is evident in the level of cognitive demand at which LAC countries intend content to be taught and learned, which is lower than in higher-performing countries. Curricula therefore might be improved to promote challenging levels of cognitive demand like those in higher-achieving countries. Specific priority areas are identified in this chapter.

4. There is persistent evidence of structural inequities in educational attainment across ethnicity and gender, and according to whether students are in urban, rural, private, or public schools. Expectations for student learning—and the resources to attain those expectations—can address the persistent inequities in the distribution of opportunities to learn in the region.

Addressing the challenges diagnosed here is likely to be a challenge. Critics of curriculum reform in the region often claim that current curricula are too comprehensive and too large for teachers to address in a school year. Yet the evidence remains that curricula in LAC attempt to teach less content and skill areas than those of higher-achieving countries outside the region. Perhaps there is an opening for innovative approaches to instruction, such as the use of computers, to enrich learning opportunities for students.

The intended curriculum promotes and constrains educational opportunities for students, and this matters insofar as it affects what students learn. As this chapter examined evidence of student learning in the region, it also documented some specific strengths and weaknesses. There is evidence that average levels of student achievement in primary school mathematics are rising throughout the region. There is also evidence that gender gaps in mathematics at this grade level are small and do not consistently favor boys. Yet, the overall status of mathematics achievement in the region is weak. This chapter and this book more generally set out to describe evidence-based approaches to help policymakers, educators, and other stakeholders confront these challenges.

TABLE 3A1.1
TERCE SIXTH GRADE MATHEMATICS PERFORMANCE COMPARED WITH PER CAPITA GNI

Country	TERCE (1)	GNI Per Capita (2)	Difference (2–1)
Chile	1	1	0
Mexico	2	5	+3
Uruguay	3	2	–1
Costa Rica	4	7	+3
Peru	6	9	+3
Argentina	5	3	–2
Brazil	7	6	–1
Colombia	8	8	0
Ecuador	9	11	+2
Guatemala	10	13	+3
Honduras	11	15	+4
Panama	12	4	–8
Nicaragua	13	14	+1
Paraguay	14	12	–2
Dominican Republic	15	10	–5

Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: GNI: gross national income.

DIFFERENCES BETWEEN LOWEST AND HIGHEST SOCIOECONOMIC QUINTILES IN SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS IN SEVEN LATIN AMERICAN COUNTRIES BY COGNITIVE AREA (IN STANDARD DEVIATIONS)
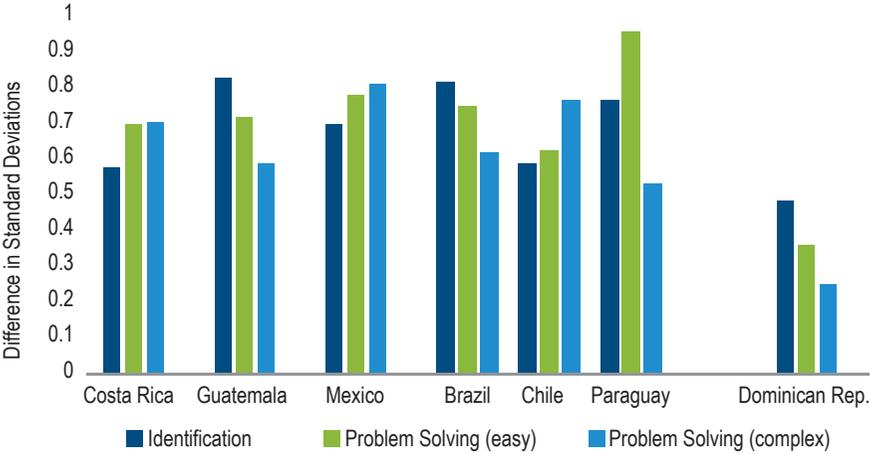
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

SIXTH GRADE MATHEMATICS ACHIEVEMENT IN URBAN SCHOOLS IN SEVEN LATIN AMERICAN COUNTRIES BY CONTENT AREA (PERCENT)
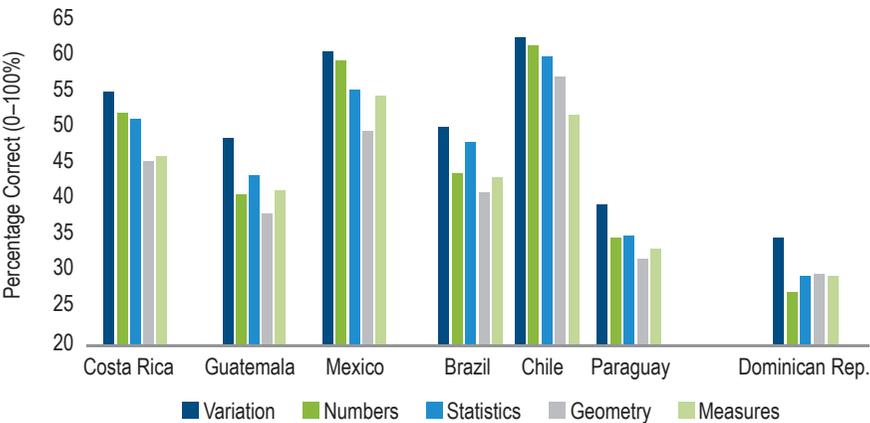
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.

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What Are the Main Challenges to Learning Mathematics in Latin America and the Caribbean?

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This chapter analyzes the challenges for learning mathematics in Latin America and the Caribbean (LAC) through a review of empirical evidence linking inputs and classroom practices with student performance. The review is guided by three fundamental questions. First, what does the mathematics teaching and learning environment look like in the region's urban primary schools? Second, what elements appear to be most important in explaining variations in student achievement, and how available are these critical inputs and processes in the average classroom? And finally, how do educational opportunities vary within countries on the basis of socioeconomic status? The review is intended to complement other chapters in this book that have described how children learn mathematics (Chapters 1 and 2), and actual student achievement levels (the second half of Chapter 3). It is restricted when possible to primary grades in urban schools, and also makes some tentative linkages with the larger question of technology solutions in mathematics by including inputs such as computer and Internet use.

It should be stated up front that this chapter has an ambitious agenda. The questions posed involve issues related to causation and how specific inputs and processes directly impact student achievement levels in mathematics. There is certainly a large amount of evidence on the covariates of mathematics achievement in LAC both on a regionwide and country basis. However, research designs with experimental (or quasi-experimental) features are much less common (McEwan 2015), and, as a result, the ability to assess the evidence on a strictly causal basis is considerably handicapped.

To address the main questions, this chapter relies on two general sources of information. The first includes data from the Latin American Laboratory for Assessment of the Quality of Education (*Laboratorio Latinoamericano de Evaluación de la Calidad de la Educación* – LLECE) that provide an empirical overview of student achievement levels, teaching and learning environments, and family background factors throughout LAC. The focus here is on the most recent LLECE application, the Third Regional Comparative and Explanatory Study (*Tercer Estudio Regional Comparativo y Explicativo* – TERCE), which is augmented with some particularly useful variables that were collected only in the previous application, the Second Regional Comparative and Explanatory Study (*Segundo Estudio Regional Comparativo y Explicativo* – SERCE). The LLECE data are valuable because they (1) allow for urban-specific analyses in mathematics using representative samples from upward of 16 countries; (2) can be used to assess issues of equity and the stratification that exists within the urban sector; and (3) include a range of questions on teaching and learning processes that can be analyzed as covariates of student achievement levels, including some questions about the actual use of technology in classrooms and the home.

However, despite the obvious appeal of multiple regionwide datasets in reviewing mathematics in the region, the LLECE data provide a fairly basic snapshot of the classroom environment itself. In other words, some of the fundamental components that determine the quality of mathematics teaching—such as different kinds of teacher knowledge of mathematics, and the interaction between teachers and students—are largely absent. This in turn requires consulting a very different information source, comprised mainly of individual, country-specific studies. Their inclusion allows for pushing the discussion of quality and effectiveness beyond the kinds of variables commonly available in large sample data sets, but there are some trade-offs vis-à-vis the LLECE data. First, these studies are not as malleable as the LLECE data sets in terms of presenting results for urban areas only, or comparing characteristics across socioeconomic (or other) strata. Second, the data are not always linked with student achievement, which limits their reach as indicators of effective practice based on student outcomes. There are also limitations in generalizability and, in the case of the more qualitative resources, the results may only come from a few schools or classrooms. Finally, some of these studies are from outside LAC, and they are not always specific to mathematics.

This chapter first provides a brief review of the LLECE data and the methods incorporated for the statistical analysis that is undertaken specifically for this chapter. The main research questions are then addressed

in three separate sections that are intended as a holistic overview of mathematics teaching and learning. These include school inputs and teacher background characteristics; teacher capacity; and teaching processes, including the use of computers and technology. The final section presents the chapter's conclusions.

4.1 Analysis of Data from the Latin American Laboratory for Assessment of the Quality of Education

The analysis of the LLECE data first involved a review of the TERCE (2013) and SERCE (2006) survey instruments to identify student, family, teacher, and school variables in various categories (school inputs, teacher characteristics, etc.).[1] Each variable was then added (individually) to a multivariate regression equation[2] that included a number of frequently analyzed student, family, and community control variables.[3] This was done for sixth grade students in urban schools only (public and private combined). The results for each variable were then summarized using a frequency table based on three categories: statistically significant (p ≤ 0.05) and positive, significant and negative, and insignificant.

The output from the regression analysis is extensive: more than 100 individual variables were chosen from the TERCE and SERCE, and each was included in as many as 16 country-specific regressions for urban sixth grade

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[1] For more information on these assessments and data downloads, see http://www.unesco.org/new/en/santiago/education/education-assessment-llece/.

[2] The presented results are based on an ordinary least squares (OLS) regression with weighted data and standard errors corrected for clustering by classroom. For the TERCE, the dependent variable is not a single measure of mathematics achievement, but rather five plausible values generated by item response theory (IRT) scaling. This requires a modified statistical specification that in effect averages the coefficients for the independent variables across the five dependent variables. Additional estimations were obtained using school fixed effects (which was only possible in the SERCE, since the TERCE collected data from a single classroom in each school), as well as random effects extensions commonly referred to as hierarchical linear modeling (HLM). The results from these statistical extensions are generally similar to the OLS results, although in most cases the selected variables are less likely to be statistically significant in the fixed effects or HLM specifications.

[3] The variables included in all models are the following (with the TERCE or SERCE indicated in parentheses when the variable is specific to that survey year only): student age and gender; student has never repeated a grade; controls for language spoken in home; working status of child; ratio of people to rooms in home (SERCE); individual student socioeconomic status based on home possessions and services; parental education taken from individual parent questionnaire (TERCE); school average for socioeconomic measure; and a private school control.

students. The variables that are presented in this chapter were chosen primarily on the basis of their applicability to a specific domain—and potential relevance as policy levers—and not on the basis of how significant or insignificant they were as predictors of student mathematics achievement (see UNESCO, 2008 and 2015, for more complete reviews). This data reduction strategy borrows from a long line of metastyle reviews in education that take a group of existing quantitative studies and summarize the most frequently significant (and insignificant) predictors of student achievement (Fuller and Clarke 1994; Glewwe et al. 2011). The contribution of this chapter to the mathematics achievement literature is an updated, systematic review of an extensive set of independent variables that is specific to one particular region (LAC). The use of a single data source (LLECE) provides some advantages. First, the results for individual variables are easier to compare, since each one is added individually to the same standard regression model in all countries. Second, by assessing the results based on all estimations, and all countries, one avoids the kind of publication bias that is inherent in meta-summaries of statistical analyses that rely exclusively on published research.

4.2 "Basic" School Inputs and Teacher Characteristics

4.2.1 School Inputs and School Climate

Figure 4.1 summarizes the statistical analysis results for some of the most commonly cited policy levers for improving student achievement in developing countries. These include school climate indicators that are related to how students get along with each other and with teachers, how teachers get along with other teachers and the degree they feel supported in the school, and measures about safety and conditions in the neighborhood. The numbers refer to individual country regression results: for example, preschool attendance is a positive (and significant) predictor of student mathematics achievement in 13 of the 15 countries that were analyzed. The results show that preschool attendance, having your own mathematics textbook, and school infrastructure are significant predictors of student achievement in about half (or more) of the countries (see the notes to Figures 4.1–4.3 for variable definitions). For class size, the results show that larger classes are actually positively associated with achievement in urban schools in the regression analysis in six countries, and negatively associated (and significant) in only three countries. This rather surprising result is a reminder of the limitations of large-sample survey work for pinpointing—with certainty—what really matters for raising student achievement levels. This topic is returned to several times in the chapter.

FIGURE 4.1
**SUMMARY OF PREDICTORS OF SIXTH GRADE MATHEMATICS
ACHIEVEMENT IN THE 2013 TERCE: INPUTS AND INFRASTRUCTURE**
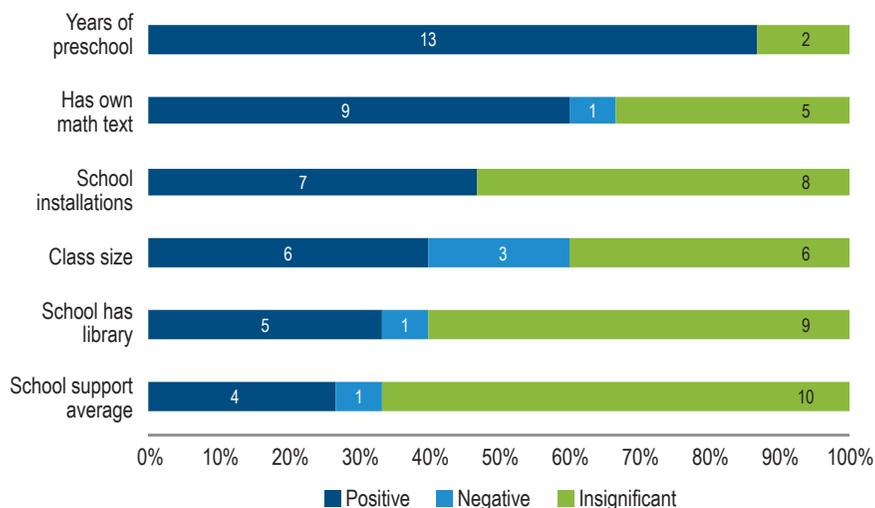
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: All results are based on country-specific regression analyses (15 in all) using weighted data for urban students; see the main text for more details on the regression model. The numbers in the bars refer to the number of countries. "Years of preschool" is total preschool attendance in years between ages 0 and 6, according to parent. "Has own math text" = 1 if students have their own math textbook, and = 0 if students have no book or share a book with other students. "School installations" is the percentage of 11 installations (e.g., director's office, computer room, etc.) that are present in the school, according to the school director. "Class size" is the number of students who took the exam, which does not account for students absent on the day of the visit. "School has library" = 1 if school has library, and = 0 if school has no library (according to the school director). "School support average" is the percentage of nine support services that are available in the school (e.g., feeding program, violence prevention, etc.).

Figure 4.2 continues with school climate. TERCE students, teachers, and parents were each asked questions about different aspects of the school climate, and all variables have been recoded so that positive scores indicate more favorable conditions. The results indicate that parent- and teacher-reported conditions are positively associated with student mathematics achievement levels in upward of half of the countries. This somewhat tentative linkage highlights the potential importance of school climate in affecting a whole range of schooling processes, and, with the spread of violence in some areas of LAC, this issue is taking on even greater importance (World Bank 2011). Student-reported conditions are less consistently associated with achievement, although it should be noted that when analyzed as individual-level student measures—instead of as a classroom average—better conditions consistently predict higher achievement.

FIGURE 4.2

SUMMARY OF PREDICTORS OF SIXTH GRADE MATHEMATICS ACHIEVEMENT IN THE 2013 TERCE: CLASSROOM, SCHOOL, AND NEIGHBORHOOD CLIMATE
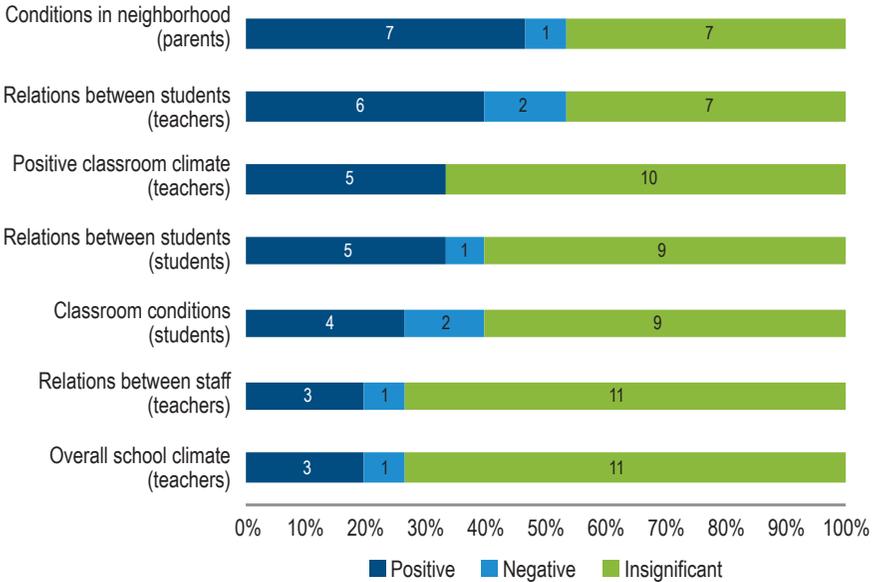
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: All results are based on country-specific regression analyses (15 in all) using weighted data for urban students; see main text for more details on the regression model. The numbers in the bars refer to the number of countries. Data source referred to in parentheses. All variables are averages for a series of questions based on three or four category scales. When necessary, the scales have been recoded so that all are positive (i.e., higher values for all variables reflect a better climate). Student- and parent-reported measures are averaged at the school level.

4.2.2 Teacher Background Characteristics

There is a strong belief in education research and policy circles that "teachers matter." This simple dictum is popular even in relatively wealthy countries where children have access to multiple learning resources (OECD 2005; Hanushek and Rivkin 2012; Rivkin, Hanushek, and Kain 2005), so there is extra reason to focus on teachers in contexts where children are exposed to few learning opportunities outside of the four or five hours a day they spend in school. But what is it exactly that makes some teachers more effective than others?

The review of teacher effectiveness begins in Figure 4.3 with the regression results from the TERCE for a group of teacher education, experience, and training indicators. Only four variables are significant predictors

FIGURE 4.3
SUMMARY OF PREDICTORS OF SIXTH GRADE MATHEMATICS ACHIEVEMENT IN THE 2013 TERCE: TEACHER BACKGROUND CHARACTERISTICS
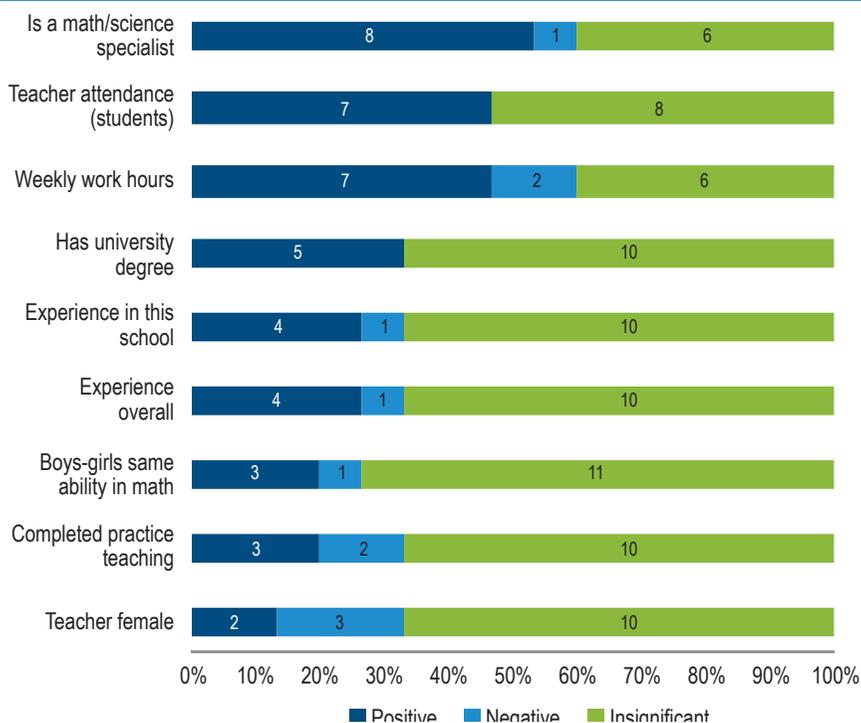
Source: Third Regional Comparative and Explanatory Study (TERCE), 2013.
Note: All results are based on country-specific regression analyses (15 in all) using weighted data for urban students; see text for more details on regression model. The numbers in the bars refer to the number of countries. All variables obtained from teacher questionnaires. "Is a math/science specialist" = 1 if teacher reports being a math or science specialist, 0 = general subject teacher or other. "Teacher attendance" is a classroom average based on student reports regarding the frequency of teacher's absence (1 = often, 2 = sometimes, 3 = never). "Weekly work hours" is hours of work in this school (reported by teacher). "Has university degree" = 1 if education is university level or postgraduate, 0 = lower than university. "Experience in this school" and "Experience overall" are measured in complete years. "Boys-girls same ability in math" = 1 if teacher indicated no difference in mathematics ability by gender, 0 = teacher indicated boys or girls are better in mathematics. "Completed practice teaching" = 1 if yes, 0 = did not complete.

of higher student mathematics achievement in five or more countries: their teacher is a math or science specialist, regularly attends the classroom (as reported by their students), reports working more hours a week, and has a university degree. Teacher experience is not consistently associated with achievement levels, and this variable was analyzed in more detail to test the hypothesis that experience is especially important at particular stages

of a teacher's career (such as during the first few years; see Chetty, Friedman, and Rockoff 2014; Ost 2014; Rice 2003). These results also did not point to a clear pattern across the TERCE countries.

One of the points of emphasis in this review is equity, so a particularly important question is how potentially important predictors of student achievement are distributed both within and between countries. Figure 4.4 provides a detailed summary of five of the most consistently significant predictors of student achievement from Figures 4.1–4.3. For each variable the average for the wealthiest (Quintile 5) and poorest (Quintile 1) schools are presented, and alongside these two averages is the "gap" between Q5 and Q1 schools (see grey bar), which is measured in standard deviations that use the right-hand vertical axis.[4] This is done for 7 of the 15 TERCE countries: in the Central America/Mexico region and the South America region the country groupings include one (relatively) high-scoring country (Costa Rica, Chile), one low-scoring country (Guatemala, Paraguay), and the largest country in each region (Mexico, Brazil). The Dominican Republic is also included as a representative of the Caribbean region, which in the TERCE does not include Cuba.[5]

The results in Figure 4.4 confirm very large gaps by social class in LAC, even when restricting the comparisons to urban areas. A substantial number of standard deviation differences between Quintiles 1 and 5 are of an order of one standard deviation or larger; note that the scale for standard deviations is 0–2.0 (right side of all figures), so grey bars that look small may actually indicate fairly large gaps. Preschool participation is especially unequal, as sixth grade students from wealthy urban families consistently spend more than one additional year in some kind of preschool/kindergarten compared with their Quintile 1 counterparts. Large gaps are also present for having your own mathematics textbook and for school infrastructure. Figure 4.4 also reveals some large differences across countries in the provision of school services. Finally, the results for teacher absences (according

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[4] The use of standard deviations makes it easier to compare the differences across variables that use different measurement scales. For example, in Paraguay the percentage of children who own their own mathematics textbook in poor (Quintile 1) schools is much lower than the average in Quintile 5 schools, and the difference comes to about 1.5 standard deviations (see panel a in Figure 4.4). This difference (1.5 standard deviations) is similar in magnitude to the difference in the number of years of preschool attendance between Quintile 1 and Quintile 5 students in Brazil (see panel b in Figure 4.4).

[5] Given Cuba's consistently high performance in previous LLECE data collection initiatives, this omission is significant but unavoidable; readers can consult earlier LLECE reports (UNESCO 2008) and other secondary sources (Carnoy, Gove, and Marshall 2007) for more information on Cuban performance.

FIGURE 4.4
EQUITY COMPARISONS OF SELECTED VARIABLES IN THE 2013 TERCE IN SEVEN LATIN AMERICAN COUNTRIES

a. Has Own Math Textbook (%)

b. Preschool Participation (years attended)

c. School Installations (%)

d. Frequency of Teacher Absence

■ School Quintile 5 (wealthiest) ■ School Quintile 1 (poorest)
■ Wealthiest school advantage (in st. deviations) ■ Poorest school advantage (in st. deviations)
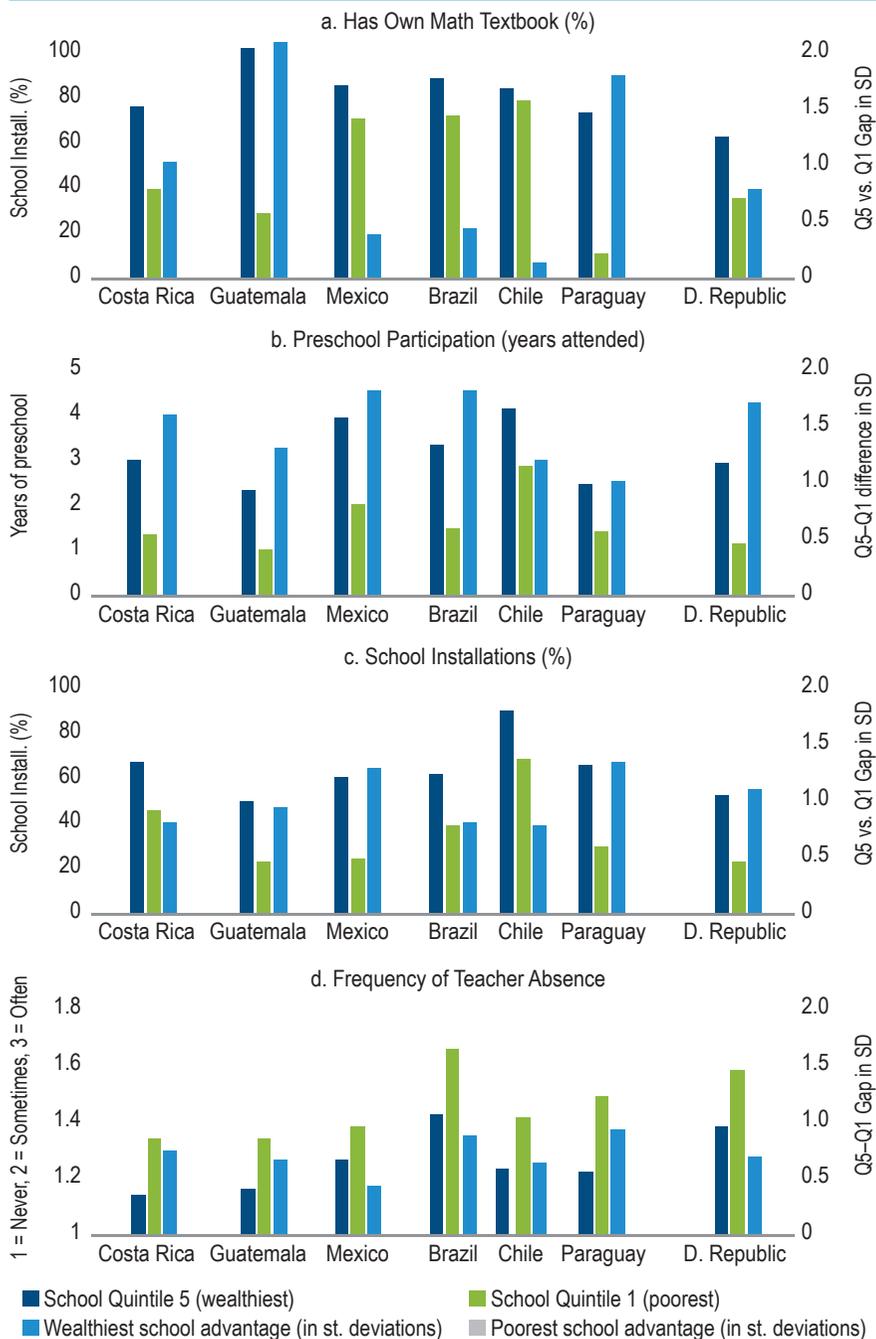
(continued on next page)

FIGURE 4.4 (*continued*)

EQUITY COMPARISONS OF SELECTED VARIABLES IN THE 2013 TERCE IN SEVEN LATIN AMERICAN COUNTRIES
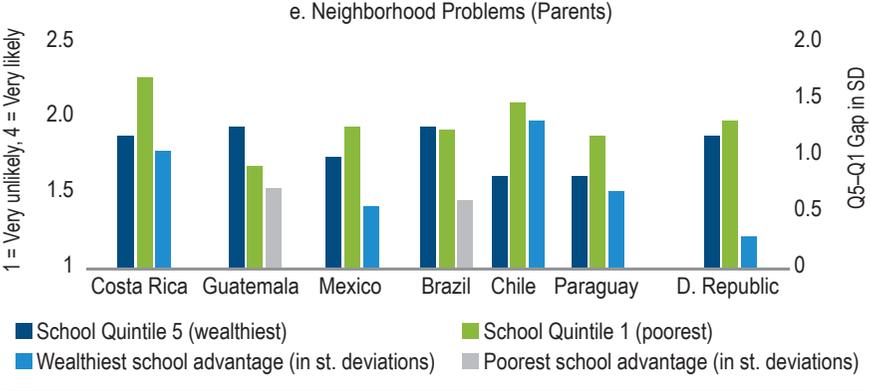
Sources: Third Regional Comparative and Explanatory Study (TERCE), 2013; and authors' calculations.
Note: SD: standard deviations.

to students) and, to a lesser degree, neighborhood conditions (according to parents), do not suggest widespread problems: the country averages are all below the midpoint of the scale ("sometimes" the teacher is absent, and neighborhood problems are "unlikely"). Nevertheless, there are some large gaps between Quintile 1 and 5 schools, which suggests that these problems are concentrated in a relatively small number of urban schools.

How much of the differences in Figure 4.4 are attributable to the presence of private schools in urban areas? A separate analysis (not presented) was restricted to urban public schools only. The results show that sizable gaps are still present, as almost half of the comparisons show wealthy school advantages of 0.50 standard deviations or more. However, in three countries (Guatemala, Chile, and Paraguay) the public-only differences are much smaller, which suggests that urban inequality in these countries is primarily a product of differences across the public and private sectors.

4.2.3 Summary

Overall, the results in this section confirm the importance of ensuring that all children have access to a basic set of school inputs and learning conditions (and opportunities). In reality these are not really basic elements, since school infrastructure, teacher attendance, preschool access, and neighborhood conditions are likely to be enmeshed in potentially

complex institutional, managerial, and contextual socioeconomic realities. The resulting gaps by social class are therefore not surprising, and serve as a strong reminder of the need for focused policies that address issues of equality, even when the comparisons are restricted to urban (public) schools. The results from this section are also notable for the number of variables that are insignificant predictors of sixth grade mathematics achievement in the TERCE. Inconsistent and even surprising results for variables like teacher education, certification, and class size are not unusual in large sample survey analyses from the developing world (Fuller and Clarke 1994; Glewwe et al. 2011). Even in industrialized contexts like the United States, where these statistical linkages can be examined using more powerful longitudinal designs, the evidence is far from conclusive, or effect sizes are relatively small (Wayne and Youngs 2003; Nye, Konstantopoulos, and Hedges 2004). That variables such as teacher education, training, and resource indicators are not dependable predictors of student achievement—and, by extension, plausible policy levers for improving outcomes and reducing inequality—has provided a major impetus for researchers to push their line of inquiry deeper into actual teaching and learning processes. The next section examines some indicators related to these processes.

4.3 Teacher Capacity

The goal in the next two sections is to move beyond basic background characteristics of teachers (experience, education) and identify features of their work that are especially important for student learning in mathematics. Of particular interest are two general elements. The first is what is loosely referred to here as teacher capacity for teaching mathematics, which includes several domains of teacher knowledge. The second (presented in Section 4.4) includes instructional practices, which encompass a large number of teacher actions that together help decide the degree of a teacher's effectiveness.

The interest in this review is in describing the full range of teacher actions observed in Latin American and Caribbean mathematics classes—the "good and the bad"—instead of focusing on the work of model teachers who appear to be especially effective. This approach is consistent with the belief that teacher capacity and actions in the classroom need to be viewed as systemic outcomes, and not simply as a collection of individual results. This also means taking into account how teaching outcomes are affected by institutional factors like pre-service training and preparation regimes, in-school support activities, and incentive and supervision structures.

4.3.1 Teacher Effectiveness and Related Factors

A detailed review of teacher recruitment and support elements is beyond the scope of this chapter, and direct linkages between these systemic features and student mathematics levels are not easily established (see Bruns and Luque, 2014, for a recent review of this evidence in LAC). But given their potential to directly affect aspects of teacher capacity and teaching practices that are included in this review, these features merit some attention. International studies of effective systems—meaning countries with very capable teachers and high scores on international mathematics tests—have identified some core features of success: (1) implement policies that make teaching an attractive career and facilitate the recruitment of high-ability students, including by paying mathematics teachers on par with scientists and engineers; (2) encourage student teachers and teachers in development to integrate content and pedagogy preparation by exposing them to classroom situations where such integration is happening (AMTE 2013); (3) establish high standards for teacher education programs, as well as for entry into the teaching profession after graduation; and (4) focus on developing and implementing strong quality-assurance mechanisms throughout the system (Ingvarson et al. 2013, 5–6; Carnoy et al. 2009; Bruns and Luque 2014).

How do education systems in LAC compare with this idealized model? For recruitment and pay, the evidence is mixed, as overall teacher salaries are lower than those of similar professionals (Mizala and Ñopo 2014), though some studies show that they compare favorably on an hourly basis (Carnoy et al. 2009; Bruns and Luque 2014). In terms of preparation, there is evidence of fragmentation: mathematics content preparation is provided by mathematics departments, and pedagogical preparation by education departments, with few linkages. There is a tendency to prepare primary-level teachers with a sort of remedial mathematics "refresher" course that falls far short of the kind of specialized content and teaching knowledge they need to be effective, even when working with young students (Rosario, Scott, and Vogeli 2015). There is relatively limited use of practice teaching and hands-on experiential learning as part of teacher preparation (UNESCO 2012), and teacher preparation programs and professional development opportunities provide very little exposure to effective mathematical instructional practices that engage students and facilitate learning (use of visual diagrams, representations, discussion around different methods and solutions, etc.) (Luschei and Sorto 2010; Sorto and Luschei 2010; Saenz and Lebrija 2014). Finally, several researchers express concern that quality assurance and support mechanisms are

inadequate, as teachers are often isolated in their classrooms, provided few opportunities to improve their practices, and work in environments where accountability is lacking and there is little pressure to maximize the use of the school day, or even come to work every day (Vegas and Umansky 2005; Bruns and Luque 2014).

Few systems in the world have all of the core quality features in place, so the kinds of problems encountered in LAC are not unusual. However, they do help explain the kinds of problems revealed by empirical analyses of teaching and learning environments in the region, the subject of the next section.

4.3.2 Teacher Capacity

Teacher capacity potentially encompasses everything a teacher knows and can do. The sections that follow focus on three specific teacher knowledge domains: mathematical content knowledge, specialized teaching knowledge in mathematics, and cognitive lesson (or task) design.

Mathematical Content Knowledge
There is little question that teachers must be familiar with the subject matter they are teaching, although the research basis for this statement is surprisingly thin. The evidence that does exist internationally is mainly from LAC, where a handful of studies have shown that higher levels of teacher mathematics content knowledge are associated with higher student mathematics scores (Harbison and Hanushek 1992; Mullens, Murnane, and Willett 1996; Santibañez 2006; Marshall 2009; Marshall and Sorto 2012; Metzler and Woessman 2012; Guadalupe, León, and Cueto 2013). Knowledge of the mathematics content that one is teaching—especially at the primary school level—may seem like a very basic measure of capacity. But even these minimum skills should not be assumed, especially in the poorest contexts. For example, Harbison and Hanushek (1992) encountered primary school teachers in rural northeast Brazil who actually scored lower on mathematics tests than their students. Teachers also need to know higher levels of content than those they are teaching.[6] When teachers understand mathematics beyond the level they are teaching, they are

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[6] In their 2001 and 2012 reports ("The Mathematical Education of Teachers") the Conference Board of the Mathematical Science, American Mathematical Society, and Mathematical Association of America recommends "a thorough mastery of the mathematics in several grades beyond that which they expect to teach, as well as of the mathematics in earlier grades" (CBMS 2001, 2012).

better equipped to tackle the day-to-day work of mathematics instruction, such as detecting and anticipating student mistakes and misconceptions (Marshall and Sorto 2012).

With the increasing professionalization of the teaching profession, results like those encountered by Harbison and Hanushek 20 years ago in rural Brazil are less and less likely. Nevertheless, there are reasons to be concerned about teacher content knowledge levels in LAC. For example, the multi-country Teacher Education and Development Study in Mathematics (TEDS-M) shows that Chilean pre-service mathematics teachers had the second-lowest average mathematics content knowledge out of 13 countries globally, scoring below relatively poorer countries such as the Philippines and Botswana (Figure 4A1.1 in Annex 4.1). Guadalupe, León, and Cueto (2013) found that Peruvian primary teachers' scores on a sixth grade mathematics test (from 2004) were normally distributed, and that the urban-rural and public-private gaps between teachers were of roughly the same magnitude as the learning gaps between students in these categories (Guadalupe, León, and Cueto 2013, Table 12).

International league tables and intra-country comparisons of averages are certainly useful, but what does low teacher content knowledge actually look like in practice? Figure 4.5 shows two questions that were asked of primary and lower secondary teachers in a comparative study carried out in Costa Rica and Panama (see Carnoy et al., 2007, for more details). Question 1 is a very basic item related to primary-level geometry, while Question 2 is taken from the lower secondary school curriculum. The

FIGURE 4.5
EXAMPLES OF MATHEMATICS CONTENT QUESTIONS FROM PANAMA AND COSTA RICA

Question 1 (level = grade 3): Which one of following angles is obtuse?

a)   b)   c)   **d)**
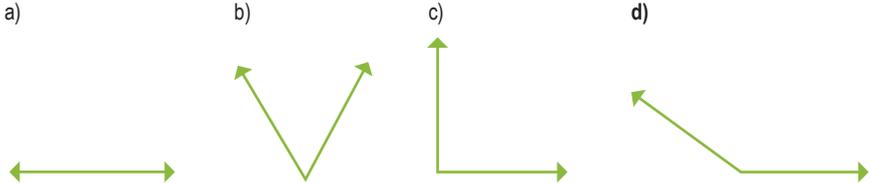
Question 2 (level: grade 7): Fourth grade students were cutting four ribbons. The yellow ribbon measured 3.2 meters, the blue one 3.18 meters, the red one 3.5 meters, and the green one 3.09 meters. The longest ribbon is:

a) Blue   **b) Red**   c) Green   d) Yellow

Source: Sorto et al. (2009).

authors found that roughly 20 percent of primary teachers in the Panama sample answered question 1 incorrectly, and another 10 percent left the item blank. Within this same sample, only 41 percent of Panama primary teachers answered question 2 correctly. By contrast, in Costa Rica, where teachers receive considerably more pre-service preparation (Sorto and Luschei 2010) and students have much higher test scores (see the SERCE and TERCE results), teacher content knowledge levels are much higher: 91 and 80 percent of primary school teachers answered questions 1 and 2 correctly, respectively.

Pedagogical Content Knowledge

Results like those in Figure 4.5 highlight the imperative of avoiding assumptions about what teachers in primary classrooms know. Meanwhile, content knowledge needs to be treated as a necessary, but not sufficient, element of teacher capacity and effectiveness. In the past 20 years, research on teacher knowledge, mainly in the United States, has produced compelling evidence that teachers need to know mathematics in a way that is specialized to their job (Shulman 1986; Ma 1999; Hill, Ball, and Schilling 2008). This specialized knowledge, or pedagogical content knowledge,[7] is at the intersection of three general knowledge domains: pedagogical knowledge, the mathematical content knowledge a teacher is responsible for teaching, and mathematical knowledge at least one level beyond that (Sorto et al. 2009; Carnoy et al. 2007).

Figure 4.6 illustrates how this knowledge is different from that used for professions outside education.

The problem shown in Figure 4.6 is not as easy as it appears, and only 32 percent of the teachers surveyed provided a correct answer to a problem of this type (Carnoy et al. 2007). First, the teacher needs to know the source of the mistake and then find an effective way to communicate to the student why his or her own representation leads to the incorrect answer. Here is where the different forms of knowledge come together: the student is using the same shaded block to represent three different quantities, (one unit, one third of a unit, and one fifth of a unit), which involves knowledge of mathematics. This identification is necessary but not sufficient to help Arnoldo understand why his reasoning is incorrect.

---

[7] Hill, Ball, and Shilling (2008) define pedagogical content knowledge as a component of a larger construct called "mathematics knowledge for teaching," which includes subject-matter knowledge as a second domain. In their depiction, pedagogical content knowledge constitutes knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

FIGURE 4.6
PEDAGOGICAL CONTENT KNOWLEDGE EXAMPLE ITEM 1

Arnoldo says that $2\frac{2}{3} = \frac{4}{5}$ and he uses the figure below to demonstrate his assertion. Why is his reasoning not correct? Do not just say how to convert $2\frac{2}{3}$; <u>explain</u> to the student what is wrong with his reasoning.

$2\frac{2}{3}$ Put blocks together $\frac{4}{5}$
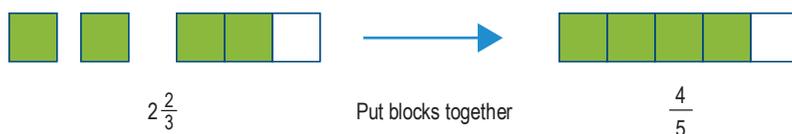
Source: Sorto et al. (2009).

Recognizing that the student does know how to represent rational numbers (the student's underlying reference unit for each quantity in isolation is consistent with the representation), and how to model the conversion algorithm based on what he or she knows, is a mixture of mathematical and pedagogical knowledge, in particular understanding the student's thinking and mathematical understanding of why the algorithm works.

Evidence from a few large-scale studies in countries in LAC suggests that pedagogical content knowledge can be measured, and that this form of capacity is associated with exposure to high levels of mathematical content (Sorto et al. 2009; Marshall and Sorto 2012; Cueto et al. 2016; Varas et al. 2013). For example, Varas et al. (2013) developed an instrument measuring an aspect of pedagogical content knowledge examined by Hill, Ball, and Shilling (2008) in the United States. Varas et al. (2013) measured "knowledge of content and students" among 83 in-service teachers and 156 preservice teachers (elementary and middle-grade levels) from Santiago and Concepción, Chile. On average, the teachers scored 45 percent correct on the instrument. The Chilean team concluded that high levels of "knowledge of content and students" are associated with high exposure to mathematics in teacher preparation programs, or in actual practice, suggesting that higher levels of mathematics predicts this specialized knowledge.

The results from the Varas et al. (2013) study are notable in part because teachers often struggled to correctly answer questions, even in urban areas of a relatively affluent and high-scoring (based on student achievement levels) country. These results are largely corroborated in the previously cited comparative study of primary and lower secondary mathematics teaching in Panama and Costa Rica (Sorto et al. 2009; Carnoy et al. 2007). Costa Rican primary school teachers scored higher than their Panamanian counterparts, and among pre-service teachers the gap was

FIGURE 4.7
PEDAGOGICAL CONTENT KNOWLEDGE EXAMPLE ITEM 2

6. Claudia has been computing the majority of her division exercises correctly, but recently she has been having difficulties. Below are some of the exercises completed by Claudia.

a.

```
413 │ 3
 3  │ 137
───
11
 9
───
23
21
───
 2
```

b.

```
815 │ 2
 8  │ 47
───
015
 14
───
 01
```

c.

```
626 │ 3
 6  │ 28
───
026
 24
───
 02
```

6.1. How do you think Claudia would do on the exercise shown if she were to continue using the same procedure for division?

```
928 │ 3
```

1. It is likely she responds to this exercise correctly using the same procedure.
2. It is likely she responds to this exercise incorrectly using the same procedure.

Source: Cueto et al. (2016).

even larger. These results, like those presented earlier for content knowledge (Figure 4.5), are consistent with more pre-service preparation in Costa Rica (college degrees, more classes in higher-level mathematics, etc.). However, it is important to restate that even in Costa Rica, a country with relatively high levels of teacher training and student achievement levels, significant deficiencies were encountered in the specialized knowledge levels of teachers.

A recent study conducted in Peru (Cueto et al. 2016) makes it possible to examine the linkages between a teacher's specialized knowledge and student achievement using a longitudinal design. The teacher instrument was designed to capture the ability of primary school teachers to recognize student error patterns, and to identify the source of the mistake. It was applied in approximately 150 fourth grade classrooms as part of the "Young Lives" longitudinal survey of student achievement and learning environments. An example of a pedagogical content knowledge question is shown in Figure 4.7. Roughly 70 percent of Peruvian teachers answered this question correctly, and there was a significant gap by socioeconomic status: 64 percent correct in the poorest schools, versus 74 percent correct in the wealthiest. Using multivariate methods, the researchers also found that student mathematics test scores were higher when their teacher had higher scores on this instrument.

Finally, teachers' specialized knowledge has also been analyzed in relation to the integration of technology into mathematics instruction, which is an important extension given the theme of this book. Technological pedagogical content knowledge refers to the "total package required for integrating technology, pedagogy, and content knowledge in the design of instruction for thinking and learning mathematics with digital technologies" (Niess et al. 2009, 7). This is a relatively new concept with few research antecedents focusing on mathematics, and no available linkages with student achievement levels. But two recent studies from Mexico (Mochon 2008, 2010) have investigated this type of knowledge by assessing how primary school teachers' technological pedagogical content knowledge is related to the use of a mathematics teaching software package. The results suggest that teachers with higher levels of this knowledge used the software as a tool to encourage students to create and share their own problem-solving strategies. Teachers having high technological pedagogical content knowledge were also more likely to consider students' cognitive growth, and to use activities to develop student concepts and ideas toward generalization. In sum, the author inferred through observation that teachers with high levels of mathematical knowledge were better able to adapt their teaching practices to align with the goals of the instructional software.

Cognitive Lesson (or Task) Design

The final aspect of teacher capacity addressed here is called cognitive lesson design, which refers to the cognitive level of the lesson as designed and implemented by the teacher in the classroom. This concept touches on elements of instructional practice (see next section), but is treated here as part of teacher capacity, since the cognitive level of the lesson is directly affected by the teacher's ability to conceptualize the learning goal and design activities that maximize the demands placed on students. The importance of cognitive design is well established in the literature in the United States. Stein et al. (2000) highlight two key findings. First, mathematical instructional tasks with high levels of cognitive demand are most difficult to implement in classroom settings, and tasks that are intended to be most demanding are often transformed into being less demanding during instruction. And second, empirical studies show that student learning gains are greatest in classrooms where instructional tasks consistently encourage high-level student thinking and reasoning. Students appear to learn less in classrooms where the instructional tasks are mainly procedural in nature.

What do LAC classrooms look like in terms of cognitive design? Some insights can be gleaned from the multi-country classroom observation

database described in Bruns and Luque (2014). Students were observed spending significant amounts of instructional time copying from the blackboard or in individual work, which suggests that instructional time is mainly spent in tasks that require memorization and procedural knowledge. The results from other more detailed classroom observation studies tell a similar story. A comparative study of mathematics teaching and learning in Brazil, Chile, and Cuba by Carnoy, Gove, and Marshall (2007) incorporated a rubric from Stein et al. (2000) that classifies lessons by higher or lower cognitive demand. Only one observed classroom (in Cuba) attained the highest score for "doing mathematics" based on complex and nonalgorithmic thinking, which also involves exploration of the nature of mathematical concepts, processes, and relationships. Cuban classrooms on average scored significantly higher than those in Brazil and Chile, in part because of more frequent use of procedures and student explanations of the procedures they were using. The authors give an example from the Cuban videos where the students were asked to indicate whether or not 430 is divisible by 10, and they were observed explaining that the zero in the units place is an indicator that 430 is a multiple of 10 and is therefore divisible by 10. This description of procedures and connections to other mathematical concepts was not typically present in Brazilian classrooms (Carnoy, Gove, and Marshall 2007), while in Chilean classrooms it was more typical than in Brazil but still not widespread.

The Cuban advantage is notable given the significantly higher scores that Cuban students have achieved on the LLECE exams in mathematics (*Primer Estudio Regional Comparativo y Explicativo* – PERCE; and SERCE). However, it is important to note, once again, that even in a high-scoring country (regionally), classroom episodes that require considerable cognitive effort—like student problem-solving independent of the teacher—were largely absent, and cognitive demand averages were in the middle to upper-middle range. At the other extreme, the overall average for the Brazilian sample of primary schools was in the lower-middle range of the four-point cognitive demand scale, which corresponds to procedures without connections. The observed lessons were focused on producing correct answers rather than developing understanding, and often consisted of a teacher writing on the board, students copying, and little interaction. There were few instances of linking concepts to procedures. Explanations were only given by teachers, and tended to focus on describing the procedure that was used (Carnoy, Gove, and Marshall 2007).

Teachers with higher levels of content and specialized mathematics knowledge are likely to design lessons with more demanding cognitive tasks, although the evidence for this kind of linkage across teacher capacity

elements is limited. One exception is provided by Sorto et al. (2009), who found that teachers' overall mathematical knowledge was associated with the level of cognitive demand of mathematical tasks with which their students were engaged. In general, teachers with higher levels of specialized mathematical knowledge tended to engage students in tasks that required them to make connections among representations, and explore and investigate the nature of concepts and relationships. For example, one high-capacity teacher (based on pedagogical content knowledge) gave her students four equal-length sticks and asked them to create geometrical shapes and explore all possible kinds and number of interior angles. Some students made a square, others made a rhombus. And a discussion followed about the kind of angles formed depending on the kind of figure (e.g., "Do angles that share a side count as one or two?"). In contrast, in a lesson with the same objective but with low cognitive task design, the students were just given a table of information about the name of the shape, the number of interior angles, and the kind of angles involved (e.g., "Square, 4 interior angles, all angles are right angles"). Higher-capacity teachers— as measured by their scores on mathematics content and pedagogical content knowledge questions—were also more likely to teach lessons that go beyond procedural knowledge, focusing instruction on conceptual understanding, reasoning, and problem-solving (Sorto et al. 2009).

Generalizations across the entire LAC region need to be handled with care, and, in the case of cognitive design, the evidence base is fairly small. But there are some common themes from analyses of cognitive demand based on a range of data sources. First, there are very few instances of students being engaged in high-level cognitive tasks. Instead, teachers tend to present knowledge with the intention of simply communicating it, which is very different (in a cognitive sense) from orienting the class around the learning of that knowledge. Instructional practices matter, too, which provides a good segue into the next section. Teachers appear to have few of the skills and tools needed to present students with a well-sequenced series of activities that might help them acquire the underlying mathematical concept. And finally, teachers do not often demonstrate an ability to effectively use models and multiple representations to illustrate abstract concepts, which is another dimension of cognitive design that is closely linked to teaching practices.

4.4 Mathematics Teaching Practices

One of the challenges of reviewing the evidence on effective teaching practices is deciding which specific aspects should be included. The

review presented here builds on the theoretical foundation for effective teaching that was detailed in Chapter 2, which includes a series of "critical elements" for effective practice that have been recently identified by the National Council of Teachers of Mathematics (NCTM 2014). Of particular interest to the analysis in this chapter are the following actions:

- Engage students in tasks that allow multiple entry points and varied solution strategies
- Engage students in making connections among mathematical representations
- Facilitate discourse among students to build a shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments
- Use purposeful questions to assess and advance students' reasoning and sense-making
- Build fluency with procedures on a foundation of conceptual understanding
- Provide students with opportunities to engage in productive struggle as they grapple with mathematical ideas and relationships
- Use evidence of student thinking to assess progress toward mathematical understanding (NCTM 2014, 10).

This hybrid framework provides a loose structure for organizing actual evidence on teaching and the degree to which the NCTM research-informed practices that support the learning of mathematics—and by extension several dimensions of the framework in Chapter 2—are present in LAC classrooms. This also includes linkages with student mathematics achievement (when possible). This begins in Section 4.4.1 with a description of activities and sequences of typical lesson archetypes. This overview of commonly observed lesson structures is useful for evaluating the degree to which key mathematical practices are being implemented. For example, for students to be able to compare approaches and arguments about mathematical procedures and ideas, they need to spend time discussing them. The second part (Section 4.4.2) describes the use of instructional tools such as textbooks and manipulatives. The frequency of the use of these tools helps us understand the implementation of tasks that promote multiple entry points, multiple solutions, and connections among representations. Section 4.4.3 then describes the nature of the interaction between teachers and students and the format for classroom work. These interactions help us understand the degree to which teachers are assessing student thinking and promoting reasoning. The format of student work

also helps define the extent to which students are engaging in productive struggle and building fluency.

The review strategy for these first three sections depends almost exclusively on existing studies. When available, this evidence comes from the LAC region, but this is not always possible and our reliance on existing research precludes tailoring the results to urban primary schools. Finally, Section 4.4.4 returns to the LLECE data (mainly the TERCE, but some SERCE data) and brings in classroom process variables. These are not observational data, but rather come from sixth grade students themselves (measured as classroom averages).

4.4.1 Structure of Mathematics Lessons

Studies of how classroom time is used are a good place to begin when describing the basic structure of the average classroom in LAC. Figure 4.8 is taken from a previously cited study of regional classrooms (Bruns and Luque 2014, Figure O.7). Based on a standard classroom observation rubric (called the Stallings instrument), the authors were able to categorize the average class for each country on the basis of a number of activities. Their results show that, on average, the time spent on activities related to learning is 65 percent or less of the total class time in all countries,

FIGURE 4.8
SUMMARY OF TIME USE IN CLASSROOMS, SELECTED LATIN AMERICAN AND CARIBBEAN COUNTRIES (PERCENT)
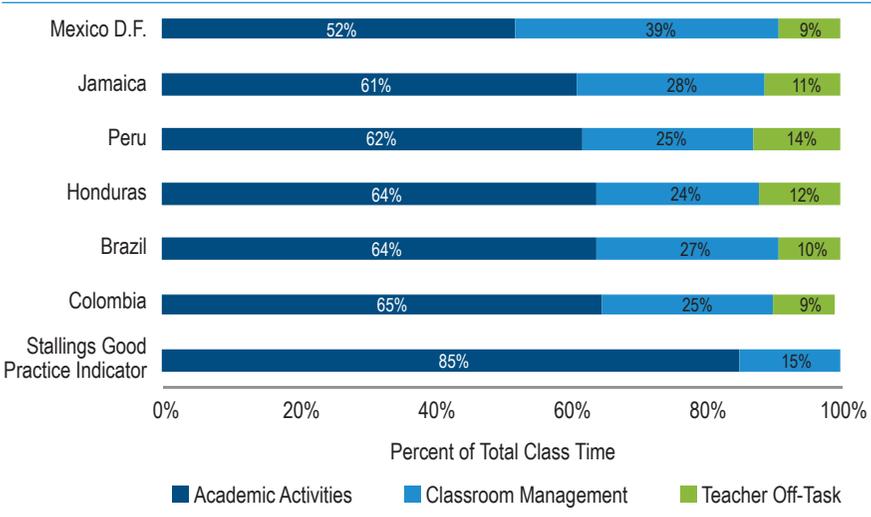
Source: Bruns and Luque (2014, Figure O.7).

with about 36 percent of this time dedicated to active instruction, like discussing and working on mathematical tasks, and 25 percent on passive instruction like copying. The percentage spent on instruction falls far short of the 85 percent benchmark suggested as part of the Stallings Instrument Good Practice Indicator (Figure 4.8).

The Bruns and Luque study samples are large enough to look at variation within and between countries, and the results highlight substantial inequalities in time-use patterns. The authors give an example from Rio de Janeiro, where, in the highest-performing schools (based on an index of test scores and pass rates), an average of 70 percent of class time was spent on instruction, and 27 percent on classroom management. Teachers were off-task in these schools only 3 percent of the time and were never absent from the classroom. By contrast, in the lowest-performing schools, only 54 percent of the time was spent on instruction, 39 percent was dedicated to classroom management, and teachers were significantly more likely to be off-task and physically absent from the classroom. Based on the authors' calculations, students in high-performing Rio schools received an "an average of 32 more days of instruction over the 200 day school year than their counterparts in low-performing schools" (Bruns and Luque 2014, 13).

Other classroom observational studies in the region have highlighted similar concerns about the amount of time students are engaged in effective instructional activities. Carnoy et al. (2007b) observed mathematics classes in primary schools in Brazil and, to a lesser extent, Chile, where significant percentages of the day were spent in "down time" with no organized activity taking place, or when students were left to copy problems and instructions from the chalkboard.

Causal linkages between classroom time-use indicators and student achievement are very difficult to establish, although the results from Bruns and Luque (2014) do suggest that test scores are higher in classrooms where more time is spent on instruction. But the time-use indicators tell only part of the story of the average mathematics lesson, and it is difficult to categorize each activity as good or bad. Instead, the collection of activities and their "flow" should be understood as the instructional structure that facilitates the implementation of effective practices, or limits student access to important mathematics content and processes.

Figure 4.9 shows the flow of two typical lessons found in LAC countries based on the kinds of classroom observation data described above (and the authors' own observations of more than 200 classes throughout the region). One is associated with more effective mathematics teaching practices (right side), the other with less effective practices (left side). The

FLOW OF COMMONLY OBSERVED LESSONS: LOW VERSUS HIGH EFFECTIVE CLASS ARCHETYPES

Low

Communicating topic or activity (3 minutes)

Explanation of concept or demonstration of procedure with questions and answers (12 minutes)

Individual work: students work individually on problems assigned (including copying problems from the board) (10 minutes)

Checking work by either sending student to the board or looking at notebooks (12 minutes)

High

Communicating topic or activity (5 minutes)

Students work individually or in groups with concrete materials (30 minutes)

Whole class discussion about the activity (15 minutes)

Individual work translating concrete models to notebooks (20 minutes)

Summary of content covered in a more abstract representation (10 minutes)

Source: Prepared by the authors.

instructional time allocation for the less effective lesson is characterized by the teacher providing a mini-lecture presenting a concept or procedure, sometimes accompanied by short-answer questions, followed by a period when students copy problems from the board (or textbook) and continue to work individually on assigned problems or exercises similar to the ones presented by the teacher. The class ends with the teacher checking student work at the students' desks or at the board with a few of the students participating. The less effective class is marked by inequality in participation, both in terms of the pace of work (i.e., some children are often far behind at the end of class) and the degree of participation in any accompanying discussion or recitation.

In contrast, the more effective lesson is characterized by the teacher introducing a task, students working individually or in groups with concrete materials or visual representations, teacher and students discussing their work, students translating their work into notebooks or onto the blackboard, and ending with a summary of the concept, procedure, or

main ideas. This type of structure is normally observed in longer lessons (about 40 percent longer than the less effective lesson prototype).

4.4.2 Use of Instructional Tools

Effective teaching elements related to exploring mathematical concepts, making connections among multiple representations, and using multiple strategies to approach problems (see discussion in the previous section of this chapter, and in Chapter 2) depend heavily on the use of learning materials. However, based on a mixture of evidence from the LAC region, manipulatives appear to be infrequently incorporated in most classrooms. First, the 2006 SERCE data show that sixth grade teachers report very little use of instructional tools in their mathematics classes, including an abacus, pattern blocks, cuisiner rods, base-10 blocks, tangrams, calculators, geoboards, or noncommercial manipulatives. The overall frequency average was roughly 1.5 on a 1–4 scale (1 = never, 2 = some classes, 3 = majority of classes, 4 = all classes).

Classroom observation studies provide similar results. Relatively small percentages of lessons use manipulatives, calculators, or computers, and teachers rely heavily on the blackboard and, to a lesser degree, on textbooks, especially at the upper elementary level (Bruns and Luque 2014; Carnoy et al. 2007; Araya and Dartnell 2008). There are certainly exceptions. For example, Carnoy et al. (2007) observed some teachers making very creative use of noncommercial manipulatives such as wooden sticks to construct regular polygons and explore their properties, or beans or lentils for counting, or as representations of points in a plane. Instead of prebuilt base-10 blocks, students were observed using their own color pencils to make groups of tens to model standard algorithms. In addition, there was evidence of the use of rulers, compasses, and protractors when learning measurement and geometry topics. But these kinds of activities appear to be the exception rather than the rule.

We are not aware of research that focuses on the linkages between student achievement and the use of learning materials in the LAC region, although this question is addressed below with data from SERCE. However, two recent metareviews from the United States show that manipulatives-based interventions are significant predictors of higher student achievement in mathematics (Carbonneau, Marley, and Selig 2013; Holmes 2013). This kind of evidence—combined with conceptual linkages between the use of manipulatives and particularly effective teaching activities (see Section 4.4.1)—raises serious concerns about the apparent lack of such interventions in the region. One obvious constraint is resource-related, as

the SERCE data show that about 40 percent of sixth grade teachers (in 2006) reported having fewer than three of the eight learning materials. But there are other kinds of systemic constraints, such as the form of the official curriculum and, more specifically, its representation in textbooks. If the tasks presented in mathematics textbooks do not focus on developing mathematical fluency based on a foundation of conceptual understanding, they are less likely to require the use of these materials. Hence teachers may not feel the materials are necessary tools for the teaching and learning of mathematics.

4.4.3 Mathematical Discourse and Questioning

Data from classroom observations can also be used to summarize mathematics discourse in LAC countries. Several formats are most common: whole class discussions, teacher and individual student interactions, and students discussing among themselves in groups (Bruns and Luque 2014; Carnoy et al. 2007; Carnoy, Gove, and Marshall 2007). On average, interactions appear to take up about one third of the average class and are concentrated in segments when the teacher is presenting material, or students are solving problems. In some of the observed countries, such as Panama and Chile, third grade students spend a substantial amount of class time (13 and 29 percent, respectively) at the chalkboard writing down their computations (Carnoy et al. 2007; Carnoy, Gove, and Marshall 2007). In general, these interactions are characterized by closed questions that require yes-no answers or a single word (e.g., "what is the place value of 5 in 1,052?") or that require students to complete sentences like "3 times 4 is…." In the case of students at the blackboard, the student is asked to write down the solution of a computation (often from his or her own notebook). If the student is correct, he or she is asked to sit down, and if the answer is incorrect, the teacher asks another student to come up to the board. Less common are interactions where teachers ask the students to explain their answers, correct one another's work, and provide explicit explanations of mathematical reasoning.

This evidence suggests that mathematical discourse and questioning in most of the classrooms observed does not have the purpose of assessing the students' thinking or promoting mathematical reasoning, but instead is more evaluative in nature, focusing on the final correct answers. Moreover, the reliance on simple questions should not be seen simply as a pedagogical choice on the part of teachers. It is intricately related to their specialized knowledge and ability to orchestrate the class in such a way that challenges their students along a range of skills.

4.4.4 Classroom Processes and Student Achievement: SERCE and TERCE Data

The review here of the evidence on mathematics achievement concludes with the same data source with which it began. In the TERCE (and SERCE), sixth grade students were asked a range of questions about classroom teaching and learning processes, including the use of computers and other technology. By averaging their responses by classroom, it may be possible to capture meaningful variation in teaching strategies, and relate these factors to student achievement levels. It should be (re)stated that these are, at best, tentative linkages: even as classroom averages the various indicators are subject to measurement error and are no substitute for actual observational data. But with so many available indicators, and in urban schools, the LLECE data on classroom processes are simply too interesting to ignore.

This last section is divided into two parts: general classroom processes and conditions, followed by a review of computer and other technology usage.

Classroom Teaching and Learning Processes
Figure 4.10 summarizes the results for a large group of classroom process variables using the same statistical analysis and reporting strategies described in Sections 4.1 and 4.2. The variables themselves come mainly from the TERCE, with five exceptions from the SERCE (flagged with an asterisk).

Overall, the results in Figure 4.10 show that few variables are consistently significant predictors of student achievement. This is another reminder of the inherent difficulties of understanding effective classroom practices on the basis of large-sample survey instruments. The result that stands out is the frequency that students report solving mathematics exercises (only available in the SERCE). As an individual student-level measure it is the most consistent predictor of mathematics achievement in Figure 4.10 (see also Cueto et al. 2014), although as a classroom average it is less often significant. There are well-founded concerns about an overreliance on solving simple mathematics exercises with low levels of cognitive demand such as memorization and procedures without connections to underlying concepts (Stein et al. 2000). This is especially true when it comes at the expense of teaching activities that expose students to mathematical reasoning and conceptual understanding. Nevertheless, the results from the statistical analysis do point to the potential importance of actively engaging students in mathematics activities of some kind, which is consistent with specific elements of an effective teaching framework. For example, Chapter 2 refers to the importance of practice during different phases of student acquisition of skills.

FIGURE 4.10

SUMMARY OF PREDICTORS OF SIXTH GRADE MATHEMATICS ACHIEVEMENT IN THE SERCE-TERCE: CLASSROOM TEACHING PROCESSES AND CONDITIONS
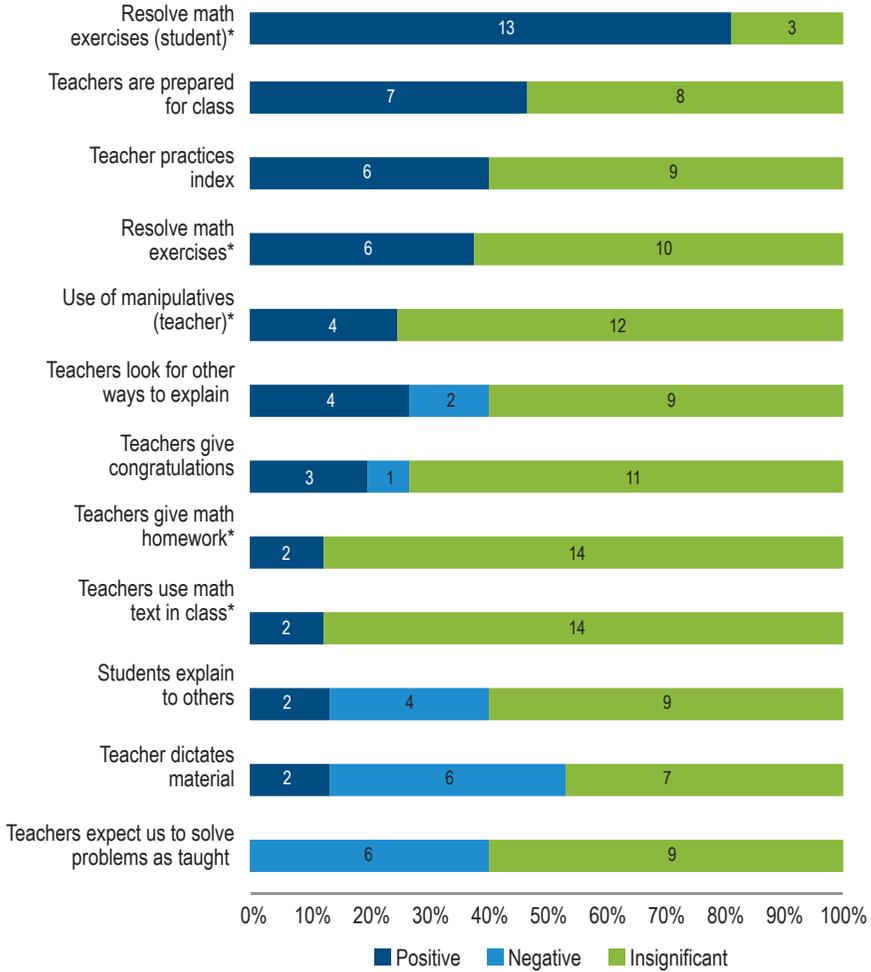
Sources: Second Regional Comparative and Explanatory Study (SERCE), 2006; and the Third Regional Comparative and Explanatory Study (TERCE), 2013.

Note: All results are based on country-specific regression analyses (15–16 in all) using weighted data for urban students; see the main text for more detail on the regression model. The numbers in the bars refer to the number of countries. Variables were obtained from TERCE student questionnaires, and are measured as classroom averages. Exceptions are noted in parentheses (for data source), and asterisks (*) refer to variables available only in the SERCE. Process measures are based on three-point scales (i.e., 1 = never, 2 = some classes/sometimes, 3 = most/all classes). "Teacher practices index" is provided by the TERCE, and is an index based on multiple classroom process variables.

Figure 4A1.2 in Annex 4.1 presents the quintile gap comparisons for four variables: availability of manipulatives in the classroom (see Section 4.4.2), frequency of solving mathematics exercises, teacher preparedness (according to students), and the teacher's use of dictation. The prevalence of gaps of at least 0.50 standard deviations (see red bars) certainly stands out, and suggests meaningful differences in classroom environments between relatively wealthy and poor urban schools in the region. Some of the largest gaps are found in the frequency that students report the use of dictation in the classroom, which is a potential sign of low cognitive task design (see Section 4.3.2). However, it is important to note that students across all quintile groups and countries report fairly frequent use of dictation (averages above 2 on a 1–3 point scale), so the gaps between quintiles—while meaningful in most countries—do not show a clear separation by school.

Technology and Mathematics Achievement

The SERCE and TERCE assessments also include a number of questions about technology. These variables fall short of informing one of the central topics of this book, which is how computer-assisted learning technology solutions can impact mathematics achievement. However, they do make it possible to assess the availability of resources like computers and the Internet in the region's urban schools, and provide at least a glimpse into how technology resources are used in classrooms.

The regression summary results in Figure 4.11 show that the strongest predictors of student mathematics achievement are the indicators for the ratio of computers (with Internet) to students, and the frequency that students report using computers in the school—but not in their classes. These results are likely related to the overall resources available in schools and communities, and say little about how the actual use of computers affects student learning.

The result that stands out in Figure 4.11 is the number of negative relationships between student achievement and technology usage. This includes classroom averages for computer usage during class and as part of homework, and the use of calculators in mathematics homework (SERCE). Sixth grade students were also asked a series of questions about computer use during their natural science class (but not mathematics). In general, the classroom averages across these various activities (look for information online, teacher uses computer to present material, etc.) were quite low: about 1.7 out of a 4 point scale, or in between "never" and "some classes." However, students who study in classrooms with more reported usage of computers in science classes have lower mathematics scores, ceteris paribus, in 8 of the 15 TERCE countries.

FIGURE 4.11

SUMMARY OF PREDICTORS OF SIXTH GRADE MATHEMATICS ACHIEVEMENT IN THE SERCE-TERCE: TECHNOLOGY AVAILABILITY AND USAGE
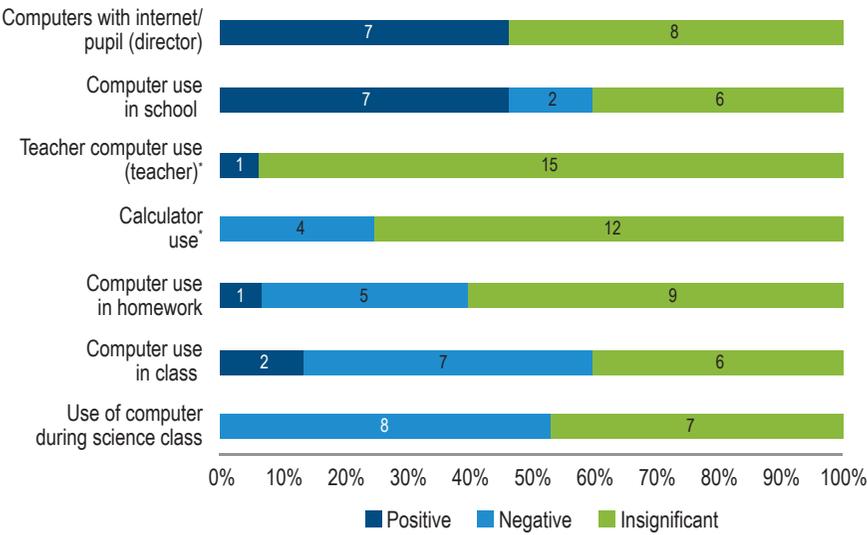
Sources: Second Regional Comparative and Explanatory Study (SERCE), 2006; and the Third Regional Comparative and Explanatory Study (TERCE), 2013.

Note: All results are based on country-specific regression analyses (15–16 in all) using weighted data for urban students; see the main text for more detail on the regression model. The numbers in the bars refer to the number of countries. Variables were obtained from TERCE student questionnaires, and are measured as classroom averages. Exceptions are noted in parentheses (for data source), and asterisks (*) refer to variables only available in the SERCE. Process measures are based on three-point scales (i.e., 1 = never, 2 = some classes/sometimes, 3 = most/all classes). "Use of computer during science class" refers to the classroom averages for a series of statements about the use of computers during science classes.

These results indicating a negative relationship between technology usage in the classroom and mathematics achievement are robust to different statistical specifications,[8] but they still must be handled with a lot of care. At the very least, the negative results are a reminder of the danger of relying on simple technological solutions to improve student achievement levels. There is nothing magical about introducing a computer (even with Internet) into a classroom, just as providing a calculator to a student will

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[8] These include adding the availability of computers in the school as a control, as well as including rural schools and dropping private schools (from the urban-only regressions). Wealthier students report much more usage of computer resources across all venues (school, home, etc.), and wealthier schools have significantly more computer and Internet resources. However, computer usage within classrooms is not much different between wealthy and poor schools within urban areas.

not automatically improve that student's mathematics achievement. These are tools that need to be used effectively in conjunction with, or better still incorporated into, learning tasks based on the critical elements referenced elsewhere in this chapter. In fact, technological aids can even work at cross-purposes with actual learning when students use them to solve problems and obtain answers without exploring concepts, gaining fluency in algorithmic types of exercises, or committing important facts to memory.

4.5 Conclusion

This chapter has focused on three interrelated questions regarding mathematics teaching and learning in LAC: What does the average classroom look like? Which input and process variables are the most significant predictors of student achievement in mathematics? And how are these critical features distributed in the region's urban schools, especially across social class differences? As stated at the beginning, this is an ambitious agenda given the region's diversity between countries, the lack of empirical research in some of the critical areas addressed, and the inherent difficulty of establishing causal relations between (1) classroom and teacher characteristics and (2) student outcomes such as achievement.

With these challenges in mind, it is important to take into account the dangers of generalizing across the entire region. Nevertheless, the main findings from the chapter are easily summarized, and are consistent with previous studies of teaching in the LAC region (Bruns and Luque 2014). The result that stands out in this chapter is the yawning gap between what an effective mathematics class should look like, and what many classes in the region actually look like. In general, the observed classrooms in LAC are too dependent on learning tasks with low levels of cognitive demand and direct teaching practices, and as a result the lessons do not challenge students to really learn mathematical concepts, which is the gateway for reaching proficiency. Few teaching materials are used, the interaction between teacher and student is too often focused on simple recitation rather than discussion, and students are not always asked to demonstrate their work, or show mastery before the class moves on to the next topic. Simply stated, this is a recipe for low average achievement, which is largely corroborated by Latin American and Caribbean mathematics scores in national, regional, and international assessments.

The chapter also highlighted concerns about equity, which of course touch on a long-standing theme in LAC. There is certainly evidence that inputs such as computers and school infrastructure, as well as some school climate characteristics, are very unevenly distributed across, and within, urban school sectors in the countries of the region. This is not a surprise, and

it bears restating that these unequal conditions generally persist even when the samples are restricted to urban public schools. However, the story is less clear for the critical teacher and teaching elements obtained from existing studies. Even amid data constraints, however, two tentative conclusions seem warranted based on the evidence that is available. First, few classrooms exhibit effective teaching characteristics, and those that are observed are likely to be in relatively wealthy schools. Second, and more importantly, the kinds of children who need especially effective teachers are not studying in classrooms where the teacher is well prepared to help them overcome disadvantages associated with poverty and low levels of parental education.

What explains the general lack of quality observed in the region's urban primary school mathematics classrooms? The answer must include references to contextual factors. Urban teachers in the region appear to be working with limited teaching materials beyond a textbook, and their students receive little help outside of class; on average, almost 50 percent of the parents in the sixth grade urban samples have not advanced past primary school. But difficult conditions alone are not responsible for this situation. Capacity limitations are also present, which in this review focused on different forms of mathematics knowledge that teachers must have to be effective, yet in many cases do not appear to have. Solutions for addressing these deficiencies are beyond the reach of this chapter, but the analysis highlights the importance—in various regions around the world, not just LAC—of creating systemic conditions where teachers are well prepared for their work, where they are adequately supported, and where quality control is paramount.

This issue of facilitating teacher effectiveness provides a useful segue into a key theme of this book, which is how computer-assisted learning can help students and teachers in mathematics classrooms in LAC. Other chapters take up this question in more detail; the focus here has been on how technological solutions align with the model of effective teaching detailed in Chapter 2. The results in this chapter raise some concerns about a simplistic reliance on technological solutions in education: the use of computers and the Internet in a classroom will not automatically improve learning, even if they provide students with a more enjoyable experience. Fortunately, current mathematical practices and established classroom structures can be leveraged to make use of new technology in a way that can be incorporated into existing pedagogical methods. To this point, we agree with the statement by Means (2010, 304) that "educators and policymakers need to stop thinking of learning software as an intervention in and of itself and to think instead of broader instructional activity systems." In sum, mathematics classrooms in LAC need to expose students to learning tasks that promote reasoning and thinking, and technology can serve as a catalyst for reaching this goal.

FIGURE 4A1.1
**MATHEMATICS CONTENT KNOWLEDGE OF FUTURE MATHEMATICS
TEACHERS (SCALE)**
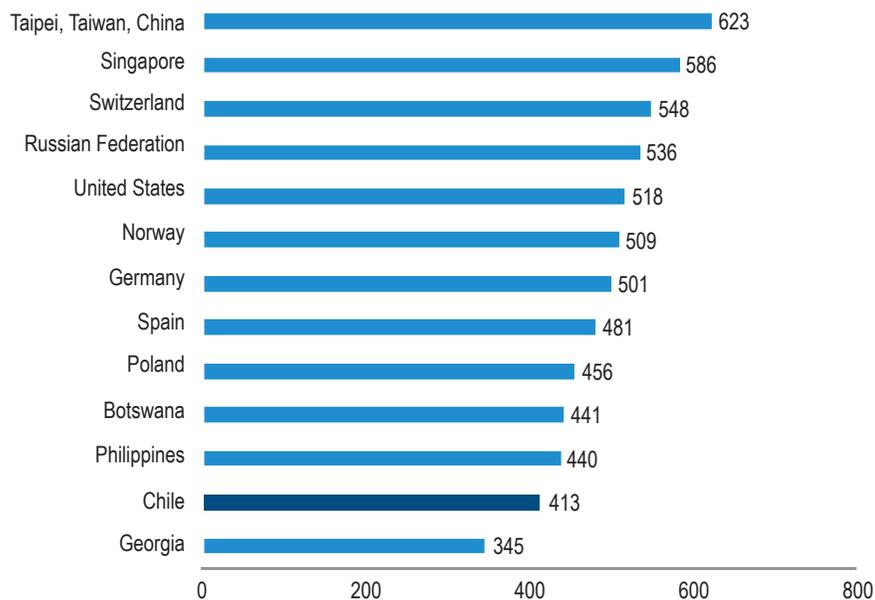
Source: Bruns and Luque (2013: Figure O.5), based on data from the 2008 Teacher
Education and Development Study in Mathematics (TEDS-M).
Note: Mathematics content knowledge is a scale measure with mean = 500.

FIGURE 4A1.2

EQUITY COMPARISONS OF SELECT VARIABLES ON THE SERCE (2006) AND TERCE (2013) FOR SEVEN LATIN AMERICAN COUNTRIES
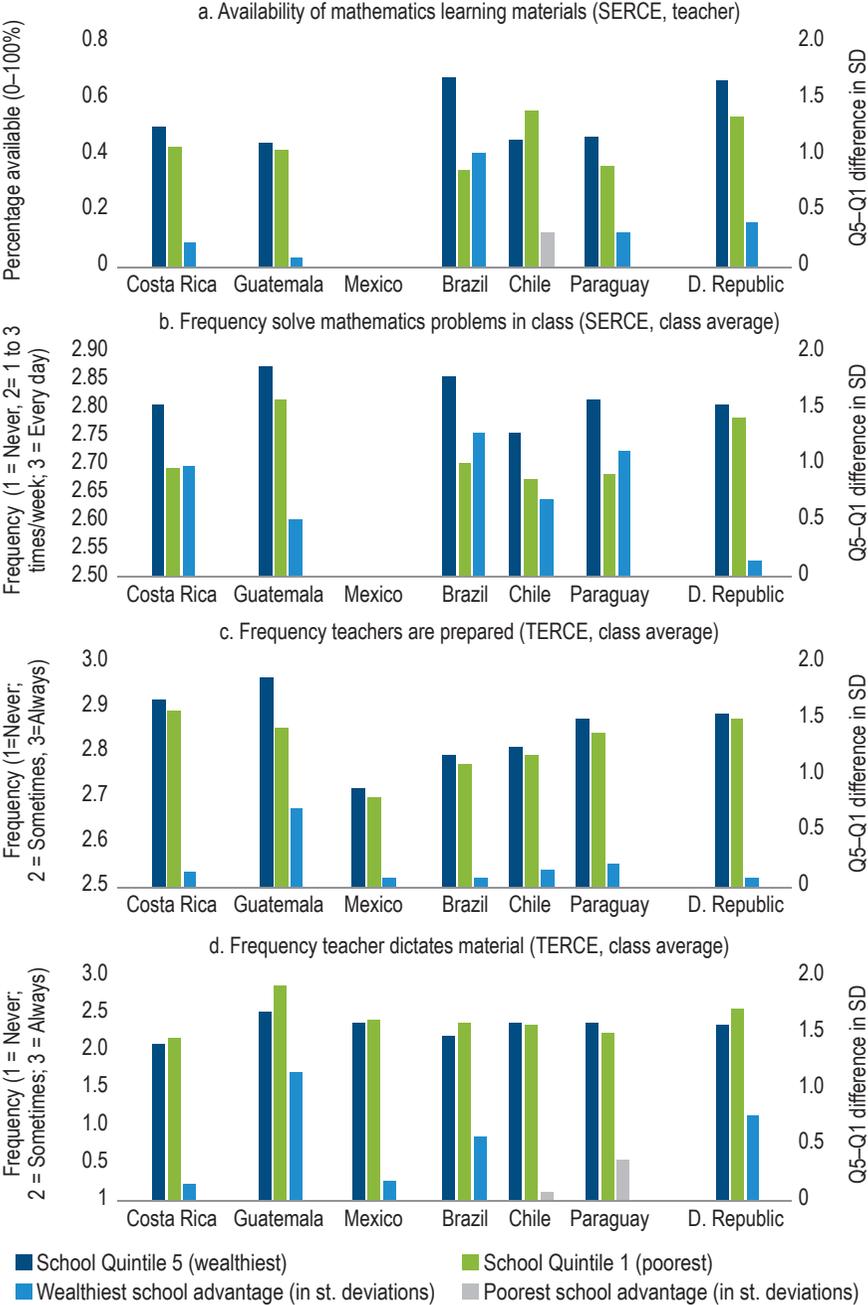
Sources: Second Regional Comparative and Explanatory Study (SERCE), 2006; Third Regional Comparative and Explanatory Study (TERCE), 2013; and authors' calculations.

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Promoting a Good Start: Technology in Early Childhood Mathematics

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Educational technology has the potential to make multiple contributions to early mathematics education in Latin America and the Caribbean (LAC). Whether this potential is realized depends on which technology is used, and how. Research on different models of educational technology has identified the specific benefits of each. These models include technology-assisted instruction (including practice, tutorials, tasks, tools, and games); technology manipulatives; programming, coding, and robotics; and combinations of these models. To realize the benefits from these different models of educational technology, teachers need support and professional development. Fortunately, there is a growing supply of such resources.

5.1 Promoting a Good Start

First grader José never talked aloud, was slow to complete his work, and was placed in a "socialization group" to "draw him out of his shell." When a computer arrived, José spent nearly 90 minutes with the machine his first day. Immediately thereafter, his teacher noticed that he was completing seatwork without prompting. Then he would slide his seat over to the technology

and watch others program (write computer code, or instructions) in the computer language Logo—directing an onscreen "turtle" to draw geometric figures. For example, the code "repeat 4 [forward 100 right 90]" would instruct the turtle to draw a straight line, then turn 90 degrees to the right, and do that four times in all, drawing a square. Soon after, José would stand beside the technology, talking and making suggestions. When others had difficulties, he was quick to show them a solution. Others started getting help on Logo from him. He began completing twice as much work per day as he had previously. He participated eagerly during class discussions and as a "crowning achievement" of sorts was given a 10-minute "time out" one day because he wouldn't stop talking. In brief, José moved up from the lowest to the highest achievement group.

Are such results merely coincidences or real benefits of certain technology environments? If the latter, how can technology be used to maximize these benefits for early mathematics teaching and learning? What are the unique characteristics of these technology environments that can be capitalized upon? What are the advantages and disadvantages of different models of educational technology for early mathematics? What are effective teaching strategies? What professional development and support do teachers require? Finally, what do the models and research suggest for effective use of educational technology in LAC? This chapter addresses these questions in turn, but first it asks: Why early mathematics?

5.1.1 The Promise, and Unrealized Potential, of Early Mathematics

Early mathematics is surprisingly important. What children know and can do in their first years in school can predict their mathematics achievement for years to come—and even throughout their school career. Moreover, what mathematics they know predicts their reading achievement. Mathematics appears to be a core component of cognition (Clements and Sarama 2011; Duncan et al. 2007; Duncan and Murnane 2014).

Further, mathematics is a critical learning area: many students perform poorly in it in schools throughout the world (Clements and Sarama 2003, 2007c), including in LAC (see Chapter 3; see also Bos, Ganimian, and Vegas 2013). For example, students in LAC are more than two years behind their student counterparts in Organisation for Economic Co-operation and Development (OECD) countries in math, reading, and critical thinking skills—and even further behind East Asian countries, including Vietnam (Bruns and Luque 2014). The lack of computer resources explains part of this gap (Breton and Canavire-Bacarreza 2018). This chapter focuses on children in the lower grades of primary schools in LAC's urban areas

because foundational competencies and dispositions are critical for success throughout students' school careers.

5.1.2 Technology and Young Children: Debates, Theory, and Research

Twenty years ago, it was argued that "we no longer need to ask whether the use of technology is 'appropriate'" in early childhood education (Clements and Swaminathan 1995, 275). The research supporting that statement was, and remains, convincing, but social and political movements follow their own cyclical course and there remain polemics against the use of technology by young children. This is important, because some teachers retain a bias against technology that contradicts research evidence (Lindahl and Folkesson 2012). Especially in mid-socioeconomic-status schools, some teachers believe it is "inappropriate" to have technology in classrooms for young children (Lee and Ginsburg 2007).

We have countered such criticisms elsewhere (Clements and Sarama 2003), and others have similarly argued against such critiques (Lentz, Seo, and Gruner 2014). In the meantime, research continues to accumulate that, for example, homes with more technology better support mathematics learning (Li, Atkins, and Stanton 2006; Navarro et al. 2012), and this is particularly so for children from minority households (Judge 2005). Having said that, it is also true that some correlations are not significant, including those from research in LAC (see Chapter 4). So, clearly, the way technologies are used matters. The sections that follow summarize some basic findings from research on young children and technology (Clements and Sarama 2010).

Children Working with Technology
Perhaps the oldest criticism is that educational technology is "developmentally inappropriate" for young children (Barnes and Hill 1983). One argument is that such technology inappropriately demands "abstract thinking" (Cordes and Miller 2000). Such criticisms are based on discredited interpretations of Piagetian theory (Gelman and Williams 1997).

Perhaps most important, these criticisms are based on an overly general and undifferentiated view of technology. The types of technologies available and their content vary widely and, when used appropriately, can benefit children from preschool through third grade (Clements and Sarama 2007c, 2010), especially for mathematics (Shin et al. 2012; Thompson and Davis 2014). The nature and extent of technology's contribution depends largely on what models of technology are used and the goals put forth for these models.

Approaches to Educational Technology: Beyond False Dichotomies

Debates also broil regarding how technology should be used in improving mathematics learning. These may involve false dichotomies. For example, some educators focus solely on drills—an approach that used alone is pernicious and even ineffective for those limited goals (see Chapter 2; see also Henry and Brown 2008). Other educators tolerate "open-ended" or (narrowly defined) "developmentally appropriate" technology applications based on a constructivist view. We believe that constructivism is an important theoretical construct (Sarama and Clements 2009b), but also that although it has important implications for teaching, it tells us more about learning than about teaching (Clements 1997). Thus, policies and practices must carefully consider how children learn mathematics and how technology might support that learning.

Learning trajectories. The theoretical foundation here is based on learning trajectories, a device for constructive-based learning and teaching. Each learning trajectory has three parts: (1) a goal, (2) a developmental progression, and (3) instructional activities. To attain a certain mathematical competence in a given topic or domain (the goal), students learn each successive level (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects that enable thinking at each higher level (Clements and Sarama 2014). Teaching strategies include early exploratory work (or play) and a range of techniques from problem-solving to a variety of explicit instructional strategies. One key is to integrate them appropriately (e.g., in phases of learning; see Chapter 2 and van Hiele 1986).

The role of technology in the implementation of learning trajectories. The first phase of learning allows children to explore a topic initially, followed by Phase 2 activities that require students to apply the concepts to solve problems. Concepts and skills are developed together and connected. Only once they are firmly established are Phase 3 tasks introduced to develop fluency. These tasks are, of course, sequenced corresponding to the developmental progressions to complete the hypothesized learning trajectory. Further, the characteristics of the tasks and their accompanying pedagogical interactions are explicitly linked to transitions between levels.

In conclusion, technology can make substantial contributions to early childhood mathematics education, if used well (Sarama and Clements 2002b; Seng 1999), with applications consistent with the phases of learning. The bad news is that reality often falls short of realizing this promise (Cuban 2001). To be effective, policies and practices need to be based

on research and the wisdom of expert practice. Simply providing hardware is not likely to increase the learning of mathematics (Ortiz and Cristia 2014) although it may increase cognitive skills (Cristia et al. 2017). Further, even if used well, technology alone cannot be expected to have more than a moderate effect (Cheung and Slavin 2013). This chapter draws implications from what has been learned from research regarding selecting models of educational technology, using effective teaching strategies, and providing professional development.

5.2 Technology Models for Early Mathematics

The introduction to this chapter described the different models of technology use in mathematics education. This section will discuss and provide illustrations on specific issues regarding the application of these models to early mathematics education. From the outset, it is important to note that the mathematics curriculum used makes a significant difference in what and how well children are learning (Agodini et al. 2010), and decisions about educational technology should be in concert with the core curriculum.

5.2.1 Technology-assisted Instruction

Even young children can benefit from technology-assisted instruction (TAI) (including any use of digital devices for educational purposes) to develop mathematical skills and concepts. One review of rigorous studies indicated that technology-assisted instruction applications that are well designed and implemented could have a positive impact on mathematics performance (National Mathematics Advisory Panel 2008), and recent studies support this conclusion (Moradmand, Datta, and Oakley 2013; Outhwaite et al. 2019; Nusir et al. 2013; Thompson and Davis 2014). Another recent review also concluded that TAI has positive effects, although they are modest, and also suggested that there can be differences depending on which TAI model is used. Supplemental TAI had the largest effect at +0.19. Two other interventions had smaller effect sizes, but still positive. Technology-management learning—that is, software that administers and scores tests and uses the information to formulate instructional decisions—had an effect of +0.08. Comprehensive programs, which integrate TAI and traditional instruction into one curricular system, had an effect of +0.07. However, another meta-analysis of educational technology for early mathematics found a larger effect of .48 (.53 for number sense, .42 for operations, .57 for word problems, and .59 for geometry and measurement) (Harskamp 2015).

Practice

A common use of technology-assisted instruction is to provide practice, for example, in skills such as counting and sorting (Clements and Nastasi 1993) or addition facts (Fuchs et al. 2006). Indeed, some reviewers claim that the largest gains in the use of TAI have been in mathematics practice for lower-primary-grade students (Fletcher-Flinn and Gravatt 1995), especially in compensatory education programs (Lavin and Sanders 1983; Ragosta, Holland, and Jamison 1981). About 10 minutes per day proved sufficient time for significant gains, and 20 minutes was even better. (Note that research recommends short repeated sessions, so for young children, 5–15 minutes in a session is suggested). Another program showed good effects in arithmetic fluency for first graders who practiced for 15 minutes three times per week for four months (Smith, Marchand-Martella, and Martella 2011). This approach to TAI may be as or more cost-effective than traditional instruction (Fletcher, Hawley, and Piele 1990) and other instructional interventions, such as peer tutoring and reducing class size (Niemiec and Walberg 1987). This approach has been successful with all children (Shin et al. 2012), with substantial gains reported for children from low-resource communities (Primavera, Wiederlight, and DiGiacomo 2001).

Technology practice can be especially helpful for children who have mathematical difficulties or mathematical learning disabilities (Harskamp 2015). However, this must come at the right point in the learning trajectory and it should be the right kind of practice. For example, "bare bones" practice, such as repeated, speed-based drills in arithmetic "facts" does not help children who are at the level of more immature counting strategies. Instead, research suggests practice that helps them understand concepts and learn arithmetic facts before any time-pressured drills (Hasselbring, Goin, and Bransford 1988). Also, practice that teaches fluency and cognitive strategies may be more effective, especially for boys (Carr et al. 2011).

How young can children be and still obtain such benefits? Three-year-olds learned sorting from a technology task as easily as from a concrete doll task (Brinkley and Watson 1987–1988). Reports of gains in such skills as counting have also been reported for kindergartners (Hungate 1982).

The position taken in this chapter is that drills should be used carefully and in moderation, especially with the youngest children, whose creativity may be harmed by a consistent diet of drilling (Haugland 1992). Some students may be less motivated to perform academic work or less creative following a steady diet of only drills (Clements and Nastasi 1985; Haugland 1992). There is also a possibility that children will be less motivated to perform academic work following drills (Clements and Nastasi 1985) and that drills on computers alone may not generalize as well as paper-and-pencil work

(Duhon, House, and Stinnett 2012). In contrast, practice that encourages the development and use of strategies, that provides different contexts (supporting generalization), and that promotes problem-solving may be more appropriate than drills, or may be best used in combination with them. To be effective, all types of practice must follow and be consistent with instruction in Phase 1 (to explore), followed by Phase 2 (to apply the concepts to solve problems), and must be appropriate for the children's culture.

Tutorials, Tasks, Teasers, and Tools

Other technology-assisted instruction models include and often combine approaches that go beyond simple practice, such as tutorials, tasks, teasers, and tools (e.g., screencasting, or using an app on a table that captures audio and video of what is written or presented on the screen; see Thomas 2017). As an example of a teaser (i.e., a puzzle or problem to solve), 5-year-old Maria was presented with the task of finding a cartoon character with big eyes and no stripes. She poised over one with stripes and said, loudly, "*Not* stripes!" She moved to another that did not have stripes. "Ha! I think . . . is this the right one! No, small eyes! [Moving again] Is this one [no stripes and big eyes]? Yes, Then I click on it." She was correct. Maria developed an understanding of and then became fluent in not only attributes and logic but also in thinking strategies and "learning to learn" skills.

As another illustration, consider a technology-enhanced learning trajectory (Clements and Sarama 2007/2013). The goal is to learn geometric composition using problem-solving abilities to put shapes together to make other shapes. The developmental progression indicates that children with an initial lack of competence in composing geometric shapes gain the ability to combine shapes into pictures—initially through trial and error and gradually by attributes, then finally by synthesizing combinations of shapes into new shapes. The levels of this progression along the third component of the learning trajectory (instructional tasks) are presented in Figure 5.1—a series of shape puzzles increasing in difficulty. Children enjoy that the blocks "snap" and stay together accurately. (Children initially solve outline puzzles with physical pattern blocks.) More importantly, they use the program's tools to perform actions on the shapes. Because the children have to figure out how to move the blocks and then choose a motion such as slide or turn, they are more conscious of these geometric motions. Four-year-old Juanita initially referred to the "spinning" tools, but later called them the "turn shapes" tools, and after several months was describing directions and quantities, such as "OK, get *this* [right or clockwise] turn tool and turn it three times!" Such choices also encourage children

FIGURE 5.1

SAMPLES FROM A LEARNING TRAJECTORY FOR THE COMPOSITION AND DECOMPOSITION OF GEOMETRIC SHAPES

Age	Developmental Progression	Instructional Tasks	
2–3	**Piece Assembler**. Makes pictures in which each shape represents a unique role (e.g., one shape for each body part) and shapes touch.	In the first level of the "Piece Puzzler" series, each shape is outlined, but touches other shapes only at a point, making the matching as easy as possible.	
4	**Picture Maker**. Puts several shapes together to make one part of a picture (e.g., two shapes for one arm). Uses trial and error and does not anticipate the creation of a new geometric shape.	The tasks at this level start with those where several shapes are combined to make one "part," but internal lines are still available. Note that turns and flips must be used.	
5	**Shape Composer**. Composes shapes with anticipation ("I know what will fit!"). Chooses shapes using angles as well as side lengths.	Puzzles at this level have no internal guidelines and larger areas; therefore, students must compose shapes accurately.	
6	**Substitution Composer**. Makes new shapes out of smaller shapes and uses trial and error to substitute groups of shapes for other shapes to create new shapes in different ways.	"Piece puzzler" tasks are similar; the new task here is to solve the same puzzle in several different ways.	
7	**Shape Decomposer with Imagery**. Decomposes shapes flexibly using independently generated imagery.	In the "super shape" series, students only have one shape in the shape palette and they must decompose that shape and then recompose those pieces to complete the puzzle.	
	Shape Decomposer with Units of Units. Decomposes shapes flexibly using independently generated imagery and planned decompositions.	In this "super shape" level, students only get exactly the number of "super shapes" they need to complete the puzzle. Again, multiple applications of the scissors tool are required.	
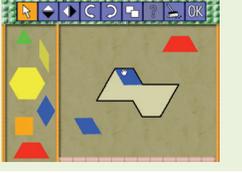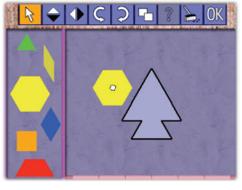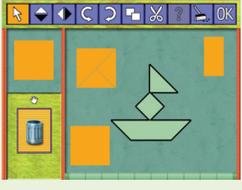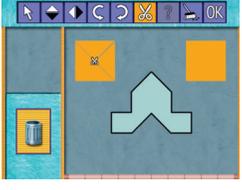
Sources: Adapted from Clements and Sarama (2014) and Sarama and Clements (2009b).

to be more deliberate. They "think ahead" and talk to each other about what shape and action to choose next. In these ways, the technology slows down their actions and increases their reflection. Just as important, using the motion tools deliberately helps children become familiar with seeing shapes in different orientations and realizing that changing the orientation does not affect the shape's name or class. In a related activity, children are challenged to build a picture or design it with physical blocks and copy it into the program. Again, this requires the use of specific tools for the geometric motions of slide, flip, and turn and encourages children to reflect on the orientation of the shapes. Note that this and other studies show that tool-type interfaces are needed to gain this benefit even though direct manipulation is possible.

Multiple studies have supported the effectiveness of this learning trajectory (Clements and Sarama 2007b, 2008a; Clements et al. 2011). The software combines the tasks (motivating puzzles) and tools (geometric motion tools) already described. In addition, brief hints and then tutorials are presented if children make several consecutive mistakes. The software used alone is effective and was particularly helpful to Hispanic dual-language learners (Foster et al. 2016; Foster et al. 2018).

Similar successes have been reported for other research-based programs. For example, TAI, even with minimal direction from a teacher, has been found to be a feasible means of helping first graders with a risk factor discover the add-1 rule (adding 1 is the same as "counting one more") by way of pattern detection (Baroody et al. 2015). The software might ask, "What number comes after 3 when we count?" and then immediately follow that by answering a related addition question, "3 + 1 = ?" Also, an "add-zero" item and an addition item (with both addends greater than one) served as nonexamples of the add-1 rule to discourage overgeneralizing this rule. A similar technology program that combined fluency and cognitive strategy use helped second graders, especially boys, improve their arithmetic achievement (Carr et al. 2011). A suite of activities increased low-income preschoolers' mathematics achievement with an effect size of 1 standard deviation, more than a year's gain over the control group (Schacter and Jo 2016).

Also important are efforts to ascertain what type of goals different types of TAI can achieve. For example, all kindergartners working in multimedia environments improved their mathematical skills more than those not working with any technology environment. Further, those working individually performed at the highest level, while those working cooperatively increased their positive attitude about cooperative learning (Weiss, Kramarski, and Talis 2006). Finally, longer tutorials are rare in early mathematics,

but some programs are developing new approaches. One used collaborative multimedia environments with problems that 4–7 year-old children solved cooperatively (Kramarski and Weiss 2007). They were given three steps of feedback to support their learning. These children outperformed those who worked collaboratively but without the multimedia environment. In another approach, children created digital images that represented a person or character and used that character to share thoughts and ideas through typed text or the computer microphone (Cicconi 2014).

Games
Properly chosen, technology games may also be effective (see Chapter 6; see also Ketamo and Kiili 2010). Second graders with an average of one hour of interaction with a technology game over a two-week period responded correctly to twice as many items on an addition facts speed test as did students in a control group (Kraus 1981). Even younger children benefit from a wide variety of technology-based as well as nontechnology games (Clements and Sarama 2008b). For example, in one simple game, young children place finger combinations on an iPad to play a game of recognizing and representing numbers before time runs out. Early pilot work with this novel interface, which also promotes use of children's most accessible manipulative, their fingers, is promising (Barendregt et al. 2012).

Again building upon learning trajectories, Figure 5.2 illustrates selected levels (Clements and Sarama 2007/2018) of a series of technological board games meant to progressively develop children's competencies in the domain of counting, leading to counting-based addition and subtraction strategies. For example, at the level of Producer (small numbers), at which they can accurately count out up to 5 objects, children may be able to solve simple arithmetic problems such as 3 + 2 by "counting all"—producing a set of 3, then a set of 2, then counting them all. An advance is made at the level of Counter On Using Patterns, at which children may add by counting on from the first addend one or two numbers, thereby solving 4 + 2 by saying "4, *5, 6,*" saying 5 and 6 in a rhythmic pattern of two beats. These children still might be unable to count on five or more, because the rhythm would be too challenging. However, when they progress to the Counter on Keeping Track level, they can keep numerical track of how many they are counting up (i.e., the second addend), thereby solving 4 + 5 by counting "4…, 5, 6, 7, 8, 9," putting up five fingers for each count to keep track. Again, the game formats and clear goals motivate children, and the instruction combines several approaches. The tasks use linked representations to ensure that children build a strong number concept. That is, children are supported in connecting images, written symbols, oral symbols, and actions, which

encourages learning and retention (Mayer 2014). The connection of the dot arrays, the length moved on the path, the time it takes to make that move, and so forth all serve to build number sense (Siegler and Ramani 2008). A management system (see below) moves children along a research-based learning trajectory, thus employing the powerful educational strategy of formative assessment (Penuel and Shepard 2016) to ensure that each child is learning new concepts and skills because the tasks are challenging but achievable (Hiebert and Grouws 2007).

Newer games can take quite different forms. For example, in one project, a robot was used to promote engagement, social interaction, and geometry

FIGURE 5.2
LEARNING TRAJECTORY FOR COUNTING

Age	Developmental Progression	Instructional Tasks	
4	**Counter (small numbers)**. Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted.	*Road Race Counting Game*. Students identify number amounts on a dot frame and move forward a corresponding number of spaces on a game board. *Road Race*. Students identify numbers of sides (three, four, or five) on polygons and move forward a corresponding number of spaces on a game board.	
5	**Producer — Counter To (small numbers)**. Counts out objects to 5.	*Numeral Train Game*. Students identify numerals (1–5) and move forward a corresponding number of spaces on a game board.	
	Counter from N (N+1, N-1). Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the *number* of counts).	*Sea to Shore*. Students identify number amounts by (simple) counting on. They move forward a number of spaces on a game board that is *one more* than the number of dots in the fives and tens number frame.	

(*continued on next page*)

FIGURE 5.2 (*continued*)
LEARNING TRAJECTORY FOR COUNTING

Age	Developmental Progression	Instructional Tasks	
6	**Counter On Using Patterns**. Keeps track of a few counting acts, but only by using numerical pattern.	*Bright Idea*. Students are given a numeral and a frame with dots. They count on from this numeral to identify the total amount, and then move forward a corresponding number of spaces on a game board.	
7	**Counter on Keeping Track.** Keeps track of counting acts numerically, first with objects, then by "counting counts." Counts up 1 to 4 *more* from a given number.	*Easy as Pie*. Students add two numerals to find a total number (sums of one through ten), and then move forward a corresponding number of spaces on a game board. *Eggcellent*. Students use strategies to identify which two of three numbers, when added together, will enable them to reach the final space on a game board in the fewest number of moves. Often that means the sum of the largest two numbers, but sometimes, other combinations allow for hitting a positive or avoiding a backward action space.	
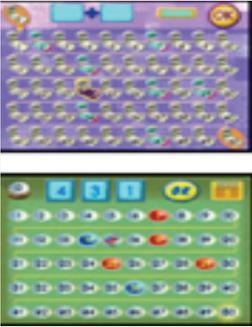
learning by engaging children in social games and activities (Keren and Fridin 2014). The robot pictured on a screen identifies a shape and asks children to find and touch the same shape on the physical robot. Evaluations revealed that these experiences improved both geometric thinking and meta-cognitive tasks in kindergartners (Keren and Fridin 2014).

Technology-enhanced Management

Many systems employ technology-managed instruction, in which computers keep track of children's progress and help individualize the instruction they receive. For example, such a system might store records of how

children are doing on every activity. It assigns them to just the right difficulty level according to past performance, using the research-based learning trajectories for each topic. Teachers can view records of how the whole group or any individual is doing at any time. The management system automatically adjusts the activity for difficulty and provides appropriate feedback and help. Some systems provide testing and worksheet generators. Such programs have been shown to increase math achievement of low-, middle- and high-performing students (Ysseldyke et al. 2003). In the future, assessment systems may integrate a curriculum-embedded benchmark, and summative assessments within and across levels, from curriculum-embedded classroom assessments to international comparisons (Quellmalz and Pellegrino 2009).

Technology-assisted Instruction—A Caveat

Policymakers and educators cannot assume that any model of TAI is effective in every instance. Whatever model is chosen, high-quality software and implementation are needed to realize effects—even moderate effects (National Mathematics Advisory Panel 2008). Consider that research reviews often report small-to-moderate effects (Cheung and Slavin 2013; Clements and Sarama 2003; Sarama and Clements 2009a). Moreover, the research reviewed may have incorporated higher-quality software than much of what is available. Therefore, specific software packages and implementation plans must be identified and piloted.

There are several other models than the TAI approach. One is technology manipulatives, as discussed in the next section.

5.2.2 Technology Manipulatives

Manipulatives are objects, often carefully structured, that children can act on to learn mathematical concepts. Technology manipulatives are similar digital objects. Preceding sections illustrated technology manipulatives (the shapes and tools in Figure 5.1 constitute a particularly good example) and showed that they may provide "concrete" representations that are just as personally meaningful to students as physical objects and potentially more effective in supporting learning. That is, especially for young children, technology manipulatives may be more manageable, flexible, and extensible. In one study, third graders working with technology manipulatives made statistically significant gains learning fractional concepts (Reimer and Moyer 2004). These manipulatives were easier and faster to use than physical manipulatives and provided immediate and specific feedback. In other examples, children may glue 10 single squares together

to create a "10" or put six equilateral triangles together to make a regular hexagon (Lane 2010; Sarama and Clements 2009a; Thompson 2012). A technology manipulative program helped second graders learning multiplication. The type of input did not influence learning, but the provision of not just visual but also auditory feedback resulted in greater learning (Paek et al. 2011). The goal of one game, for example, was to reveal a hidden scene by combining groups of blocks. To create a group of six, children add two blocks three times. As children move the blocks, they receive visual feedback about the value of the blocks and the arithmetic operator (2 + 2 + 2—symbolically and aurally). A recent meta-analysis of 66 studies found positive effects for the use of technological manipulatives (Moyer-Packenham and Westenskow 2013).

The following list summarizes seven interrelated affordances, emphasizing young children (for discussions and similar summaries, see Moyer-Packenham and Westenskow 2013; Anderson-Pence and Moyer-Packenham 2016; Sarama and Clements 2009a; and Sarama, Clements, and Vukelic 1996).

1. *Bringing mathematical ideas and processes to conscious awareness*. Even young children can move puzzle pieces into place without conscious awareness of the geometric motions that can describe these physical movements. Using technology-enhanced tools to manipulate shapes brings those geometric motions to an explicit level of awareness (Clements and Sarama 2007a).
2. *Encouraging and facilitating complete, precise explanations*. Even young children use accurate mathematical ideas more often when discussing their work with technology manipulatives.
3. *Supporting mental actions on objects*. Children can break technology base 10 blocks into ones, or glue ones together to form tens. Such actions are more in line with the mental actions that students are to learn. As another example, using manipulatives supporting mental composition and decomposition of shapes, kindergartner Alvaro started making a hexagon out of triangles on the computer (Sarama, Clements, and Vukelic 1996). After placing one, he counted with his finger on the screen around the center of the incomplete hexagon, imaging the other triangles, saying, "this is only one of what I need—two more!" Off-computer, Alvaro never made such statements.
4. *Changing the very nature of the manipulative*. Technology manipulatives allow children to explore geometric figures in ways that they cannot with physical shape sets. For example, children can change the size of the technology-enhanced shapes, altering all shapes or only

some. One linguistically and economically diverse population of kindergartners made more patterns and used more elements in their patterns when working with technology-enhanced manipulatives than when working with physical manipulatives or drawing. Finally, only when working with technology-enhanced manipulatives did they create new shapes (Moyer- Packenham, Niezgoda, and Stanley 2005).

5. *Symbolizing mathematical concepts*. Technology allows the manipulatives to be connected to symbols, as when a child adds digital apples to a basket and hears each counting number ("...two, three...") and sees the corresponding numerals (2, 3). Technology-enhanced manipulatives can have just the mathematical features that developers wish them to have and just the actions done to them that developers want to promote—and without having additional properties that may be distracting.

6. *Linking the concrete and the symbolic with feedback*. As an example, the number represented by the base 10 blocks is dynamically linked to the students' actions on the blocks, so when the student changes the blocks, the number displayed is automatically changed as well. This helps students make sense of their activity and the numbers. It also helps connect objects that students make, move, and change to other representations (Anderson-Pence and Moyer-Packenham 2016). For example, when students draw rectangles by hand, they may never think further about them in a mathematical way. In the Logo environment, however, students must analyze the (visual/concrete) figure to construct a sequence of (symbolic) commands, such as "forward 75 right 90 forward 30 right 90 forward 75 right 90 forward 30 right 90" to direct the Logo turtle to draw a rectangle. So, they have to apply numbers to the measures of the sides and angles (turns). This helps them become explicitly aware of such characteristics as "opposite sides equal in length." The link between the symbols, the actions of the turtle, and the figure are direct and immediate (Clements, Battista, and Sarama 2001). Similarly, children connected base 10 symbols to manipulatives more often in a technological than a physical environment because of the "natural consequences" feedback—that is, when students manipulated the technology-enhanced manipulatives, the connected symbols provided immediate feedback on their actions (Thompson 1992). Technology helped students link sensory-concrete and abstract knowledge, enabling them to build integrated-concrete knowledge.

7. *Recording and replaying students' actions*. Technology allows students to store more than static configurations; it also enables them to record sequences of their actions on manipulatives and modify or reflect on them at will.

Technology and Cognitive Play

Children can use extensions of the TAI approach to foster deeper conceptual thinking, including a valuable type of "cognitive play"—self-directed intellectual explorations with the software environment. Indeed, the dynamic aspects of TAI often engage children in mathematical play more than do physical manipulatives or paper media (Steffe and Wiegel 1994). For example, two children were playing with the free exploration level of a set of activities called "Party Time" (Clements and Sarama 2007/2018) in which they could put out any number of items, with the software program counting and labeling the items for them. "I have an idea!" said one girl, clearing out all the items and dragging placemats to every chair. "You have to put out cups for everybody. But first you have to tell me how many cups that'll be." Before her friend could start counting, she interrupted—"And everyone needs one cup for milk and one for juice!" The girls worked hard cooperatively, at first trying to find cups in the house center, but finally counting two times on each placemat on the screen. Their answer—initially 19—wasn't exact, but they were not upset to be corrected when they actually placed the cups and found they needed 20. These children played with the mathematics in the situation, with solutions, as they played with one another.

Final Words: Concrete Manipulatives and Integrated-Concrete Ideas

Manipulatives are meaningful for learning only with respect to learners' activities and thinking. Physical and technology manipulatives can be useful, but they will be more so when used in comprehensive, well-planned instructional settings (see the discussion of orchestration in Chapter 8). Their physicality is not important—their manipulability and meaningfulness make them educationally effective. In addition, some studies suggest that technology manipulatives can encourage students to make their knowledge explicit, which helps them build integrated-concrete knowledge, but rigorous causal studies of this have yet to be conducted.

5.3 Programming/Coding and Robotics

Kindergartner Chris is making shapes with a simplified version of Logo (Clements, Battista, and Sarama 2001). He has been typing "R" (for rectangle) and then two numbers for the side lengths. This time he chooses 9 and 9. He sees a square and laughs.

> Adult: "Now, what do the two nines mean for the rectangle?"
> Chris: "I don't know, now! Maybe I'll name this a square rectangle!"

Lower-primary grade children have shown greater explicit awareness of the properties of shapes and the meaning of measurements after working with the Logo turtle as José did in the introduction to this chapter (e.g., "repeat 4 [forward 100 right 90]"). They learn about the measurement of length and angle (Sarama et al. 2003). Especially now with new versions of computer languages, such as Scratch Jr. (Flannery et al. 2013), young children can learn related language and transfer their knowledge to other tasks, such as map reading and interpreting the right and left rotation of objects. For example, first-grader Ryan wanted to turn the turtle to point into his rectangle. He asked the teacher, "What's half of 90?" After she responded, he typed rt 45 (for "right turn 45°"). "Oh, I went the wrong way." He said nothing, keeping his eyes on the screen. "Try left 90," he said at last. This inverse operation produced exactly the desired effect (Kull 1986). These effects are not limited to small studies. A major evaluation of a coding-based geometry curriculum included 1,624 students and their teachers (Clements, Battista, and Sarama 2001). Across grades K–6, students who wrote such codes scored significantly higher than control students on a general geometry achievement test, making about double the gains of the control groups. These are especially significant because it was a paper-and-pencil test, not allowing access to the technology environments in which the experimental group had learned, and because the curriculum was a relatively short intervention, lasting only six weeks.

Finally, computer coding should not be considered work in virtual worlds only. For example, in robotics environments (e.g., Nao; see Crompton, Gregory, and Burke 2018) or the older LEGO-Logo, children create Lego structures, including lights, sensors, motors, gears, and pulleys, and they control their structures through computer codes. There are only a few studies of LEGO-Logo, but they indicate that such experiences can positively affect mathematics and science achievement and competencies in higher-order thinking skills (Browning 1991; Castledine 2011; Enkenberg 1994; Flake 1990; Weir 1992). LEGO-Logo appears to provide authentic learning tasks (Lafer and Markert 1994), motivate and empower students, and possibly develop self-esteem as well (Silverman 1990; Weir 1992). This may be because LEGO-Logo provides an academic setting in which students can develop their own goals (Browning 1991; Lai 1993; Weir 1992). This may be especially true for students at risk for academic failure (Day 2002). If started as young as in kindergarten, few differences appear between boys and girls, and both benefit from work with robots (Sullivan and Bers 2013). One study shows how 5–7 year-old students learned modeling, exploring, and evaluating building and programming Lego robots in Australia (McDonald and Howell 2012).

Today's robots, such as CHERP (Flannery and Bers 2013), and programming environments extend the earliest work with turtle robots and offer even more flexibility and opportunities (Mousa, Ismail, and El Salam 2017). That these systems support learning is backed by empirical evidence (e.g., of sequencing; see Kazakoff, Sullivan, and Bers 2013). More research is needed before firm conclusions can be drawn about any one particular application, but it is clear that there is no dichotomy between computers and hands-on learning environments (Keren and Fridin 2014). Recent work has described how very young children at different developmental levels approach programming a robot, a promising path for designing future experiences (Flannery and Bers 2013). Teachers will require professional development to implement such teaching (Kim et al. 2017).

Beyond mathematical concepts and skills, such work has been shown to increase creativity by a variety of measures (Alchin 1993; Clements, 1986, 1991, 1995a, 1995b; Clements and Gullo 1984). Once again, high-quality software, implemented well, can have multiple benefits (National Mathematics Advisory Panel 2008). One way to achieve such benefits is to combine different models of educational technology.

5.4 Combining Models of Educational Technology

Chapter 2 of this book provides a useful framework regarding the three phases of effective instruction. Recall that the first phase involves students' initial exploration of the topic, the second phase involves activities designed to promote and connect understanding and fluency, and the third phase is focused on developing fluency. These phases are consistent with both the theoretically and empirically based approaches of Dina and Pierre van Hiele (van Hiele 1986) and our own theory based on learning trajectories (Sarama and Clements 2009b).

This chapter has already provided examples of educational technology that supports learning in different phases. This section will provide two illustrations of how to combine different models of educational technology to support all three phases.

5.4.1 Combining Technology-assisted Instruction Models and Technology Manipulatives: Geometry

The first example of combining different models of educational technology involves a more complete discussion of the teaching and learning of geometric composition. The learning trajectory supports children in progressing through the levels of geometric composition with tasks, teasers (puzzles),

and some tools (e.g., geometric motions) and simple tutorials. However, there are other uses of educational technology that are blended into the curriculum. These activities teach two different topics in geometry. First, children progress through the learning trajectory for shape matching, recognition, and naming, initially with only simple familiar shapes (e.g., circles and squares) and then by expanding their knowledge of shapes (e.g., rhombi), so they simply match the shape to an outline shown (Figure 5.3). When they do, they hear the name of the shape (repeatedly, as the mystery picture is built up), and the program becomes a simple tutorial for shape naming. When they complete the mystery picture, they see an animation (panel b in Figure 5.3). Progressing to the following level, they do not see the outline, but instead hear the name of the shape (and a size, if there are two sizes), and they must identify that shape (panel c in Figure 5.3).

Such shape matching and naming are the main goals of this sequence. However, the same activities serve as an experiential introduction to

FIGURE 5.3

"MYSTERY PICTURES" SETS THE FOUNDATION FOR A LEARNING TRAJECTORY IN GEOMETRIC COMPOSITION
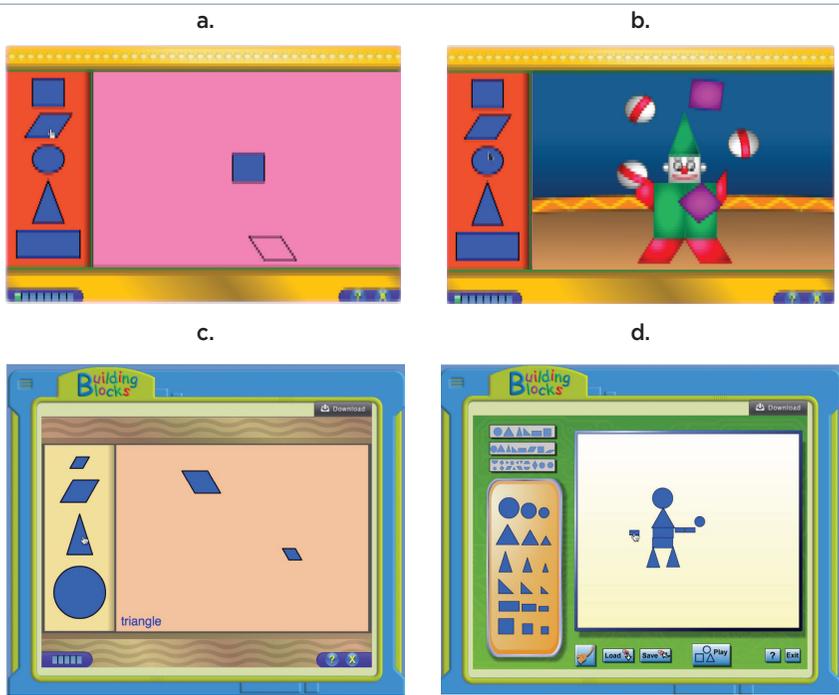
Sources: Adapted from Clements and Sarama (2014) and Sarama and Clements (2009b); software from Clements and Sarama (2007/2018).

the topic of geometric composition (putting shapes such as tangrams together to make other shapes). Children see examples of how shapes can be combined to make new pictures and shapes. More importantly, these are mystery pictures, so children are constantly guessing what the resultant picture is. That motivates them to mentally "complete" the picture and thus anticipate the placement of the succeeding shapes. Consistently, their mental images are either confirmed or not confirmed and must be updated. These are dynamic experiences of shape composition, powerful precursors of the upcoming Piece Puzzler series that requires that they compose the shapes themselves.

This is not all that Mystery Pictures contributes to this second learning trajectory. After completing any level (not just the final one), children are invited to freely explore environments like "playgrounds" with mathematical tools. Illustrated in panel d of Figure 5.3, children make their own mystery pictures by dragging shapes to form designs or objects. When they press the "Play" button, their composition becomes a new "Mystery Picture" for others to solve.

Notice that in this early exploratory phase, any picture is acceptable and there are no turns or flips (which are introduced in the Piece Puzzler free exploration environment, later in the sequence). However, children do explore putting shapes together to make pictures and new shapes, laying the foundation for understanding geometric composition; the second-phase understanding and eventual third-phase fluency promoted by the Piece Puzzler series was illustrated in Figure 5.1. In other words, Mystery Pictures' tasks and free exploration with tools set the foundation for children's progress through the geometric composition learning trajectory. Children only match or identify shapes, but the results of their work are pictures made up of other shapes—demonstrations of composition.

Finally, students work at the second and especially third phase, developing fluency in the "Super Shape" series. Here, children solve similar puzzles, but they only get one shape to use; thus, they must decompose and transform that shape. One transformation, performed with the "axe tool," decomposes shapes into their canonical components (e.g., symmetrical halves). In later problems in the Super Shape suite, students use the scissors tool, which requires them to cut the shape from one vertex or midpoint to another. Thus, they have to create shapes they have not seen before. Students then use the "Create a Scene" program, in which they create their own pictures using the mathematical ideas and skills they have developed. That is, they turn, flip, resize, glue, and even cut shapes to create objects for their pictures. These are examples of extensible manipulatives, embedded in a progression of TAI activities based on learning

trajectories. (The field needs more examples of inquiry-based educational technology; see Wang et al. 2010.)

In sum, without technology manipulatives, learning from TAI can be limited. Students do not always learn to manipulate mathematical objects to solve problems independently. On the other hand, without TAI, students often do not learn to use the features of technology manipulatives, or they explore their surface characteristics only in a trivial manner.

5.4.2 Combining Multiple Technology-assisted Instruction Models and Technology Manipulatives: Numbers and Arithmetic

As a second, more concise, illustration, consider arithmetic. First, children explore adding and subtracting small numbers in multiple environments that provide tools and linked, multiple representations (panel a of Figure 5.4 presents one example where adding a dinosaur to one of two boxes increases the numeral on that box and the sum). Second, they carry out guided, motivating tasks. In panel b of Figure 5.4, children engage in a simple simulation, working in the dinosaur shop and receiving "orders"—they must label the third box with the sum after the customer asks for both orders to be placed in the same box. Again, linked representations help connect ideas and processes (including connecting counting and arithmetic, and linking images, symbols, and oral words such as "three" and "add"). Multiple environments such as these build understanding, fluency, and generalization. Finally, drill

FIGURE 5.4
THE "FREE EXPLORATION" ENVIRONMENTS OF DINOSAUR SHOP

Sources: Adapted from Clements and Sarama (2014) and Sarama and Clements (2009b).
Note: In panel a, children can put as many (up to 10) of any type of dinosaur in any location in each of two boxes and see the sum. The series begins with the simplest example of addition. For example, in panel b, students are asked to join disjoint sets with simple numbers and provide a solution that they can determine with simple counting.

FIGURE 5.5
PRACTICE PROGRAMS FOR ADDITION

a.	b.

Sources: Adapted from Clements and Sarama (2014), Sarama and Clements (2009b), and Clements and Sarama (2007/2018).
Note: Panel a: In "double compare," children add the two cards and then indicate which sum is greater. Panel b: In this drill program, children try to get the highest score by answering as many addition questions as possible in a short time period.

programs build fluency in the third phase, but only after the child has shown full competence at phases one and two (Figure 5.5).

This is just an example. New programs and technologies are being created every year. For instance, one study shows numerous ways that new technologies might support children's development of a "mental number line" (Moeller et al. 2012), and another describes the use of multitouch technology in Malaysia to teach arithmetic (Tyng, Zaman, and Ahmad 2011). The goal should be to achieve the potential of educational technologies, including promoting children's active engagement with mathematical tasks, frequent feedback, and increased opportunities for collaborating and communicating around mathematical ideas.

5.4.3 Evaluation of the Combined Approach

Is such a synthesis of models effective? As part of a comprehensive curriculum, the combined approach has been shown to be effective (Clements and Sarama 2007b, 2008a; Sarama and Clements 2002a, 2009c). Further, the research shows the special role the Building Blocks software played in that curriculum (Clements and Sarama 2007/2018). That is, in each of these studies, software was shown to be a strong correlate—and sometimes the single highest—with child gains in mathematics achievement. One study used a counterfactual in an evaluation of Earobics software and found significant positive effects on mathematics achievement (Anthony et al. 2011).

5.5 Choosing Models: The Educational Technology Environment

Given the many models of educational technology and the mixed research on each, what models are most likely to benefit teachers and children in schools in LAC? This section first considers the educational environment and then issues of access and equity. It then turns to professional development and support for teachers before addressing the question of choosing among the models.

5.5.1 The Importance of the Technology Environment

A recent survey reported that lack of funding for equipment (including inadequate numbers of computers for the number of children in class), lack of technical and administrative support, and inadequate training (leading to lack of confidence) were the major perceived barriers to the use of computers in early childhood settings (Nikolopoulou and Gialamas 2015). This section provides a brief overview of what is required of schools, classrooms, and teachers for success in educational technology.

In early childhood and early-primary classrooms, two working devices (computers or tablets) is a workable minimum for classrooms of 18–22 children (and with efficient scheduling, should serve even larger classrooms). Too few devices with too little access can lead to tension and aggressive behavior.

The arrangement of technology in the classroom can enhance its social use. Putting two seats in front of a computer and one at the side for the teacher can encourage positive social interaction. Of course, programs with technology-managed instruction may need to be used by individual children (for assessment purposes), but problem-solving environments benefit from the math talk that technology actually encourages (see Chapter 2; see also Clements and Sarama 2003). Placing computers or tablets close to each other can facilitate the sharing of ideas. Technology that is centrally located in the classroom invites other children to pause and participate. Such an arrangement also helps the teacher remain near enough to provide supervision and assistance, but not so close as to inhibit the children.

Interactive boards are particularly appropriate for early childhood use. They can engage students in a variety of TAI programs, allow teachers to monitor children's activities (Carey 2009), and promote mathematical practices such as reasoning and problem-solving activities (Bourbour, Vigmo, and Samuelsson 2015). However, it is critical that the type of use be carefully considered and planned; otherwise, "entertaining" aspects can overshadow the mathematics learning (Serow and Callingham, 2011, note that this study involved older primary grade students).

5.5.2 Equity

Descriptions of this increasing array of available technology raise the critical issue of inequity in access to technology (hardware and software).

First, what can be done with limited resources? Many schools in LAC do not have anything close to the technology described in the previous section (unsurprisingly, especially poorer schools; see Chapter 4). Schools with predominantly indigenous student populations in Mexico, and in LAC countries generally, have fewer computers than regular public schools. However, there are some positive findings. First, broadband access is expanding, and a study in Brazil indicates it can raise achievement for older students (Silva, Milkman, and Badasyan 2016). Second, the One Laptop per Child Program in rural Peru successfully increased the ratio of computers per student from 0.12 to 1.18 (Cristia et al. 2017). Third, there are still experiences that even limited technology can provide. Even quite old computers without Internet connections can run programs (from older disk and CD-ROM drives) that remain available. This includes almost all the types of software discussed here—TAI, manipulatives, and coding (e.g., computer programming Logo). Such software is older, and teachers and children deserve a greater choice, but even with limited resources, educational technology can provide substantial benefits. Consider that most of the research with positive effects was conducted with these types of software programs. With adequate professional development and support for teachers (see the next section), for example, half-century-old versions of Logo can provide useful mathematical experiences for children, just as centuries-old unit blocks can.

Even with one device in a classroom (or school, with rotations among classrooms), teachers can either present TAI simulations to an entire classroom or rotate children through individual or paired use throughout the school day. If teachers are provided with resources and information on how such simulations can be integrated into the curriculum, children can benefit greatly. Even inexpensive, self-powering calculators can be used (with adequate teacher support) to provide exploratory and problem-solving experiences that are of considerable benefit to children (Khoju, Jaciw, and Miller 2005; National Mathematics Advisory Panel 2008).

Second, and perhaps more important in terms of inequities in access to technology, addressing limited resources is a critical policy issue for LAC not only because there are inequities in school resources, but because schools serving children from lower-resource communities need adequate technology even more than schools in higher-resource communities. As an example, consider that the availability of a computer at home and high socioeconomic status are significant predictors of children's baseline

computer skills in kindergarten (Saçkes, Trundle, and Bell 2011). Meanwhile, the availability of computers in kindergarten is a significant predictor of the development of children's computer skills from kindergarten to third grade. Thus, the availability of an adequate amount of computers in early childhood classrooms helps close the gap in children's computer skills due to socioeconomic status and lack of computer access prior to entering school. Ensuring that all early childhood classrooms have an adequate amount of computers may positively contribute to children's long-term development of computer skills (Saçkes, Trundle, and Bell 2011). These schools need hardware. Also, although there are many experiences children can have without the Internet, access to it in schools can greatly expand applications and knowledge resources. Tablets and phones are mobile and less expensive than most new computers and can access the Internet in an increasing number of geographic regions across LAC. Work should be done to extend that access to all schools. Policies need to be put in place to move toward adequate educational technology environments. When computers are in the hands of many students, motivation and achievement can increase, as long as the programs are part of balanced and comprehensive initiatives that address changes in educational goals, curricula, assessment, and teacher training (addressed in the following section) (Zucker and Light 2009) .

As a final note, space constraints do not permit a description of the use of educational technology for children with special needs, but those advantages should not be overlooked. For example, a large-scale, multiyear study showed conclusively from every data source—interviews, observational data, and scores on a developmental measure—that every one of the study's 44 3-to-5 year-old special-needs children gained substantially and significantly in social-emotional development from their work with computers. The quantitative measure of development showed that, upon joining the program, children were making an average gain of less than half a month per month in social-emotional development. While participating in the program, children were making an average rate of progress of 1.93 months per month (Hutinger and Johanson 2000). As another example, technology facilitates social interaction between children with disabilities and their normally developing peers (Spiegel-McGill, Zippiroli, and Mistrett 1989). These and other publications can serve as resources supporting the use of technology for special-needs children (Edyburn 2000, 2002).

Another promising finding is for children who are dual-language learners. A correlational analysis of the Early Childhood Longitudinal Study data showed positive effects of home computer access and computer use. Importantly, computer use for mathematics was associated with a

reduced gap in math achievement between native English-speaking and dual-language learner students (Kim and Chang 2010).

5.6 Teachers and Technology

Different models of educational technology, and combinations of them, can be effective, but only if teachers receive adequate professional development and support. Space constraints do not allow for describing the research on these critical issues. However, resources can be provided to help guide teachers and teacher trainers on such matters as managing the technology environment (Sarama and Clements 2006) and effective strategies for teaching with technology (Bourbour, Vigmo, and Samuelsson 2015; Clements and Sarama 2008b 2010). These issues are critical. One survey of the level of adoption of information and communication technologies in teaching in three Latin American countries found that most of the teachers who participated in a project focused on using educational technology in mathematics education rated themselves at the highest levels of technology adoption (Salinas et al. 2017). However, there were differences between countries in teachers' perceptions and training and other factors (such as the people in policy and administration roles), and these differences affect technology adoption. Cultural and individual factors have complex interactions with technology use (Salinas et al. 2017).

5.6.1 Effective Teaching Strategies

Critical to the effective use of technology is teacher planning, participation, and support. Optimally, the teacher's role using educational technology should be that of a facilitator of students' learning, such as by establishing standards for and supporting specific types of learning environments (see Chapter 4). When using open-ended programs such as technology manipulatives, for example, considerable support may need to precede independent use. Other important aspects of support include structuring and discussing technology work to help students form viable concepts and strategies, posing questions to help students reflect on these concepts and strategies, and "building bridges" to help students connect their technology and nontechnology experiences.

Teachers whose students benefit significantly from using technology are active. A teacher or assistant working with three to six children at a time functions well for introducing technology use or new applications (Aronin and Floyd 2013). This small group can then help the teacher introduce the technology to their classmates.

Then, as children use the program, effective teachers ensure that each child works on the programs at least 5–10 minutes, two times a week in preschool and up to 10–20 minutes per day in kindergarten to third grade. Teachers monitor and guide children's learning of basic tasks and encourage experimentation with open-ended problems. They engage in encouraging, questioning, prompting, and demonstrating, but without offering unnecessary help or limiting students' opportunity to explore (see Chapter 2; see also Hutinger and Johanson 2000). They redirect inappropriate behaviors, model strategies, and give students choices. They focus attention on critical aspects and ideas of the activities. When appropriate, they facilitate disequilibrium by using the technology feedback to help students reflect on and question their ideas and eventually strengthen their concepts. They also help students build links between technology and nontechnology work. Such teaching strategies lead students to reflect on their own thinking behaviors and bring higher-order thinking processes to the fore. Such meta-cognitively-oriented instruction includes strategies of identifying goals, active monitoring, modeling, questioning, reflecting, peer tutoring, discussion, and reasoning.

Whole group discussions that help students communicate about their solution strategies and reflect on what they have learned are also essential components of good teaching with technology. Effective teachers avoid overusing directive teaching behaviors (except as necessary for some populations and on topics such as using the technology equipment). Instead, they prompt students to teach each other by physically placing one student in a teaching role or verbally reminding a student to explain his or her actions and respond to specific requests for help. Also during such discussions, effective teachers make the mathematics to be learned clear and extend the ideas students encounter.

Students work best with open-ended software when projects are suggested and guided rather than when they are told merely to explore freely. They spend longer time and actively search for diverse ways to solve the task. Children told to just explore freely quickly grow disinterested. Providing models and sharing students' projects may also help guide and maintain their focus on learning mathematics.

5.6.2 Professional Development

There is evidence that the more teachers receive support in using technology, the more their students learn, especially if the support is targeted at students' effective use of technology (Fuller 2000). Research has described features of effective professional development (for more

detailed descriptions see Clements and Sarama 2008b, Clements and Sarama 2010, and Sarama and Clements 2006).

Many agree on the general characteristics of effective professional development. For example, professional development should be multi-faceted, extensive, ongoing, reflective, focused on common actions and problems of practice (and especially students' thinking), grounded in particular curriculum materials, and, as much as possible, situated in the classroom (Sarama and Clements 2013). With regard to technology, professional development must be "characterized by access to high-quality software, ongoingness, curriculum and instruction embeddedness, a variety of learning partners (e.g., coordinators, other teachers), a variety of learning formats (e.g., visits, workshops, meetings, group, one-to-one), opportunities for practice-practice-practice and feedback, and data on the impact" (Fullan 1992, 46). It should also involve participants in teams from the same school, model constructivist approaches to learning, and promote ongoing conversations and reflections about practice, theories of learning, and how classroom practice might change in the context of technology (Dwyer, Ringstaff, and Sandholtz 1991). Technology is a particularly challenging field because the learning task is daunting, the vision of high-quality use is not clear, and well-designed, intense, relevant, sustained assistance is critical (Fullan 1992).

Research has suggested that less than 10 hours of teacher training in technology can actually have a negative impact on these teachers' students (Ryan 1993). Whatever time is available should be dedicated to hands-on training on the hardware and software to be used and its connection to the curriculum. The goal is to develop comprehensive technological pedagogical content knowledge (see Chapter 4; see also Jaipal and Figg 2010).

Effective technology use depends on establishing a functional, well-trained, on-site technology support team at the school that provides leadership and support that hold the system together. This can lead to institutionalization of the program after external funding ends (Hutinger and Johanson 2000).

Again, however, there are issues of equity. Many teachers have not had the opportunity to learn the mathematical content and pedagogical strategies that underlie effective teaching and learning (see Chapter 4). For example, in Guatemala, teachers had one more year of education than high school. But starting only recently, colleges are starting to train teachers, so there are new opportunities to influence teachers' knowledge and skills. Some countries in LAC include funding opportunities from nongovernmental organizations for professional development, and policies should support these greater opportunities. Here the focus is on

professional development in the use of educational technology, but teachers need to learn, and learn to apply in classrooms, all three components of learning trajectories: the goal (i.e., understand the mathematics content), the developmental progressions, and the instructional tasks and strategies (including, but palpably not limited to, technologies).

Also, a teacher with Internet access (in school or out) may find several resources that help fill those gaps. As an example, we are working with a team from the University of Michigan on free online resources for professional development in mathematics.[2] Other discussions are available of educational technologies that support teachers' learning using representations of practice and of the challenges involved in understanding students' thinking (Herbst et al. 2010).

Our TRIAD (Technology-enhanced, Research-based Instruction, Assessment, and professional Development) Project enhances professional development with a variety of technologies, including discussion boards, e-mail, distance-learning centers, and websites and applications, increasing the scalability of professional development. The most important of these is the "Learning and Teaching with Learning Trajectories" [LT[2]] web application,[3] which provides scalable access to the learning trajectories via descriptions, videos, and commentaries. Each aspect of the learning trajectories—the developmental progressions of students' thinking and connected instruction—are linked to the others. Of course, the [LT][2] application is only a tool. All teachers were provided a full range of professional development opportunities, based on the research previously described. They participated in a credit-bearing course with several components, including two full-day sessions in summer and a one-day follow up each month, electronic communications, and coaching and mentoring within each teacher's classroom. All of these components use the [LT][2] web application as a tool.

Funded by the Heising-Simons Foundation and the Bill and Melinda Gates Foundation, we are currently redeveloping this research-based tool for wider access.

5.7 What Educational Technology Models Suit Latin America and the Caribbean?

What models of educational technology might be recommended for LAC? What has been learned from research and practice on models of

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[2]   See http://www.umich.edu/~devteam/.
[3]   See LearningTrajectories.org.

technology, technology environments, and teaching must be considered together. For example, although the benefits of computer programming are promising, especially in an increasingly complex technological age, the requirements for the effective use of coding are considerable. If hardware is scarce and teachers have not received sufficient professional development and support, this may be an unproductive and frustrating model to implement. However, long-range policy could be crafted to reach toward such sophisticated applications over time.

In the context of limited resources, policymakers face difficult choices. When teachers are underpaid and the physical environment lacks basics such as books and mathematical manipulatives, the moderate effects of educational technology may not justify the cost. If funds are available, simple applications such as TAI tutorials and practice applications—explicitly aligned to extant standards (goals) and curricula—may represent a tractable approach for teachers. Again, even for these simpler applications, teachers must receive training and in-class support (e.g., as seen earlier, drill TAI is inappropriate and ineffective unless children have moved through the first two phases of instruction). Research suggests implementation of a software system that:

- Combines models of TAI
- Moves children through learning trajectories
- Features introductory exploratory activities
- Includes explicit instruction and then practice
- Includes technological manipulatives and guidance in their use
- Manages instruction and its impacts for the teacher (technology-managed instruction).

Such a system may support children directly and have the additional benefit of allowing teachers to learn more about mathematics, and the teaching and learning of mathematics, as they observe and support children using the system.

If hardware is scarce, models that use one computer per classroom may be necessary. For example, if children cannot use the computer for at least 5–10 minutes two times a week, applications for individual-use-only may not be effective.

Once that minimum usage level has been established, inclusion of other models of educational technology is possible. However, teachers will need substantial professional development, including reflecting on and restructuring their teaching to effectively use simulations, tools, and computer programming (Clements and Sarama 2002).

TABLE 5.1

CONCLUSIONS AND IMPLICATIONS FOR POLICY AND PRACTICE WITH EDUCATIONAL TECHNOLOGY IN LATIN AMERICA AND THE CARIBBEAN

Conclusions	Implications
1. Effective use of educational technology demands connected hardware, software, curriculum development, and professional development.	To begin with scarce resources, it is useful to have technology-assisted instruction (TAI) tutorials and practice applications that are carefully aligned to standards and curricula. Long-range planning to incorporate the use of more sophisticated applications, such as computer programming and design, could also be initiated.
2. Research has identified characteristics of educational technology that are effective in helping practitioners teach better and children learn better and more mathematics.	Procure or design educational technology that combines different models of TAI consistent with learning phases (see Chapter 2) and learning goals, moves children through learning trajectories, and provides management and record-keeping.
3. Continual formative evaluation and professional development ensures continued success.	Educators and policymakers at all levels should plan a system, first for choosing and implementing an approach to educational technology, then for scaling up, and continually for monitoring and improving the intervention.

As noted in Sarama and Clements (2013), one cannot simply choose a model of educational technology—the work starts, not ends, with such a choice. What is imperative is the efficacy of the particular software package, its appropriateness for the intended audience of children, and its requirements (hardware, other resources, support for teachers, etc.). Once selected, a careful, monitored plan for implementation, piloting, and—only if these are successfully accomplished—scaling up the intervention are requisites for success (Sarama and Clements 2013). A summary of the implications of research is provided in Table 5.1.

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Guiding Technology to Promote Student Practice

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Governments in Latin America and the Caribbean (LAC) are seeking educational programs that improve student learning, are afford-able, and can be scaled up with existing resources. This chapter argues that programs that guide the use of technology to promote stu-dent practice, if well structured, can satisfy these three requirements. Such programs provide clear instructions on how to use relevant technological resources. More specifically, these programs clearly define the subject to be targeted, the software to be used, and the schedule of the use of the resources. That is, the three "S"s are clearly defined: subject, software, and schedule (Arias Ortiz and Cristia 2014). Moreover, there is a strong focus on developing skills through practice.

This chapter presents evidence and theoretical reasons to back the claim that guided technology programs focused on practice can be effec-tive, efficient, and relatively easy to scale up. Because they require limited investments in hardware, software, and pedagogical support, these programs are efficient and can be afforded by most countries in the region. Finally, since these programs reduce the workload of teachers (by saving time on grading exercises and exams), their implementation requires few extra skills or changes in the way instruction takes place. The programs also have the potential to be scaled up, but there are challenges to do this that need to be considered, and the chapter discusses strategies for tackling them.

Guided technology programs are not a panacea, however: evidence shows that though they do produce improvements in learning, they can-not by themselves solve the significant educational problems of the LAC region. Their basic limitation may lie in the fact that they can support but never replace good teaching.

The central question this chapter seeks to answer is: How can the effects of these programs be maximized? To answer this question, the chapter analyzes 10 key design decisions that need to be made when structuring these programs. The first decision pertains to defining the objective of the program—that is, which mathematical skills will be targeted for improvement? Next are four key design decisions on how computers are to be used during technology sessions. These decisions involve defining which type of learning activities will be performed during the sessions, how many hours a week computers will be used, what role teachers and technology coordinators will play in the process, and whether the learning activities will be personalized for each student or include all students in a session on a particular topic. Finally, the chapter discusses decisions related to inputs, including where the computers will be located, whether students will share them, what software will be used, how the staff will be trained and coached, and how the program will be managed.

Clearly, the chapter cannot provide optimal decisions valid in all contexts. In fact, it is difficult (even impossible) to ascertain the optimal solution for one specific context without detailed evidence of the effects and costs of all the relevant options. Acknowledging these limitations, the chapter seeks to provide an exploratory analysis of each of the 10 key design decisions mentioned by exploring these questions:

1. What is the nature of the design decision?
2. What are the relevant options and their theoretical advantages?
3. What evidence exists of the effectiveness of each option?
4. What choices were made in a set of effective technology programs that produced large positive effects?
5. What choices were made in the program ConectaIdeas in Chile, considered a best-practice example?

Questions (1) to (3) involve theoretical and empirical analysis of the different options for each design decision. Question (4) points to lessons from past choices made in programs reviewed on the website SkillsBank under the category "Guided technology with extra time," as described in Table 6.1.[1]

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[1] SkillsBank summarizes rigorous evidence on how to develop skills along the life cycle. It can be accessed at: www.iadb.org/skillsbank. The website includes three categories of programs related to technology. Programs in the category labeled "Guided technology with extra time" were found to be effective, based on the meta-analysis presented in the website. In contrast, the other two program types were not found to be effective.

TABLE 6.1
PROGRAMS THAT GUIDE TECHNOLOGY TO PROMOTE STUDENT PRACTICE

Evaluation	Effect	Grade	Students	Country	Weekly Time with Technology
Banerjee (2007)	19	4	11,255	India	1 session of 120 minutes
Lai et al. (2013 i)	12	4	2,613	China	2 sessions of 40 minutes
Lai et al. (2013 ii)	21	3	1,717	China	2 sessions of 40 minutes
Lai et al. (2015)	16	3	2,425	China	2 sessions of 40 minutes
Linden (2008)	25	3	1,114	India	5 sessions of 60 minutes
Mo (2013)	16	4	3,592	China	2 sessions of 40 minutes

Source: www.iadb.org/skillsbank.
Note: The programs presented in this table are included in the SkillsBank under the category "Guided technology with extra time." Evaluations (i) and (ii) by Lai are both reported in Lai et al. (2013). Effects are expressed in learning points. One learning point is equivalent to 0.01 standard deviations. To benchmark the effects, note that the average third grader in the United States improves about 40 learning points in one year. The average grade of participants in the evaluation is presented. All evaluations met the criteria for inclusion in the SkillsBank. For example, all the evaluations used experimental methods. The interventions focused on improving learning in mathematics, except for Lai (2013 ii), in which the subject targeted was reading.

Finally, question (5) refers to the choices made by the team implementing the program called ConectaIdeas. This program was designed and implemented by a multidisciplinary team of researchers at the Centro de Investigación Avanzada en Educación at the Universidad de Chile. The team was led by Roberto Araya, who is one of the co-authors of this chapter. The program seeks to increase learning in mathematics in primary schools located in disadvantaged areas in Santiago, Chile. It was implemented in 11 schools in the Comuna Lo Prado from 2011 to 2015, and during this period the program was continuously refined to enhance its effectiveness. In a review of about 90 initiatives aimed at improving mathematics learning in LAC, the editors of this book considered ConectaIdeas to be

The programs in the "Guided technology without extra time" category include those that guide the use of technology but keep constant the instructional time devoted to mathematics and language. The programs in the category "Computers" include programs that basically provide access to technological resources without guiding their use.

the most promising. Therefore, the editors invited Roberto Araya to contribute to this chapter and to convey the experience accumulated by the team in developing an effective program for mathematics learning in the region. Hopefully, this experience can inform the development of other initiatives in LAC.

6.1 Why Guided Technology Programs Focused on Practice May Work

This section describes the status quo of mathematics instruction in LAC and how a guided technology program focused on practice may change how students learn. It then reviews empirical and theoretical evidence for considering these programs as effective, efficient, and easy to scale. Finally, the section analyzes the limitations of these programs.

6.1.1 The Status Quo in Mathematics Instruction and Its Challenges

This section starts with a stylized description of how mathematics instruction takes place in schools similar to those targeted by ConectaIdeas in Chile. In these schools, fourth grade students have three weekly mathematics sessions of about 90 minutes each in their regular classrooms. (Note that in some schools, there is an additional fourth mathematics session of 90 minutes.) A regular class may start with the teacher presenting a certain topic (e.g., adding fractions with the same denominator), then providing key concepts and examples, and asking students to solve some exercises. While students are solving the exercises, the teacher walks around the classroom to make sure that all the students are working, answer questions, and provide feedback. At the end of the class, the teacher summarizes the topic covered, emphasizes key takeaways, and assigns homework.

There are several challenges with this learning session, including:

- *Low engagement*. Many students may find the learning activities less engaging than game-based activities or those involving the use of technology.
- *Limited practice*. Partially due to the lack of engagement, the amount of practice that students may perform could be limited. And this is problematic because practice is key for skills development.
- *Sporadic feedback*. Feedback is also key for skills development. But because teachers have limited time and many students to support, they can only provide sporadic feedback.

- *Costly monitoring*. In this learning session, it is difficult for teachers to monitor the work of students and identify who needs extra support. Hence, teachers cannot use their time strategically.
- *Little collaboration*. There may be students in this session who have mastered the concepts and who could provide support to those struggling to understand. This peer-tutoring process could be beneficial for both sets of students because research suggests that explaining concepts and discussing strategies are potent ways to improve understanding (Okita and Schwartz 2013). Moreover, collaboration and communication are considered critical skills in the 21st century, and practicing them in school could be a good way to develop them.

6.1.2 How a Guided Technology Program Focused on Practice Can Improve Instruction

Against this backdrop, the team at the Universidad de Chile developed the ConectaIdeas program, which seeks to leverage technology to tackle these challenges. Since the introduction of this program, there have been some changes in how mathematics instruction takes place in local schools. To start with, one of the three mathematics sessions that previously took place in the classroom is now conducted in the computer lab. An additional fourth mathematics session also takes place in the computer lab. This session replaces time that was previously devoted to learning computer skills or music, or it uses some other unused slot in the regular schedule. Hence, since the introduction of the program the total weekly time devoted to mathematics has typically increased from 270 to 360 minutes. Moreover, the time spent in the classroom has been reduced by 90 minutes and the time spent in the computer lab has increased by 180 minutes. Note, however, that in certain cases schools already had four sessions, in which case the total time devoted to mathematics learning remains unchanged, but two of the four sessions moved from the regular classroom to the computer lab.

During the two mathematics sessions in the classroom, the teacher continues to provide instruction in the same way that he or she did so before implementation of the program (though the teacher might devote a little less time to solving exercises and more to presentation and discussion). In the two mathematics sessions at the computer lab, a computer lab coordinator is responsible for instruction, though in many cases the coordinator works together with the teacher. During these sessions, students mainly work to solve mathematics problems that are aligned with the topics covered in the classroom sessions. These problems seek to develop a

range of mathematical skills including computation, modeling, representation, analysis, and problem-solving.

Also, skills related to communication and collaboration are developed during these sessions. For example, teachers ask open questions to all students, who have to provide answers and also analyze and provide feedback to the answers provided by their peers. Peer tutoring is facilitated by technology. In particular, students who show mastery of the concepts covered in a class are identified by the platform, and computer lab coordinators can assign them to tutor students asking for assistance. These activities can improve students' conceptual understanding and also strengthen their meta-cognitive and socioemotional skills.

Finally, a number of strategies are implemented to increase the motivation of students. These strategies seek to leverage the positive social dynamics associated with competition among teams. In particular, since students can see how their class is ranked (in terms of the number of exercises performed in a week) compared with other classes in other schools, it motivates them to exert greater effort to solve the exercises assigned to them. To enhance student motivation, tournaments can be organized every two to four months in which students from different schools all connect to the platform at the same time and compete in teams in real time.

6.1.3 Guided Technology Programs Focused on Practice: Effective, Efficient, and Easy to Scale?

Effective

There is strong evidence that programs that guide the use of technological resources and provide additional learning time generate important learning gains. In fact, as shown in Table 6.1, six evaluations of programs reviewed in SkillsBank produced gains of 12 to 25 learning points in students' average mathematics and language test scores.[2] The average learning gain is 16 points. To put this in perspective, the average third grader advances about 40 learning points in a year (Hill et al. 2008). Another way to benchmark this effect is by comparing it with the documented difference in the effectiveness of classrooms led by teachers in the top and bottom quartile—this stood at 33 learning points in a study in the United States (Kane, Rockoff, and Staiger 2008).

Finally, to benchmark the effects of the programs reviewed in this chapter, consider that the average student in LAC is lagging behind his

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[2]   One learning point corresponds to an effect of 0.01 standard deviations.

or her counterparts (that is, in countries with a similar GDP per capita) by about 50 learning points. Hence, the programs implemented in China and India analyzed in this chapter can close about one-third of this gap.

There is some evidence suggesting that the ConectaIdeas program in Chile also generates important learning gains. One study shows that in the 11 schools where the program was implemented, average test scores in math increased by 33 learning points in one year (Araya et al. 2015). The authors of that study report that the average yearly mathematics gain in a neighboring district was 7 learning points. Using this neighboring district as a comparison group, the authors conclude that the program produced an increase of 26 learning points in mathematics test scores.

Efficient

A program is efficient if it achieves its objective at a reasonable cost. In this context, the question is whether a program is cost-efficient based on the learning gains it generates. An analysis presented in Busso et al. (2017, Chapter 7) indicates that the type of programs analyzed in this chapter are in fact cost-effective. Specifically, the analysis showed that guided technology programs that offer students extra learning time require an annual increase of less than $5 to achieve an increase of 1 learning point. In contrast, other popular educational interventions, such as reducing class size and extending the school day, require $47 and $210, respectively, to achieve an increase of 1 learning point.

(Relatively) Easy to Scale

A program that yields substantial learning gains at low cost is not really relevant if it cannot be scaled up given the context and existing resources in an educational system. Therefore, it needs to be determined whether the programs covered in this chapter can be implemented on a large scale in countries across the region. To do this, it is necessary to theoretically explore the capacity requirements that scaling up these programs would demand, the changes in behavior that scaling up would require from relevant actors, and whether or not these changes can be achieved.

When considering capacity requirements, a basic input that these programs need is electricity. Because the vast majority of public urban schools in various countries in the region have electricity access, this does not seem a challenge to scaling up. Technology in education programs requires the provision of hardware, software, and other infrastructure arrangements. Importantly, all of these inputs need to work well for instruction to take place. Consequently, these programs require the capacity of implementers to ensure that computers are operating adequately. There

are different models to ensure this, such as developing special units within the education ministry or contracting out these services to the private sector. No matter the specific model implemented, it is paramount that there is capacity to ensure that all inputs work adequately.

Implementing technology in education programs has a unique feature compared with other educational interventions, which is the possibility of monitoring how and how much technology is used. This provides invaluable information for teachers, principals, and program managers. Knowing which schools or students are using the technological resources suboptimally provides an opportunity to think about strategies to tackle these problems.

Other important aspects of scaling up relate to whether the key stakeholders will support changing practices to accommodate the program. As opposed to many other programs seeking to improve mathematics instruction, a guided technology program does not impose further requirements for teachers' content and pedagogical knowledge as long as specialized staff are available to train and support existing teachers.

However, it is critical to gain the support of teachers, principals, and other actors to ensure the adoption of these programs and to ascertain that the expected changes in mathematics instruction take place. This is not an easy task. Teachers may fear that the new program will bring in new responsibilities and more work for them and that these changes will expose their inadequacy in the skills required to use the technology effectively in the classroom. Hence, it is critical to provide teachers with the necessary support so they can transition effectively into their new roles.

Maybe the question that arises here is why these programs have not been already implemented at scale in LAC? This is particularly relevant given the clear interest expressed by governments across the region in implementing technology in education programs (Arias Ortiz and Cristia 2014). One reason could be that until recently there was no strong evidence of the effectiveness of various technology models. This uncertainty paved the way for the introduction of large-scale interventions that increased access to technology but produced little effects on learning. This lack of information is starting to disappear, at least in certain circles, given the rigorous evidence that is emerging. While a decade ago there was no experimental evaluation of technology in education programs in developing countries, now the situation has radically changed. Still, more work is needed to generate and disseminate rigorous and relevant information to governments and other relevant actors so they can use updated evidence for policy decisions in this area.

6.2 Ten Key Design Decisions

This section analyzes 10 key decisions on how to structure programs that guide technology use to promote student practice. As mentioned, the goal here is to guide critical design decisions with the aim of maximizing the effects of these programs while keeping costs low.

Table 6.2 presents the 10 key design decisions to be analyzed. These decisions are classified in three areas: objectives, processes, and inputs. To start with, the first decision is related to the objectives of the program, that is, what the program seeks to achieve. Decision 1 entails defining which types of mathematical skills will be targeted with the program.

Next, there are four decisions on how the process of learning with technology will unfold. That is, decisions 2 to 5 entail defining which learning activities will be implemented, the length of time the computers will be used, the roles of the teachers and lab coordinators, and whether instruction will be personalized to each individual student or will unfold at a common pace.

Finally, there are five decisions that are related to program inputs. That is, decisions 6 to 10 are related to certain resources or services that the program will directly provide. In particular, these decisions involve different aspects such as the location of the computers, whether there will be one or more students per computer, what software will be used, what training and coaching will be provided, and how the overall program will be managed.

TABLE 6.2
TEN KEY DESIGN DECISIONS

	Design Decision
Objective	1. Which skills?
Processes	2. Which learning activities?
	3. How much time?
	4. Roles for teachers and lab coordinators?
	5. Personalizing or common pace?
Inputs	6. Computers in the classroom or in the lab?
	7. One or more students per computer?
	8. Which software?
	9. How to train and coach?
	10. How to manage?

Source: Prepared by the authors.

Decision 1: Which Skills?

Since the 1970s, technology in education programs focused on mathematics have been traditionally targeted to develop mathematical fluency. That is, they have sought to develop fluency in performing basic operations, such as single-digit multiplication. Mathematical fluency is important because it provides the necessary skills that are used for other higher-order tasks such as problem-solving. Moreover, computers are particularly well suited for developing these skills because they provide engaging activities to promote sustained practice and provide automatic feedback about whether the answers provided by students are correct.

However, as has been pointed out in Chapters 1 and 2, changes in the 21st century are increasing the need for students to be able to deeply understand mathematical concepts and apply them successfully to real-world situations. That is, although mathematical fluency in basic facts is still needed, there are increasing demands for students to solve mathematical problems related to real-world situations, such as those emphasized in the Programme for International Student Assessment.

There is a near consensus among experts in mathematical education that to develop this deep understanding, instruction needs to change substantially (NCTM 2014). In particular, more emphasis should be given to learning activities that promote the development of multiple solution methods, students' explanations of how they reach an answer to a problem, the use of mathematical graphic diagrams, and the establishment of strong connections between multiple areas of mathematics (e.g., arithmetic, measurement, geometry) and problems in the real world.

Against this backdrop, it is difficult to envision that traditional technology in education programs that emphasizes intensive practice to achieve mathematical fluency in basic facts will play a major role in facilitating this transition. Technology can be effectively used for students to practice and achieve fluency in basic facts, which are still the necessary building blocks for higher-order tasks. But if all computer instruction is devoted to only mathematical fluency, then it is difficult to expect effects in other areas. This is well reflected in the evaluations of traditional programs that show that the learning gains from them are heavily concentrated in computation and not problem-solving (Slavin and Lake 2008).

Software developers recognize this challenge and are introducing new activities that could also contribute to developing better mathematics understanding. But many of the activities promoting the new ways of teaching mathematics (e.g., the use of multiple solution methods, students explaining how they reach an answer, and the use of diagrams) do

not seem to be well suited to the types of interactions enabled by traditional software. This is well exemplified by an evaluation by Wang and Woodworth (2011) of the software DreamBox, which has a number of well-developed features and has been explicitly designed to emphasize understanding and not only fluency. Though the evaluation found positive effects of the use of this software on mathematics learning, the effects were concentrated in computation rather than problem-solving (13 and 6 learning points, respectively).

Consequently, an important challenge is to ensure that the skills developed in the technology sessions emphasize other skills that go beyond the development of mathematical fluency. The Universidad de Chile team has sought to tackle this challenge by developing a range of mathematical skills during the technology sessions. These include basic computation, modeling, representation, analysis, and problem-solving. The underlying strategy is to ensure that the activities performed are contributing to the development of conceptual understanding and number sense. This involves presenting problems in different ways, relating abstract concepts to practical applications, and applying strategies to novel situations.

Decision 2: Which Learning Activities?

Traditional technology in education programs emphasizes learning activities, such as interactive games, that seek to develop fluency in basic operations. Additionally, in some program implementations, short, engaging videos (3–5 minutes in length) are provided at the beginning of the technology sessions to review important concepts covered during learning sessions in classrooms. In some cases, these sessions typically close with general explanations and conclusions by teachers (together in discussion with students) on the important concepts and algorithms reviewed during the day.

However, to develop a range of mathematical skills that go beyond fluency in basic operations, other activities need to be included. To tackle this challenge, the Universidad de Chile team noted that interactive games to develop mathematical fluency of basic facts should not take more than 10 to 15 minutes in each session. Additionally, they have noted that teachers do not tend to use videos, probably because they think that they should be the ones providing explanations of new concepts to children.

Most of the learning activities involving mathematical exercises on computers should seek to develop students' conceptual understanding and number sense. In some cases, students need to solve standard multiple-choice questions, but in other cases, students have many different potential options to choose from when solving an exercise.

One example of an exercise that departs from the standard multiple-choice format is presented in Figure 6.1. In this exercise, students need to help a dog by guiding it through different cells in search of a bone. Students start at the cell located at the left-bottom corner and need to choose which contiguous cell the dog will move to next, taking into account that the intensity of the smell is proportional to the fraction presented in the cell. The top panel shows the exercise as it is presented to the student, and the bottom panel shows the correct trajectory chosen by a student. This exercise involves a student making multiple comparisons of fractions in the context of a real problem.

In other problems, students need to choose among infinite possibilities. For example, Figure 6.2 shows an exercise in which a student is presented with the panel on the left and is asked to connect a line to a

FIGURE 6.1
EXERCISE IN WHICH STUDENTS NEED TO COMPARE FRACTIONS
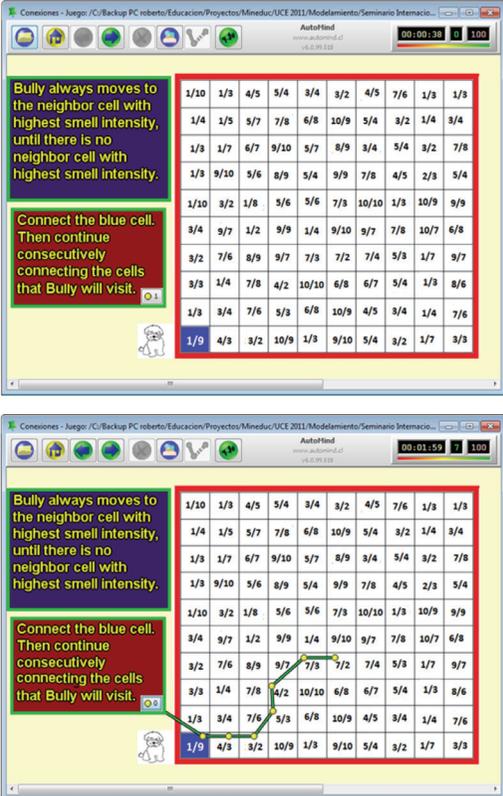
Source: Screenshot of the ConectaIdeas platform.

FIGURE 6.2
EXERCISE IN WHICH STUDENTS DEVELOP NUMBER SENSE ABOUT FRACTIONS
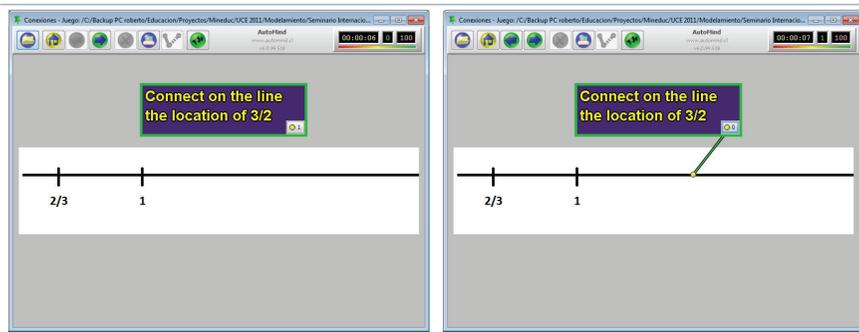
Source: Screenshot of the ConectaIdeas platform.

point that is roughly in the location of 3/2. Hence, this exercise does not involve the application of a standard algorithm. Rather, it seeks to develop the number sense of the student, who will need to use the information about the distance from 2/3 to 1 to understand the approximate size of the unit and then locate the 3/2 point.

One challenge faced by the Universidad de Chile team was that some teachers are not comfortable with the nontraditional exercises in Figures 6.1 and 6.2. In many cases, they had to be convinced that introducing these exercises was relevant and that students could solve them. This process may constitute professional development in the sense that it shows teachers how new exercises that go beyond the application of standard algorithms can be introduced in instruction.

As emphasized in Chapter 2, experts in mathematics learning recommend that students spend time providing explanations about mathematical concepts and ways to solve different problems. The Universidad de Chile team tried to implement this recommendation by requiring that all students in each session answer at least one question that requires them to think about a certain problem and explain why a certain strategy is adequate for solving it. These types of questions seek to develop meta-cognitive skills, that is, skills related to recognizing problems, thinking about different strategies to solve them, designing a plan, implementing it, and evaluating at the end whether the chosen strategy worked.

One example of a question that seeks to develop meta-cognitive skills is the following: "In which situations of daily life would you need to apply the concept of area?" When this question was posed to a class, students provided a range of answers, such as:

1. I don't know
2. When buying something big
3. When there is a perimeter
4. When I buy fruit
5. When I have to buy wood for the floor of a room
6. With my toys or cars
7. Never

Among all the answers reported only (5) shows a correct understanding of how to apply the concept of area. This shows the difficulties that students face in understanding the applications of the concepts taught in school and the importance of providing them opportunities to practice these skills. Moreover, the platform forwards answers provided by one student to other students so that they can reflect on them and provide feedback. These processes are not only useful in developing mathematical understanding, but also in building teamwork skills. In fact, when working on a team, there are many instances when participants need to assess answers provided by teammates and make (constructive) comments about them.

Another important activity facilitated by technology is peer tutoring. As mentioned, the ConectaIdeas platform identifies those students who have mastered the concepts at hand, and then the lab coordinator assigns them to assist students who are asking for help. Experience shows that struggling students seem to value the support provided by other students and are willing to request such support. Also, some struggling students may feel more comfortable requesting support from a peer than from a teacher.

The basis for the programs analyzed in this chapter is that practice is key to developing mathematical skills. However, practice requires motivation. Technology can help make learning more engaging, but experience shows that once the novelty of using a particular technology wanes, complementary actions are needed to maintain high student involvement. Embedding games in technology (usually called "gamification") is a useful strategy to increase motivation and sustain practice; however, this strategy has limitations because, again, students may lose interest as time goes by.

One strategy that seems particularly promising involves introducing team-based tournaments. Evidence from different disciplines indicates that people are particularly motivated in team sports (such as soccer or basketball) or even by the mere presence of other persons when performing a task (Zajonc 1965). In the realm of education, researchers have long pointed to the great potential of introducing team-based tournaments to promote student motivation (Edwards, Vries, and Snyder 1972; Slavin 2010). However,

this is difficult to implement in regular classrooms because of the logistics involved. Additionally, making one group of students in the classroom compete with another group may create harmful classroom dynamics.

The Universidad de Chile team has sought to leverage the promise of team-based tournaments in education using technology. In particular, the team introduces regular tournaments in which students in a section of one school compete with students in sections of other schools. These tournaments involve a collection of individual games played by pairs of students (a student from one school competing with a student from a different school) who try to win and obtain points for their section. The tournaments involve the simultaneous participation of many schools, which makes the events more engaging for students. The tournaments not only increase motivation, but also provide a concrete incentive to engage in intensive practice in the weeks leading up to the tournament. Moreover, these tournaments can promote collaboration within the section because winning or losing depends upon the performance of all the students, not just the best ones in the class or school, as is the case in the International Mathematical Olympiad and other competitions.

Decision 3: How Much Time?

Instructional time is the central resource in learning and, hence, needs to be managed carefully. A key design decision is how much time should be devoted to technology sessions.

One source of evidence relevant to this decision comes from analyzing the results of evaluations of educational technology programs that aim to increase mathematics learning. For example, a meta-analysis of 71 evaluations in the United States presented effects by weekly time of use (Cheung and Slavin 2013). The study shows that for evaluated programs where students spent less than 30 minutes a week, the average effect was 6 learning points; where students spent between 30 and 75 minutes, it was 20 learning points; and where students spent more than 75 minutes, it was 14 learning points. This suggests that the effects seem to be higher for programs that devote at least 30 minutes to weekly practice. These results, though informative, should be interpreted with caution because the interventions evaluated in these studies vary not only in terms of the weekly time spent on the technology sessions, but also along many other dimensions such as the software used, the quality of implementation, and contextual conditions.

A review of experimental evaluations in developing countries included six programs focused on mathematics (Arias Ortiz and Cristia 2014). Three of the programs in China included 80 minutes of weekly computer time

on mathematics with an average effect of 14 learning points. One program in India specified 120 minutes of weekly practice and had an effect of 41 learning points, whereas another entailed 300 minutes spent using computers during after-school sessions and had a positive effect of 25 learning points. Finally, a third program in India also entailed 300 minutes of weekly practice, but that practice replaced regular mathematics instruction and generated a negative effect of 48 learning points. The authors of this last study attributed the negative effects to the fact that the technology sessions were not well designed and replaced highly effective instruction in schools. These results suggest the need for careful thought on how much time is devoted to technology sessions.

Clearly, this decision should depend upon the types of skills that are to be developed and the learning activities involved. In particular, experts in mathematics education suggest that it should not take more than 10 to 20 minutes a day to develop mathematical fluency. There may actually be potential detrimental effects of devoting too much time to drills to promote mathematical fluency.

A related question is whether the weekly time should be spent in one, two, or more sessions a week. Here there are two potential issues to bear in mind. On the one hand, shorter sessions may help provide more varied and engaging learning activities to students in a day. On the other hand, starting a technology session may require substantial time for set up, and so fewer and longer sessions may be more practical.

Hence, a key issue here is how much time is needed to start a technology session. This depends on a variety of factors including technical issues, difficulties in ensuring that all students transition adequately to the computer lab, and even cultural issues related to whether or not groups start activities punctually. These factors may explain why the length of sessions may differ across contexts. In particular, several evaluations that demonstrated effectiveness in China involved two weekly sessions of 40 minutes each. In contrast, the Universidad de Chile team advises against having such short sessions, because it has documented that normal set-up time requires about 25 minutes. Hence, in the case of Chile, longer sessions may be more appropriate (which is why the program's sessions take about 90 minutes of instructional time).

Decision 4: Which Staff?

For technology sessions to be effective, it is critical to consider which staff members are willing and able to conduct them. There are three basic models. The first involves employing a specialized staff member—a lab

coordinator—to conduct the session. This person can be a former teacher or a community member with a certain basic level of education. This model has been used in India in the programs reviewed in Table 6.1. The lab coordinator receives specialized training in how to deal with common technical issues and how to conduct the sessions. This model works well if the technology sessions are done after class. If they are done during regular school hours, it is necessary to find time when the students are not receiving instruction from the regular classroom teacher. For example, the sessions can take place during teacher planning periods (i.e., time that teachers have during the regular school schedule for preparing classes or grading exams). The advantage of this model is that the lab coordinator can specialize in the task at hand, but the disadvantage is that it is more difficult to coordinate the timing of technology and nontechnology sessions. Moreover, the sessions need to take place after class or during empty slots in the school schedule.

A second model involves hiring a lab coordinator who shares the responsibility of conducting the session with the regular teachers. The advantage of this model lies in the fact that the technology and non-technology sessions are well aligned and coordinated. Moreover, this model involves teachers in the process without asking them to take up new responsibilities, and so it is usually well received. However, the disad-vantage relates to cost: now there are two people being paid to provide instruction to one class.

A third model involves only the teacher conducting the technology ses-sion. The advantage of this model is that the technology and nontechnology sessions are well coordinated and that costs are less than for the second model. However, this model faces several challenges. To start with, the teacher needs to be trained in solving technical issues and using the plat-form. But teachers have many other responsibilities and, hence, it is difficult to ensure expertise because of the lack of specialization. Moreover, teach-ers may get frustrated because of problems using technology and could stop performing the technology sessions when certain problems (e.g., tech-nical issues) emerge. Potentially, as governments invest in ensuring better technological infrastructure in schools (e.g., by making Internet connections more reliable), teachers may be more willing to take up this task.

In terms of cost-effectiveness, the first model seems ideal. The advan-tages of specialization can be exploited, and costs remain low because only one person conducts technology sessions. In this model, the issue of coordination with the teacher can be resolved by establishing good com-munication between the teacher and the lab coordinator. But this model requires finding a time slot when the students are in school but are not in their regular classroom sessions (e.g., after hours or other unused slots).

For its part, the third model is clearly less expensive than the second (because it does not require the hiring of a lab coordinator) but it is problematic to give teachers new responsibilities and expect that technological resources will be adequately used.

If the first model cannot be implemented, a cost-effective alternative involves a hybrid between the second and third model under which some sessions are conducted by both the lab coordinator and the teacher, but most sessions are conducted during regular hours by teachers. The strategy involves supporting the teacher to take up most of the sessions, but conducting the sessions with both the teacher and the lab coordinator when the teacher is not able to conduct the session alone. For example, at the beginning of the year, teachers may not know how to use the platform and solve technical issues, and so some joint sessions could help jumpstart the process. Similarly, if teachers do not conduct sessions for an extended period (e.g., after vacations or medical leave), a joint session could also be helpful. Under this hybrid model, the ratio of teachers per lab coordinator may decrease over time as teachers become more capable of conducting sessions by themselves.

The Universidad de Chile team has adopted the second model as a way of ensuring that technology is well used and that teachers are involved in and supportive of the changing mathematical instruction. However, because of the high costs involved in this model, the team is currently considering shifting to the first model, taking advantage of the policy trend in Chile to provide teachers with more planning time. Consequently, the lab coordinators can conduct the technology sessions during teacher's planning periods during the school day. Another possibility involves shifting to the hybrid of the second and third models described above as a strategy to reduce costs without suffering substantial decreases in effectiveness.

Beyond who conducts the technology sessions, an important issue is what role that facilitator conducting the sessions should play. Here there are two basic options. In the first, the facilitator provides only technical support, helps students understand the software, and encourages their participation in learning activities. In other words, the facilitator does not play a pedagogical role. In the second option, the facilitator offers both technical support and also plays a pedagogical role by providing explanations, asking students questions, and directing the entire class. Moreover, a number of variations beyond these two options may involve, for example, one person being in charge of the technical support aspects and another in charge of the pedagogical aspects.

In the intervention implemented in India, evaluated by Banerjee et al. (2007), the facilitator plays only a technical support role and is not

expected to provide any type of pedagogical support to students. In this case, the facilitator is a community member with at least a secondary education. A similar model is used in the interventions in China included in Table 6.1. That is, a specialized person is in charge of the sessions and this person is not expected to provide pedagogical support. In the intervention in India evaluated by Linden (2008), several students use computers in their classroom while a teacher provides instruction to the rest of the students. Also in this case, the teacher is not expected to provide pedagogical support to students while they are using the computers.

The model developed by the Universidad de Chile team also involves hiring specialized staff to be responsible for the sessions. However, this lab coordinator is expected to solve technical issues and also provide pedagogical support to students. Moreover, teachers are invited to participate in providing pedagogical support to children during the sessions. In practice, most though not all participate actively during the sessions and share pedagogical roles with the lab coordinators.

Decision 5: Personalized Instruction or a Common Pace?

Another key decision is whether instruction in technology sessions should be personalized to each individual student or whether all students should be instructed on the same topics in each lesson. There is no consensus on this issue yet. On the one hand, the potential of technology to personalize instruction to the individual student has long been recognized. Personalization is beneficial: one of the central principles of learning is that the difficulty of a task should be aligned with the level of the student. If the task is too challenging, the student gets frustrated; if it is too easy, the student gets bored. Hence, providing challenging but achievable tasks for students seems optimal. This is the central motivation for introducing "tracking"—that is, assigning students to classrooms based on baseline achievement levels, which has been shown to have positive effects on learning (Duflo, Dupas, and Kremer 2011; Duflo et al. 2015). However, tracking is highly controversial because of its potential negative effects on the self-esteem of students with lower levels of baseline achievement. Hence, computers are seen as a strategy of teaching students at their level while avoiding the potential detrimental effects of tracking.

On the other hand, personalizing instruction complicates the coordination between traditional and technology sessions. Because all students cover the same topics during the traditional sessions but different topics during the technology sessions, it is not possible to align the work in both types of sessions for all students. Moreover, personalization

reduces the scope for leveraging social strategies during instruction. For example, if students are working on different topics, then peer tutoring across students gets complicated. Also, the attractiveness of all students working on the same concepts in preparation for a tournament cannot be implemented with personalization. On a more general level, learning approaches implemented in high-performing Asian countries favor whole-class activities.

The empirical evidence on whether personalization or a whole-class approach is more effective for technology interventions remains scarce. A recent study in the Netherlands shows that students assigned a prespecified set of exercises learned slightly more than those for whom exercises were chosen depending on their level of academic achievement. However, the difference was not statistically significant (Van Klaveren, Vonk, and Cornelisz 2017).

In the six evaluations reviewed in Table 6.1, five followed a whole-class approach in which all students advance at a common pace. The only exception is the evaluation implemented in India, reported by Banerjee et al. (2007), which used software to provide exercises to students based on their achievement levels. The Universidad de Chile team followed a whole-class approach because a central element in its strategy involves leveraging positive social dynamics in learning. Consequently, all students in the program ConectaIdeas advance at a common pace.

Decision 6: Computers in the Classroom or a Lab?

There are four main options regarding where computers are used. The first involves a computer lab that is shared by students from different classes. In the second, portable laptops are located in a cart with wheels and are moved across classrooms and shared by students in different classes. In the third, there are four to six desktop computers located in a corner of the classroom. Finally, in the fourth, each classroom has a large number of dedicated laptops that students use when necessary.

In the context of LAC, the last two options can be quickly eliminated. The outlay involving four to six desktop computers is relevant for educational contexts such as the United States, where different learning centers are set in the classroom and groups of students transition across the centers. This type of a learning approach is almost nonexistent in LAC. Also, if many laptops are assigned to each classroom they will remain unused for long stretches of time given that the model described here involves using computers for a maximum of three hours a week. Therefore, this is clearly an inefficient approach.

As regards the first two options, the Universidad de Chile team considers a dedicated lab more efficient because fixed desktops are easier to maintain, can be protected against theft, are less expensive for the same level of computing power compared with laptops, and can facilitate the work of the person in charge of ensuring that all computers are working adequately. On the other hand, having portable laptops (or tablets) can be a feasible alternative for schools that do not have available space for a dedicated computer classroom or as a temporary solution.

Decision 7: One or More Students per Computer?

During a technology session, one student can use one computer or, alternatively, two or more students can share a computer. Sharing computers decreases costs and may be a good strategy to promote cooperative learning. However, sharing computers precludes monitoring how much each individual student is practicing, and some students may take a free ride on the efforts of their partners. Moreover, providing one computer per student motivates students to participate in tournaments and ensures the accountability of each member of the team. This has been pointed to as an important design principle when introducing tournaments for enhanced learning (Johnson, Johnson, and Johnson, 1984).

All evaluated programs presented in Table 6.1 have implemented the approach of students sharing computers. However, the Universidad de Chile team provides individual computers to students because it considers monitoring the progress of each student to be critical, both in itself and also in preparing for tournaments.

Decision 8: Which Software?

Among the desirable features that the selected software should have is that it should:

1. Provide immediate feedback
2. Present varied activities
3. Seek to develop different skills
4. Engage students
5. Provide a balanced coverage of the curriculum
6. Allow real-time monitoring by teachers and administrators
7. Be well regarded by teachers
8. Allow flexibility for teachers to add or at least select items

9. Be relatively inexpensive to buy or develop
10. Use a moderate level of Internet bandwidth

Features (1) to (3) are related to basic principles of effective learning. These emphasize providing feedback, varying activities to foster a deeper understanding, and developing a range of skills that go beyond achieving fluency in the application of standard algorithms. Feature (4) is linked to the importance of fostering motivation to achieve sustained practice. Features (5) to (8) are important to facilitate the work of teachers, and hence, adoption of the software. Finally, features (9) and (10) are related to controlling development and recurring costs.

It is rare to find software that includes all these features. Hence, program managers face trade-offs. More generally, there are two important decisions that program managers need to make. The first involves whether to buy off-the-shelf commercial software available on the market or engage in the development of customized software. On the one hand, readily available software typically has nice graphic design features that may engage students, and the start-up costs (in terms of money and time) are relatively low. On the other hand, developing customized software facilitates a balanced coverage of the curriculum, allows for monitoring all student activities in one place, and provides flexibility to add or select new items.

All the evaluations listed in Table 6.1 are of customized software, suggesting that, in practice, customization seems to the best option. Particularly telling is the program evaluated by Banerjee et al. (2007), which used standard commercial software during the first year of implementation and switched to customized software in the second year. Similarly, the Universidad de Chile team decided to develop customized software because of its important advantages and the expertise of the team in developing software. Moreover, the team agreed that it was more important to motivate students by integrating elements of play (e.g., tournaments) rather than by presenting nice graphic design features, whose motivational benefits have been shown to decrease markedly over time.

The second decision faced by program managers involves whether to use online software that requires Internet access. In many contexts, such as in rural areas, this is moot simply because there is no Internet access or that access is prohibitively expensive. Where there is access, online software is advantageous because it enables the monitoring of key indicators (such as the average number of exercises done per student in each classroom every week) in real time and also facilitates the provision of technical support (such as installing fixes and updates). In turn, software

that does not require Internet access has lower costs and reduces the chances of system malfunction due to unreliable access.

In all the evaluations reviewed in Table 6.1, the software used did not require Internet access. In some cases, this had to do with the contexts of the evaluation—for example, in poor neighborhoods in India in 2005 (Banerjee et al. 2007) where Internet access was not available at a reasonable price. In the interventions implemented in China, on the other hand, Internet access was intentionally blocked from computers to eliminate the possibility of students wasting their time surfing the web or having access to potentially harmful content.

In contrast, the Universidad de Chile team decided that it was desirable to use online software. To start with, Internet access in the schools in Santiago de Chile where the program operates is quite consistent and improving over time. Moreover, the team considered it critical to monitor the learning process in real time to quickly detect problems and implement corrective actions. Additionally, the use of tournaments to sustain motivation over time was a core strategy of the program, and this was easily done with an online platform. Finally, the team found that in cases where regular Internet access was unreliable, disruptions could be minimized using wireless Internet access coupled with a potent router that allowed the sharing of one Internet access connection among all the students in the classroom.

Still, access to a stable Internet connection remains an ongoing challenge, and the Universidad de Chile team has adopted several strategies to deal with it. First, the software does not involve any video because there is not enough bandwidth available. Second, at the beginning of the year all directors and district administrators were asked to make the necessary arrangements to ensure reliable Internet access. Third, the team developed protocols on what is to be done if Internet access is down—such as available printed exercises that students can do instead of using the platform. All in all, stable Internet access remains a challenge that will hopefully become less of an issue as the infrastructure at schools is upgraded over time.

Decision 9: How to Train and Coach

The teachers or lab coordinators who conduct the technology sessions play a central role in the success of these initiatives. To perform this role, they need to be well prepared. But which skills should they have? This depends on the actual role that the person is expected to play. As mentioned in design issue 5, in some cases the person conducting the session has responsibility for only technical support. In other cases, the person

is also expected to perform pedagogical activities. Hence, the training should be aligned with the specific role that the persons involved will play.

To prepare this person to perform the expected tasks, programs usually involve some training and coaching. For example, in the intervention reported by Banerjee et al. (2007), the individuals conducting the sessions received one week of training as well as regular visits from supervisors. Also, in the evaluated cases in China, the person conducting the technology sessions participated in two days of training and received regular supervisory visits.

The Universidad de Chile team has developed a strategy for computer lab coordinators that involves a day's training in the use of the platform, followed by extensive coaching during weekly meetings (described in design decision 10). Teachers do not receive training; rather their skills are expected to be developed through practical experience as they co-lead the technology sessions together with the computer lab coordinators. In conclusion, the pedagogical support in the model implemented by the Universidad de Chile team relies almost exclusively on the accumulation of practical experience alongside coaching sessions.

Decision 10: How to Manage the Intervention

The management of any intervention plays a critical role in its effectiveness, and the programs analyzed in this chapter are no exception. Management involves the usual phases of planning, execution, monitoring, and identification of corrective actions. In the technology programs covered in this chapter, management involves a number of tasks in key areas such as:

- Human resources—defining profiles, hiring, and supervising staff
- Software—generating exercises to ensure relevance, balance, and quality
- Technical support—ensuring that devices work as expected
- Relations with stakeholders—maintaining strong support for the project
- Monitoring—continuously checking indicators of the quantity and quality of use
- Administration—making purchases, payments, and keeping records.

Among the empirical evidence highlighting the importance of good management, the review in Cheung and Slavin (2013) documents that programs that report a high quality of implementation had average effects of 26 learning points compared with only 12 learning points for those that

reported a medium or low quality of implementation. Though this evidence has to be considered with caution for various reasons, it is important to note that strong management is needed to produce significant results.

An invaluable tool for monitoring technology in education programs involves the use of real-time data on the quantity and quality of the use of the technological resources. Important indicators can be constructed to measure key elements such as the number of exercises solved, percentage of exercises solved correctly, time spent on each exercise, whether students asked or received support from peers, and whether the amount of practice is well balanced in terms of the topics in the curriculum. Moreover, all these indicators can be computed for each student, class, school, and group of schools (e.g., those assigned to the same computer lab coordinator), and also for different time periods such as weeks or months. This provides invaluable information not only for program management but also to computer lab coordinators, teachers, principals, and even parents and students to identify and troubleshoot problems.

The Universidad de Chile team has developed a number of different reports that are tailored to specific audiences. For example, there are reports that show the number of exercises that each student has done per week and a comparison with the class average. This shows students how much practice they have accumulated and provides them with an incentive to practice more. Similarly, there are also specific reports that focus on elements relevant to parents, principals, teachers, and lab coordinators.

Finally, the Universidad de Chile team convenes a weekly meeting attended by the program coordinator and all the lab coordinators to discuss the progress made and next steps. A number of statistics for the week are reviewed (e.g., the number of exercises performed by each student from different schools), challenges are discussed, and potential solutions are analyzed. These meetings are central for monitoring advances, but, more importantly, they provide an excellent opportunity to provide feedback to lab coordinators to support them in adopting effective practices. That is, these meetings play a central role in terms of the professional development of the staff.

6.3 Summing Up the Design Decisions and a Final Note on Coherence

This chapter provided an in-depth analysis of 10 key decisions that need to be considered when designing guided technology programs focused on student practice. For each decision, the chapter laid out options, considered theoretical issues, reviewed the available evidence, and discussed in

detail how a number of effective programs in China and India, as well as a program in Chile, have tackled each decision.

There are two central themes that emerge from this analysis. To start with, in many of these decisions there are important trade-offs that need to be considered, but which may not be recognized at first. To make the best decisions on these trade-offs, it is important to analyze the options carefully by considering not only their potential benefits but also the potential challenges that will be faced in reality. For example, for a long time it was considered that the strength of these programs lay in the possibility of personalizing the instruction. Though this idea is very appealing, personalizing instruction has important drawbacks in practice. Therefore, the Universidad de Chile team decided that all students in a session would review the same topic and thus refrained from personalizing the instruction to each student.

The experience of the Universidad de Chile team is potentially informative because the team developed a program model that is highly promising. Table 6.3 summarizes how this team tackled the 10 key decisions analyzed in the chapter.

TABLE 6.3
THE 10 KEY DECISIONS IMPLEMENTED BY THE UNIVERSIDAD DE CHILE TEAM

	Area	Decision
Objective	1. Which skills?	Computation, modeling, representation, analysis, and problem-solving
Processes	2. Which learning activities?	Varied problems, metacognitive questions, peer tutoring, games, tournaments (no videos)
	3. How much time?	180 minutes per week (90 minutes of additional mathematics instruction)
	4. Which staff?	A specialized lab coordinator (who co-leads the sessions with the teacher)
	5. Personalized or common pace?	Common pace
Inputs	6. Computers in classroom or lab?	Computers in the lab
	7. Computers shared during a session?	Each student has a computer
	8. Which software?	Customized software (not off-the-shelf) and online
	9. How to train and coach?	Very short training, learning-by-doing with coaching, and regular meetings
	10. How to manage?	Strong management team, continuous monitoring, and weekly meetings

Source: Prepared by the authors.

Another important theme that emerges from the analysis in this chapter pertains to the importance of assuring coherence across the decisions made. That is, it is not enough for each individual decision to make sense; it needs to make sense when seen in conjunction with all the other decisions made. A good example is the importance of coherence between decisions 7 (whether computers should be shared) and 8 (about software, in particular whether to use online or offline software). If computers are shared, then the advantages of using online software are reduced because it is difficult to construct individual-level statistics on use and advances in learning. In other words, these two decisions are linked. This may explain why the programs in China have typically involved sharing computers and using offline software, whereas the program developed in Chile involves individual computers and online software. Ensuring adequate coherence across all decisions needs to be a central guiding principle to ensure that the technology used in education programs lives up to its promise.

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Mathematically Open Learning Technologies: Tools for Student-Centered Mathematics

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This chapter considers the use of powerful, open-ended, and student-centric mathematically open learning technologies (MOLTs), which are considered to be among the most effective and high-impact technology applications available in today's mathematics education setting. Through three essential (definitional) features, this broad class of technologies seeks to cultivate the mathematical understanding, competence, confidence, and proficiency of its student users.

Perspectives that position mathematics less as inert "knowledge" or "content" to be transmitted from teacher to student, and more as the actively constructed and emergent result of individuals' experiences manipulating mathematical ideas, practices, representations, and tools, imply that any technologically-based effort desire to improve students' mathematical ability ultimately depends on a technology's ability to support students in the act of doing mathematics. MOLTs are technologies that engage students in the active pursuit and construction of knowledge and experience in a diverse range of mathematical topics and practices. While existing in many mathematical subject areas, across different grade levels, and for various hardware and software platforms, MOLTs exhibit common characteristics both in their form as technologies, and in their impact as educational practices.

The goals of this chapter are twofold. On one level, the aim is to introduce MOLTs, document these technological forms and their impact on practice, and discuss example models of their successful use, both at a categorical level and through detailed case studies focused on particular MOLTs. On another level, the aim is to usefully direct the discussion of MOLTs toward two potential readerships. For the policymaker or technology coordinator

who is often one step removed from the lived experiences and intellectual challenges of students in the mathematics classroom, this chapter aims to illustrate how the key ingredients of mathematics education (the student and the math) shape, and in turn are shaped by, technologies in education. For the practitioner working with specific technologies, and managing the complexities of their use by children and their impact on classroom dynamics, the chapter aims to help identify models of contemporary best practice and—more importantly, given the relatively short life span of specific technologies compared with the longer social arc of the classroom's technology-based transformation—help foster critical evaluations of the potential contribution of the many new technologies such practitioners will encounter in their future careers. By considering the relevance and impact of MOLTs at multiple levels, ranging from specific examples to categorical attributes, the aim is to connect both audiences not just to MOLTs but, in some sense, to each other.

The chapter begins by discussing different priorities and voices within the educational technology landscape, and how these differences can fracture debate and productive effort in planning implementations. The suggestion here is that focusing on technologies centrally concerned with students learning mathematics helps simplify or orient this chaotic landscape. Two classroom examples of such technologies in action are then put forth in order to provide a concrete foundation for the discussion that follows. Generalizing characteristics of these examples allows for defining a broad category of tools that share an emphasis on student-centricity, open-ended mathematical inquiry, and technologically innovative treatments of mathematics. Many specific software technologies that have adopted this approach over the past few decades are identified. Turning to policymakers' concerns, the chapter reviews the state of research on the effectiveness of tools in this category, while recognizing that the category itself is a post-hoc organization of specific technologies that precede it. From there, common potentials and pitfalls realized in the large-scale deployments of such technologies are identified. The chapter concludes with recommendations for implementers drawn from these collective risks and rewards.

7.1 So Many Problems, So Many Solutions

7.1.1 A Chaotic Landscape

The policymaker, school technology coordinator, and classroom teacher trying to make smart, well-informed decisions about educational technology focused on mathematics face a bewildering variety of choices. They

are increasingly surrounded by high-impact digital technologies—in their offices and homes, and in the pockets and backpacks of their students—and by ever-growing evidence of the profound transformations digital technologies have brought about in our work, communication, and play. Relevant research—even large-scale case studies and clinical evaluations—is often contradictory. This is made painfully clear in a recent report by the U.S. Department of Education entitled *Expanding Evidence Approaches for Learning in a Digital World*, which states "evidence [for technology's impact] has been relatively scarce in education. And the quality of the best available evidence has [...] been disappointingly weak" (Office of Educational Technology 2013, vii). Correspondingly, the mathematics curriculum—and in many schools, mathematics instruction and the physical mathematics classroom itself—remain surprisingly untouched by the digital developments of the past 40 years.

One can understand this situation structurally, and also predict that it will continue for some years to come. Understanding comes in the realization that the separate metabolisms of technology and school radically differ. Technological innovation (at the most fundamental level, of new hardware and software paradigms) is driven in society at large primarily by business and market considerations, where disruptive change corresponds directly to opportunities for competitive advantage and opportunity for profit in the marketplace. School, by contrast, is inherently resistant to change, with teachers often perpetuating—across their careers—models of teaching they learned in a pre-service phase. Research and assessment designs often increase this resistance: an emphasis on comparability across time (long-term trend analysis) and geography (international performance comparisons) often willfully ignores evidence of disruptive local change. And one can expect this situation to continue, since only now—well into the 21st century—are teachers and policymakers beginning to enter the workforce from the pioneering generations that have grown up with ubiquitous digital technology (Internet, mobile telephony), and that perceive it as basic cognitive infrastructure rather than some form of midlife intellectual novelty.

7.1.2 Clarifying Stakeholder Concerns

Yet all is not lost. One way of coming to terms with the plethora of different tools and research findings is to acknowledge the fundamentally different purposes they serve. Students, teachers, subject-specific curriculum coordinators, school-specific administrators, and district, provincial, and national educational officials are all potential clients of educational technology—just as technology developers, academic researchers, and a

wide range of school authorities, from classroom teachers to high-level government overseers, are potential clients of distinct types of research into technological effectiveness. These different actors in different roles bring different questions and objectives to the table, and motivate both different tools and different research approaches. Thus, differing and even contradictory "solutions" or "recommendations" can be equally valid (even if not equally useful) if on inspection they align themselves with different problems and questions. By clarifying and particularizing one's own questions and objectives before turning to the vast marketplace of technological promises, one can use those questions and objectives as filters to consider only those possibilities that have the potential to become relevant solutions.

This chapter adopts exactly that philosophy by proposing to investigate only technologies that take helping students learn mathematics as their central and defining purpose. Stated so bluntly, this proposition appears so broadly unobjectionable—even grandiose and generic—as to be useless in restricting discussion or in filtering the research literature of proffered technological solutions. But it is offered not as a banal rhetorical goal but rather as an operational definition that can usefully differentiate certain types of technology in education—or certain aspects of technology—from other types and aspects. Thus, a central technological preoccupation of "helping students learn mathematics" is clearly different from that of "helping students practice or demonstrate mathematics they have already learned," just as it is different from "helping deliver pretechnological instructional materials to students" (such as delivering teacher lectures by Internet, video, or traditional textbooks in e-book wrappers). And it is very different from "helping teachers teach mathematics" or "helping educational authorities deliver specific curricula or manage student enrollment and attendance and performance data."

All of these and other needs are real concerns of stakeholders within educational settings, and all of them pose problems that a variety of technologies seek to address. This chapter puts many of those other needs aside, and restricts the discussion to technological tools that help students learn mathematics, and to investigating the impact of such an operative restriction. At the same time, while only one of many goals, helping students learn mathematics is a fundamental one that many of the others (teaching, assessment, content delivery) seek to support indirectly. Thus for policymakers attempting to balance the needs of many stakeholders while responding to, if not integrating, diverse and even contradictory research evidence, adopting such a restricted goal can be both simplifying and productive.

7.1.3 Mathematically Open Learning Technologies in the Historical Development of Educational Technology

History is relevant here. The technologies described in this chapter—student-centric, open-ended, mathematically innovative learning tools—emerge within a chronology of educational technology in mathematics shaped on the one hand by the exhilarating promise of the digital era's ever-changing landscape of technological innovations, and, on the other, by a maturing awareness of the complexity of the interplay between technologies, learning, students, teachers, mathematics, and the diverse expectations and mandates of classrooms.

While a rich history of the evolution of educational technologies—even just of digital math technologies—is beyond the scope of this chapter, it has been argued elsewhere that significant milestones in the educational understanding of digital tools parallel significant milestones in the development of representational paradigms for technology input and output (Roschelle et al. 2016). Thus, the 1960s and 1970s plaintext drill-and-practice environments of early "computer assisted instruction," which emerged in the era of line-printer output devices, were modelled on the printed student workbooks and exercise books of the day, cross-bred with the limited multiple-choice response capabilities of alphamerical keyboard input. That era offered a very rigid notion of student learning (i.e., students learn by reading short paragraphs of text, all students learning equally with the same paragraphs) and very crude right-or-wrong assessments. A next-generation movement focused on constructionist technology experiences such as Logo (Papert 1980; Papert and Harel 1991), where open-ended, student-centered technologies involving student programming offered project-based opportunities for discovery learning. These environments took advantage of the graphical output screens of the late 1970s and 1980s to enrich mathematical depictions and enable more interactive forms of learner engagement. They marked the first real mathematics learning technologies to be widely influential in Latin America's first wave of digital technology adoption, when the Programa Nacional de Informática Educativa initiated by the Omar Dengo Foundation, IBM Latin America, and the Costa Rican Ministry of Education became a model for similar endeavors in a dozen other countries. In the 1990s, MOLTs evolved into numerous and sophisticated approaches that, though still open-ended and student-centric, were more calibrated to the entire school milieu, and to more school-friendly expressions of instruction and curriculum. Of these, dynamic geometry software is perhaps the most widely known example, again with significant

adoption in Latin America. The Instituto Tecnológico de Costa Rica was an early advocate of The Geometer's Sketchpad in pre-service education, and Cabri enjoyed significant uptake in Mexico and Brazil. In dynamic geometry, the computer mouse—as a (then-novel) two-dimensional, planar input device—offered the same sort of immediate, concrete access to the mathematical abstraction of the geometric plane as the (2D, planar) graphical output screen in a very satisfying fusion of technology and content, capable of illuminating (rather than, as for example Logo, replacing) large swaths of the mathematics curriculum. Finally, the advent of the Internet shifted technology conversations once again, rapidly solving a host of long-standing problems about scalable deployment (distribution, installation, and software maintenance problems) while introducing new challenges (connectivity on a societal scale) and even rolling back decades of progress in computer's representational capabilities (with early web-systems taking us back to the plaintext mathematics and the limited-bandwidth interaction possibilities, of the 1970s).

Thus when considering MOLTs, one is describing technology innovations across a spectrum from the late 1970s to the present. While the Internet has furthered the impact of those developments tremendously, it has not itself significantly revolutionized their mathematical content or representations (which are seen often as an application of hardware innovation), their student-centrism (which is an instructional design philosophy, rather than a technology artifact), or their mathematical openness. While many MOLTs are already more than a decade old as technologies, the educational sector moves more slowly than the technology sector, as argued previously, and so the social uptake of MOLTs and a mature understanding of their contributions to the classroom are very much works still in progress.

7.2 Two Examples of Learning with Mathematically Open Tools

Before isolating the definitional ingredients of such learning technologies, or prescribing specific practices and recommendations for their use, this section briefly visits two classrooms that use such technologies in not atypical ways. These are descriptions of actual classrooms using hardware and software that are widely available today. The physical particulars of these classrooms, their curricular contexts, and the social backgrounds of their students are less significant to the purpose of these examples than is the manner in which they suggest technology can serve the mathematical goals of whole-class activity. The two examples are chosen for contrast: they are situated at two disparate points on the primary school curriculum

spectrum; one deals with geometry while the other with numbers; and the first involves one of the most popular, ubiquitous mathematics software packages of the past 20 years, which runs on conventional desktop platforms, while the second features innovative multi-touch tablet software that has only recently emerged from the research lab. Yet at the same time, the pedagogical and mathematical roles of technology appear very similar across the examples.

7.2.1 Fifth Graders and Dynamic Geometry Software

A fifth grade review lesson on the properties of triangles begins with the teacher presenting a premade sketch of a triangle to the entire class via an interactive whiteboard and The Geometer's Sketchpad.[1] The triangle appears as in Figure 7.1, with its vertices labeled and a collection of measurements to the side displaying its edge lengths and angles numerically. As the teacher begins dragging various vertices of the triangle, it stretches and shrinks to track the teacher's moving fingers, with the numeric measurements dynamically updating to track changes to the moving triangle. While Figure 7.1 shows two examples of the triangle in different configurations, this is a limitation of print: students instead see a seemingly unlimited number of continuously related examples as the teacher pulls one of the vertices. They quickly notice both the visual behavior of the triangle and that the first two values in the top group of measurements (that is, two sides) and the last two values in the bottom group (that is,

FIGURE 7.1
TWO VIEWS OF THE TEACHER'S CANONICAL TRIANGLE

$m\overline{AB} = 5.1$ cm
$m\overline{AC} = 5.1$ cm
$m\overline{BC} = 5.1$ cm

$m\angle A = 32º$
$m\angle B = 74º$
$m\angle C = 74º$

$m\overline{AB} = 3.7$ cm
$m\overline{AC} = 3.7$ cm
$m\overline{BC} = 5.4$ cm

$m\angle A = 94º$
$m\angle B = 43º$
$m\angle C = 43º$
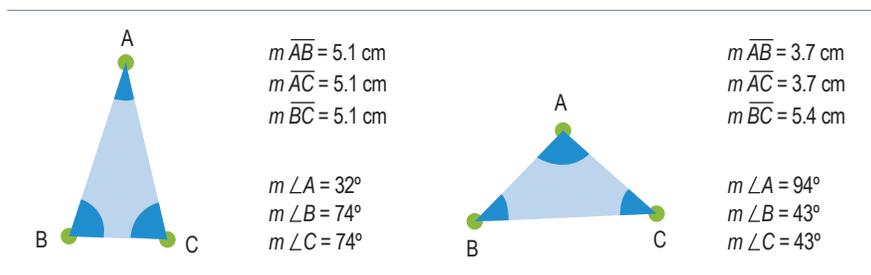
Source: Prepared by the author.
Note: In the figure, at right, vertices *A* and *C* have been dragged from their original position, but as each is dragged (at least) one other vertex moves as well to preserve certain invariants in the overall figure.

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[1]   See http://www.dynamicgeometry.com.

two angles) remain equal. This is enough to suggest to several students that this must be an isosceles triangle—they encountered that term and its definition in the fourth grade. These students call out that word and the evidence they're seeing to justify it, and the teacher reminds other students of a formal definition and demonstrates how the dynamic triangle meets that definition.

Then the teacher challenges students to construct an isosceles triangle, using the various geometric construction tools (an electronic compass and straightedge) to which they have been introduced in two previous classes with Sketchpad. Students are arranged in collaborative groups of four students each and are given several minutes to discuss the challenge, then take turns at the whiteboard attempting to develop their solution. They are given no written instructions or other task materials, though the preconstructed triangle (Figure 7.1) remains visible on the whiteboard until their group work activity is complete. During this time, the teacher circulates among groups, posing questions and helping with technology skills. When it comes to demonstrating on the whiteboard, one student ambassador from each group drives the demonstration. This process is at first error-prone but rapidly convergent thanks to ample spirited input from the rest of the class. Figure 7.2 shows three successful solutions the teacher captures from these small-group presentations.

The leftmost construction in Figure 7.2 uses mirror symmetry to define point *D* as the reflection of any point in the plane *C* over some given line *AB*, and then builds the triangle *ABD*. Since *AC* and *AD* are mirror images, the student authors claim, they have the same length, which "makes an

FIGURE 7.2
STUDENTS' PROPOSED CONSTRUCTIONS OF AN ISOSCELES TRIANGLE
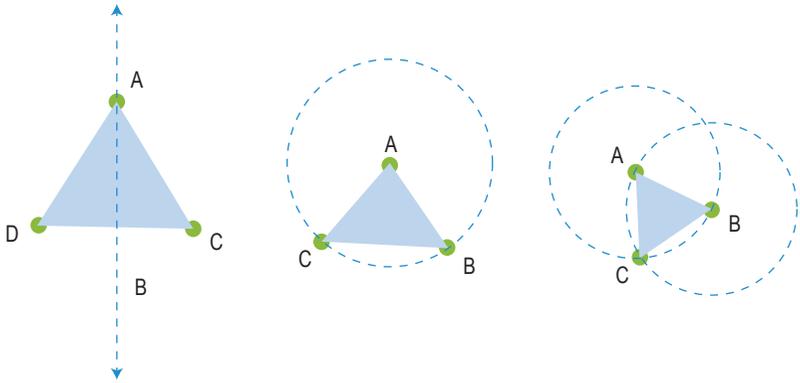
Source: Prepared by the author.

isosceles triangle." The middle construction begins with a circle *AB* and some point on its circumference *C. AB* and *AC* are both radii of the same circle, so they are the same length, and they are both edges of triangle *ABC*, so *ABC* "must be" isosceles as well. The final construction, on the far right of Figure 7.2, is more complex. Here, two circles (*AB* and *BA*) are both defined on a common radius (segment *AB)*. These two circles cross at a point *C* that is on each circle. Since the two circles have the same radius, *AC = BC*, its student authors claim the construction meets the definition of an isosceles triangle.

The teacher provokes and coordinates a brief classroom discussion of these constructions after each is complete, and again once all three are on the whiteboard. A rich discussion of the properties of triangles and also about mathematical aesthetics ensues. Students are surprised but somewhat delighted by the symmetry construction (at left), which "feels very different" from the compass-and-straightedge construction in the center. For one thing, no circles are involved, and students are unsure how much they like involving circles in the definition of triangles. For another, though, in Sketchpad's dynamic geometry, mirror images are strict peers, and so the figure at left "behaves" differently than the figure in the center when dragged. In the symmetry construction, when you move *C, D* moves opposite; when you move *D, C* moves opposite; and neither motion moves *A*.

By contrast, in the center construction, point *B* defines the radius (and so the size) of the circle, whereas point *C* is constructed as a point on a circle of size fixed by *A* and *B*. Thus, dragging *B* changes the distance of both *B* and *C* from *A* (while of course keeping those two distances equal), whereas dragging *C* simply rotates it around the circle, which does not change size. Students vastly prefer the symmetry of the left construction to the asymmetry of the right when it comes to the perceived effect of dragging the two "equal angle" vertices. However, an argument is also offered that the center construction involves only the three points that construct the triangle, whereas the leftmost construction also involves point *B,* which sets up a mirror of reflection (through *A*). It "feels weird" to need this fourth point, whose behavior is empirically described as determining "how tilted the triangle is." By contrast, in the center construction, "the tilt" is controlled equally by *B* and *C,* which are "part of the triangle."

Finally, no one appreciates the final construction, which they recognize as one introduced earlier in the discussion of equilateral triangles (which have all three sides equal, rather than at least two). While students agree that an equilateral triangle is itself isosceles, they are quick to point out that there are an unlimited number of isosceles triangles that this construction cannot make, because it will always enforce all three sides being equal. The

teacher introduces the term "overconstrained" to describe this approach, and compares it to a print triangle in the students' textbook: it shows an example (or in this case, a number of possible examples) of the concept of isosceles, but—unlike the first two solutions—is not as general as that concept. Returning to the first two constructions, the teacher demonstrates how each covers both equilateral and nonequilateral isosceles configurations. Then the teacher concludes the lesson by overlapping the two constructions into one, demonstrating how the center of the binding circle falls on the mirror of symmetry, and suggesting other ways that these two approaches to isosceles triangles might inform one another (as indeed they usefully do in a variety of dependent theorems students will encounter later in their mathematical careers). Finally, two groups that did not originally create functional constructions are given a brief opportunity at the whiteboard to rework their original triangle constructions to demonstrate isosceles properties.

7.2.2 Third Graders and the TouchCounts App

The second mathematical moment is drawn from much earlier in the curriculum. In this example, the teacher has decided to use TouchCounts to review skip-counting with students as an introduction to multiplication at the start of third grade.[2] TouchCounts is a multi-touch tablet-based software environment supporting finger counting and an embodied, gestural approach to whole number arithmetic (Sinclair and Heyd-Metzuyanim 2014). Its more typical milieu is in kindergarten or with preschool-age children. One of the experiences it offers is of a "Counting World" in which successive on-screen finger touches create sequentially numbered tokens immediately at the tips of one's fingers, which then "fall away" (pulled as if by gravity off the bottom of the display) when the finger is removed from the screen. As new tokens appear, TouchCounts also audibly names the corresponding number in a child's voice, though when multiple fingers touch the screen simultaneously—producing multiple successive values— only the highest-valued token is audibly named (Figure 7.3 illustrates this process). Thus, work in TouchCounts brings together four different ways of representing and indicating quantities—through physical pointing gestures ("this" one, "this" one, and "this" one), by iconic representation (the visible group of three tokens), by written numeral ("1," "2," etc.), and by audible number-names ("one," "two," etc.). Just as dynamic geometry manipulation—dragging shapes—provides the central mathematical experience

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[2] See http://www.touchcounts.ca; see also Sinclair and Jackiw (2011).

FIGURE 7.3
COUNTING OPERATIONS IN TOUCHCOUNTS
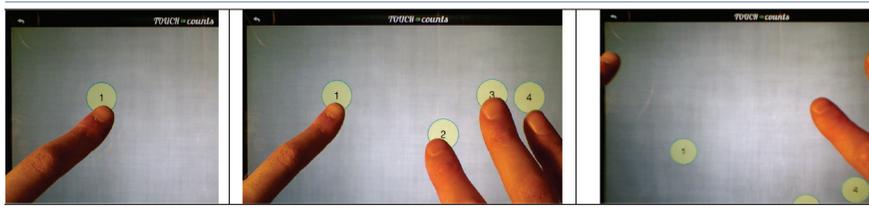
Source: Prepared by the author.
Note: Left: A user touches the screen with the left index finger and the token "1" appears at the user's fingertip; the software announces "one." Center: Three more fingers have been pressed on the screen simultaneously producing tokens "2," "3," and "4;" the software says "four." Right: A user lifts the fingers off the tokens, and all four of them fall off the bottom of the screen.

in environments like Sketchpad, the coordination and synchronization of these different representations of quantity provide the common foundation of diverse mathematical activity in TouchCounts.

Handing out a class-set of eight iPads, one to each pair of students, the teacher briefly explains the software simply by demonstrating how one can "count to 20" by tapping the screen repeatedly, and by pointing out two screen buttons that reset the Counting World's number sequence to start again at 1, and which turn on, or off, the "gravity" that wipes numbers away. Then the teacher establishes an open-ended task proposed in Sinclair and Zazkis (2015): use TouchCounts to "figure out—and show the rest of us—how to count by threes."

Students rapidly discover that the software responds to multiple simultaneous finger presses regardless of which finger—or whose hand—is doing the pressing. This means there is no need to "take turns" and instead both partners can interact with the tablet at the same time. This leads in some situations to momentary chaos; in others, to more deliberate collaboration; and in one group, to a quick division of labor in which one student produces numbers while the other takes responsibility for frequent use of the reset button. The teacher and the teacher assistant circulate and help address both questions about software mechanics and students' initial frustration with the open-endedness of the task. Students are somewhat familiar with counting by threes—most of them can readily count by twos—but they point out that no matter what they do, TouchCounts only counts by ones! (That is, the next token the software produces is always only one higher than the previous token.) The teacher encourages them to accept this behavior but work creatively with the challenge, and look for ways they can count by threes even though their tool only counts by ones.

After 10 minutes of open exploration—during which there is much excited sharing of discoveries and strategies across groups—a variety of approaches emerge, including those outlined in Figure 7.4.

In this class, there is less significant discussion comparing different approaches, but given the relative ease of producing each variant, most students try all methods rather than just one. The teacher readily

A. Students repeatedly touch the screen with two fingers and a thumb tapping "all together." Each tap produces three tokens that fall away when the fingers are released, and the software names aloud the highest token in each trio. Thus, over repeated taps, the software counts "three… six… nine… twelve… fifteen…" etc. (Here we see the screen just after the software has counted by threes to 27.)

B. Here a similar strategy has been applied, but with gravity turned "off" so that repeated triples of numbers do not fall away. The screen records each triple-tap in a cluster of tokens—the screen has been "tapped" eight times here. (Within each cluster, we can see the precise order in which the users' "simultaneous" finger taps actually touched the screen.)

C. Students have discovered a shelf in the software, on which they can "park" certain values so they don't fall away when released. Then they've developed a rhythmic approach to tapping where they drop the first of every three tokens onto the shelf and the next two below it, for a repeated gesture sequence of "one above, two below; one above, two below." The values accumulated on the shelf start at one and "count by three."

D. The same students who propose approach C evolve their strategy to repeated taps of "two below, one above," so that every third value rests on the shelf. They prefer this approach for its closer similarity to the sequence they know for counting by twos (which begins with 2 rather than 1).

Source: Prepared by the author.

FIGURE 7.5

SKIP-COUNTED TOKENS ARRANGED INTO AN AREA MODEL OF (*n* X 3) MULTIPLICATION
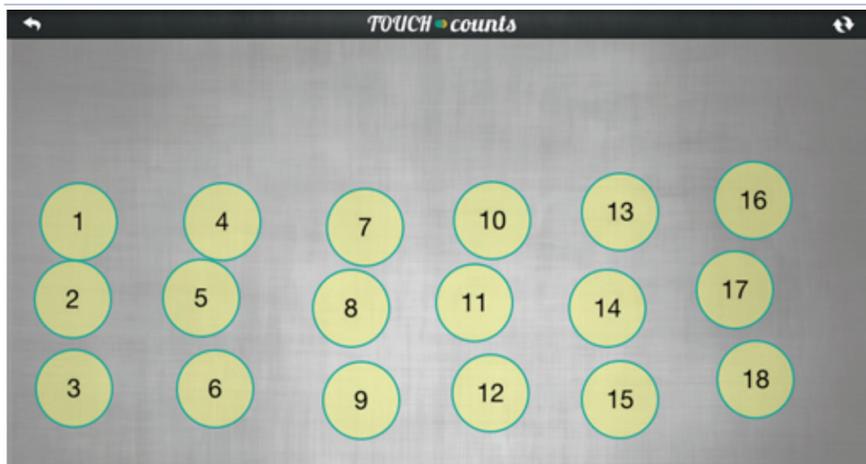
Source: Prepared by the author.

establishes a canonical sequence of counting by threes—3, 6, 9, 12...—and uses the grouped representation of approach *B* to point out how "counting by threes" and "grouping by threes" is the same thing, and these counts enumerate the total in one group of three, two groups of three, three groups of three, and so on. The teacher suggests a slight rearrangement of the students' work in approach *B* so that values are sequentially ordered inside groups, and so each group—which is arranged vertically—appears next to the previous group, as in Figure 7.5. This allows students to quickly see they are counting (vertically) by threes, and that counting horizontally—that is, counting columns—tells how many groups of three. There are 18 tokens altogether, which is six groups (columns) of three tokens. Focusing on this rearrangement of *B*, the teacher points out how it also contains both representations *C* and *D*, and begins asking questions that develop and unpack its multiplicative structure.

7.3 Generalizing a Model of Mathematically Open Learning Technologies

Let us step out of these two classrooms and briefly compare them. Despite being drawn from contrasting contexts, many similarities are clear, and from those similarities one can distill a general model both of MOLTs and their effective use. In each example, the technological representations that

students develop are motivated, organized, and critiqued interactively by a mathematically able and dexterous teacher. (Critically, exploration in these environments, though student-motivated and student-driven, is not unguided; the teacher sets the stage for, and facilitates the development of, student activity.) The technology augments and occasionally elevates the mathematical focus of classroom conversations—among students in their groups, or between students and the teacher—rather than supplanting them. Also, the structure of the activity in both classes is similar. Students in small groups use powerful digital representation possibilities to explore, in their own voices, a mathematical constraint or definition, and then synthesize their findings in a whole group discussion led by the teacher. If this activity is considered through the lens of a three-phase *exploration* ⇒ *understanding* ⇒ *fluency* model of activity structure (see Chapter 2), students are *exploring* as they construct their separate representations, but they are also developing *understanding* as they interact with the mathematically specific dynamic digital representations of their respective technologies, and gaining *fluency* in the mathematical model-making techniques central to each technology (geometric construction in Sketchpad; simple finger counting in TouchCounts). Analogies might be made to predigital general resources like pencils or the blackboard: we use them throughout our mathematical travels, and, indeed, it becomes hard to imagine doing mathematics without them. But these digital materials add significant mathematical structure to their use.

From a learning perspective, students in both classes are clearly engaged with mathematics on multiple levels. Consider first those in the geometry class. At a curricular content level, their activity focuses on properties of isosceles triangles, and on how those properties can be used to construct functional mathematical definitions. At the level of curricularly endorsed mathematical practices and processes, they are constructing not only geometric diagrams but also arguments, and reasoning through such arguments in their mathematical support or critique of peers' solutions. They are analyzing situations for exploitable structure and using mathematical tools appropriately—broadly, Sketchpad and a whiteboard, but more critically and specifically to each student, the individual construction and dragging of tools within Sketchpad's diverse functionality. These practices align with policy recommendations such as the U.S. Common Core State Standards for Mathematical Practice, which encourage teachers to help students of all ages develop the mathematical ability to "construct viable arguments," "use tools strategically," and "look for and make use of structure" (CCSS Initiative 2010). At an even higher level, students here are not just engaging in mathematics, but are behaving as

mathematicians, pursuing a heated argument about mathematical values and aesthetics through a variety of claims concerning functional symmetry, computational efficiency, and logical sufficiency. While the children in the younger classroom are less sophisticated in their justifications, their work demonstrates keen engagement on the first two of these levels, and perhaps even—at least in the case of the pair of students abandoning their own shelf strategy *C* for a preferred approach *D*—on the third level.

Likewise, the technology is scaffolding students learning mathematics on at least three distinct levels. First, it supports students through a student-centric technology design that in its simplest terms positions an actual student (or group of students) as its user, rather than a teacher or stakeholder outside the classroom; and it imagines that student user acting as a mathematical author and producer rather than only as a passive reader or consumer of prefabricated mathematical truths. The technology acts here as what Hoyles and Noss (2003) call an "expressive tool" in fostering students' cultivation of their own voice—their own creativity and self-expression—within the domain of mathematical knowledge and activity.

Second, in both examples, the technology supports authentic learning through its open-ended approach, which always allows numerous—perhaps even unlimited—different actions or manipulations at any juncture, rather than restrict progress to a gated, linear trajectory or set of narrowly branching, predetermined options. Such an open-ended disposition enables the common activity structure seen in the examples, where different students pursue mathematical challenges based on what they think might work (based on an assortment of prior knowledge and experience) rather than by identifying and repeating what they've been told (in the form of a single solution strategy). Feedback is constant, and always mathematical rather than moral in nature: the software responds continuously to students' work by exposing its mathematical implication, and judgment of the "rightness" or "wrongness" of those implications is left to the student. More generally, open-endedness in tool design orients the tool's suitability both to the needs of the same learner in different mathematical situations, and to the needs of different learners in the same mathematical situation.

Finally, in each case, the technology uniquely supports mathematics itself. In other words, there is a distinct mathematical contribution made by the technology, that is, a technology-specific, mathematically pedagogical idea. TouchCounts aims at young learners whose mathematical ideas both about quantity and operation are still very much grounded in their own fingers, and its technological contribution is to endow these fingers both with unlimited mathematical quantity (by allowing users to count

"past 10" on their fingertips) and with canonical mathematical identities by enumerating fingertaps ordinally (associating with each tap both a conventional audible, spoken number-name and a conventional visual, written number-symbol). Thus, TouchCounts attempts to make one's actual fingers more mathematically functional, while simultaneously attempting to make mathematical abstractions more physically graspable, more "touchable."

In a different mathematical domain, the compelling contribution of dynamic geometry's paradigm similarly redefines the border between concrete and abstract. Dynamic geometry permits users to transform any geometric figure or other mathematical diagrams (by dragging) into any other figure or diagram that shares the same mathematical definitions and properties. Figures in a dynamic geometry technology have a different epistemological and mathematical status than their print-based predecessors: where a printed figure is always a single illustration of, or an example of, a general case, the dynamic figure over time actually *is* that mathematical general case. Thus the technology speaks to the (recurrent) demand placed on learners across their mathematical careers to navigate conceptually from the specific to the general, and from the concrete to the abstract. And it does this in a physically tangible, intellectually insightful, and visually compelling fashion largely unimaginable before the advent of digital technology.

These three attributes—student-centric design, open-ended activity structure, and technologically specific mathematical contribution—can thus be taken as individually reifying a more general interest in and orientation toward students, learning, and mathematics, respectively. By defining a category or genre of educational technologies that are explicitly oriented to those interests, these attributes help students learn and do mathematics. Tools in this category act in the classroom as resources of mathematical agency and insight (through their technologically specific mathematical contribution), empower mathematical self-expression by a broad group of students (through their student-centric design), and finally (through their open-ended activity structure), accommodate a diverse set of mathematical goals set by students themselves as well as curricular and instructional goals set by teachers or policy frameworks.

As described early on, a variety of existing educational technologies fit this broad definition. And similarly defined or kindred-themed categories of technology have been identified by previous research under different names, of which "tools" and Papert's (1980) "objects-to-think-with" are perhaps the simplest and most general. Pea (1987) calls "cognitive technologies" those that "transcend the limitations of the mind, in thinking, learning, and problem-solving activities." Zbiek et al. (2007) call them

"cognitive technological tools." The preference here is to call them MOLTs so as both to emphasize their direct engagement with mathematical representation—critically, mathematics is not simply generic content that is dropped into these technological "wrappers"—as well as to explicitly orient them toward the learning context.

7.4 Varieties of Mathematically Open Learning Technologies

Table 7.1 highlights several MOLTs, many of which have had sufficient impact—both in schools and in scholarship—to spawn entire genres of conceptually similar learning technologies.

Identifying the degree to which each genre is student-centered (as opposed to teacher-centered or curriculum-centered) is straightforward—even when, as in the case of spreadsheets, the original intended user was not a student. Each genre subscribes to a similar vision of the user

TABLE 7.1
EXAMPLES OF MATHEMATICS LEARNING TECHNOLOGIES AND GENRES OF MATHEMATICALLY OPEN LEARNING TECHNOLOGIES

Early Exemplars	Emergent Genre	Technology-specific Mathematical Contribution	Initial Curricular Topics
Logo (1967); Scratch (2003)	Student-centered programming languages	Functional/imperative contexts for working with mathematical abstraction and encapsulation as well as exposure to problem-solving through iterative or recursive techniques	Varied. It should be noted that this genre has been criticized historically for its lack of immediate adaptability to topics in traditional math curricula.
VisiCalc (1979); Microsoft Excel (1985)	Spreadsheets	"What-if?" style mathematical model-making through simple arithmetical formulae; also, grid-based exploration of numeric patterns and iterative numeric approaches	Arithmetic, pre-algebra, algebra
Casio fx-7000G (1985); TI-81 (1990)	Graphing calculators	Convenient, powerful, and fast visualization of graphs of functions, refocusing curricular attention on graphs (as mathematical objects) rather than on graphing (as a mechanical/computational skill)	Pre-algebra, algebra

TABLE 7.1 (*continued*)

EXAMPLES OF MATHEMATICS LEARNING TECHNOLOGIES AND GENRES OF MATHEMATICALLY OPEN LEARNING TECHNOLOGIES

Early Exemplars	Emergent Genre	Technology-specific Mathematical Contribution	Initial Curricular Topics
Cabri Géomètre (1989); The Geometer's Sketchpad (1991)	Dynamic geometry software	Real-time, continuous, and unbounded variation of mathematical diagrams—geometric figures, graphs of functions, other visual representations of mathematics—through all mathematically equivalent configurations	Geometry, algebra, pre-algebra, number and operations
Fathom (1995); TinkerPlots (2005)	Dynamic data software	Visualization of data analysis techniques and measurements through structured and real-time variation of individual data	Graphing, data analysis, statistics, data-driven approaches to algebra
SimCalc MathWorlds (1997)		Concretization of the mathematics of rate, proportionality, and change through replicable digital simulations of motion-based phenomena	Pre-algebra, algebra, calculus
TouchCounts (2011)		Cognitively embodied mathematical generalization of early-childhood "finger counting" strategies	Counting, addition, subtraction

Source: Prepared by the author.

as empowered and intellectually curious, bringing a question, problem, or ambition to the expressive environment offered by the software. The software situates the user as a producer, rather than a consumer, of content. While in most cases the digital artifact that a student creates is some form of mathematical model (an imperative model, in the case of programming languages; a numerical model, in the case of spreadsheets; a geometric model, in the case of dynamic geometry, etc.), it is not coincidental that many of the genres adopt the "document-creating productivity software" paradigm familiar from desktop tools such as word processors. (The blank page that first greets students in a spreadsheet or a dynamic mathematics program acts as an invitation to their own creative self-expression.)

The technology genres in Table 7.1 are, likewise, "open-ended" in permitting equal access by a wide range of users to a wide—and potentially unlimited—range of applications. Open-endedness of application is of

course inherent in the idea of mathematical model-making, since a constant set of sufficiently expressive mathematical ingredients can be combined into an unlimited number of author-specific models, and as already noted, many of these technologies are mathematical modeling tools of some form. But there is also open-endedness toward the presumed user of MOLTs, and it is characteristic of many of these genres that they find application across a wide range of age levels and mathematical contexts. Where Papert (1980) uses Logo with elementary school students, his colleagues Abelson and diSessa (1986) use it to develop university mathematics topics. While dynamic geometry tools like Sketchpad seem aimed at the secondary-level course on plane geometry, reports find it often deployed in lower-primary and middle-school classrooms (Sinclair and Crespo 2006; Yin 2002), as well as in graduate-level courses and in mathematical research (Schattschneider and King 1997).

Finally, among MOLTs, the technology itself actively mediates and extends the nature of mathematical activity in the environment beyond what users could achieve in "nontechnological" mathematical environments (such as pencil and paper). MOLTs are not simply expensive and inconvenient means for restating or assessing forms of knowledge equally accessible without them; they are tools that harness computational potential to create uniquely new ways—through, for example, digital calculation, construction, visualization, and simulation—of building, expressing, evaluating, and applying such knowledge. Thus, MOLTs increase, rather than replicate, the reach of the collective instructional opportunities and strategies a teacher can make available in the classroom to diverse learners. Indeed, environments like SimCalc demonstrate that when MOLT-like technologies introduce fundamentally new mathematical representations, rather than digitally repackage conventional, pre-digital representations, they can profoundly alter the intellectual or material thresholds at which such mathematics becomes educationally accessible. Kaput and Roschelle (1998) describe how SimCalc's approach allows ideas and topics from calculus—from the traditional capstone of the advanced mathematics curriculum in secondary school—to become accessible at the middle school level once some of the "representational infrastructure" of the subject traditionally serviced by 18th century symbol manipulation mechanics is upgraded to 21st century digital representation. These representational shifts can overcome physical barriers as well as cognitive ones, as when Fernandes et al. (2011), working with blind students in São Paolo, report on developing mathematical technologies in which (digitally-produced) audible attributes replace traditional graphical and symbolic ones in communicating the behavior of algebraic functions and calculations.

The nature of these mathematical contributions has varied over time and across MOLTs, often (as described earlier) with developers finding innovative ways to interpret new advances in hardware from specifically mathematical perspectives. Table 7.1 lists examples of distinct mathematical contribution or innovation of MOLTs on a genre-by-genre basis, and it suggests appropriate content areas for the implementer considering where best to map the mathematical openness of MOLTs to specific topics in a given curriculum, while noting that MOLTs often serve far broader ranges of mathematical interest.

7.5 Perspectives from Classroom Research and Practice

Because the operational definition here of MOLTs spans decades of hardware variety as well as diverse curricular contexts, it is inherently more diffuse than the technologies described by most educational studies of controlled effectiveness. Still, its many individual, constituent technologies have been well studied and provide a deep body of evidence for considering classroom practice and impact. And this is where the value of a broadly categorical definition, like that of the MOLT, may prove its value. Typically, educational research on effectiveness greatly lags behind technological innovation; by the time technologies are well considered from a research perspective, they risk being obsolete from the perspective of the school market. By identifying temporal invariants such as the three ingredients constituting MOLT, however, the policymaker can extrapolate from research evidence and findings about past MOLTs to future ones emerging—or soon to emerge—on the innovation horizon.

Despite the problems of a broad categorical definition, some of the largest-scale research on educational technology—both on academic achievement and on perceived appeal by educators—appears to identify a space very similar to that which we claim is occupied by the MOLT. At the end of the 1990s, two separate projects concluded large-scale assessments of the state of educational technology adoption and impact across the United States, 20 years after the dawn of the microcomputer era. Wenglinsky (1998), studying technology effectiveness, found that the use of computers to teach the higher-order thinking skills that MOLTs routinely encourage—reasoning, problem-posing, and problem-solving—is positively related to both academic achievement in mathematics and the social environment of school. In contrast, the study found that the use of computers to teach lower-order thinking skills (e.g., learning facts, practicing drills) is negatively related—and actually harmful—to the same two outcomes. Becker, Ravitz, and Wong (1999) found that MOLT-like applications of

technology that focused on student self-expression and student-centered learning were most predictive of important learning according to teacher's beliefs. Turning specifically to mathematics technologies, the survey found greater ratification, among math teachers, of a single piece of software as "the most valuable software for students" than existed among teachers of any other subject for any other piece of subject-specific software. That software was The Geometer's Sketchpad, an essential MOLT.

Though these were associative analyses, not causal findings, more recent broad-scale quantitative research appears to reach compatible if not identical conclusions. Cheung and Slavin's (2013) meta-analysis of research on the use of technology in mathematics education, which considers only studies meeting a high standard of rigor, focuses primarily on non-MOLT-like technologies, that is, on curriculum/technology hybrids rather than mathematically open learning tools. Of these, the one demonstrating the most significant effect on student achievement was the one in which the curriculum-technology coupling was least essential, perhaps pointing to an even greater possible effect of more open (less curricularly coupled) learning tools. Another large and rigorous study of commercial mathematics software products (Campuzano et al. 2009) found no significant effect on achievement, but, again, included no software that could be construed as a MOLT. Thus the quantitative research argument for MOLTs is mixed, with studies suggesting, but not clearly isolating, the impact and effectiveness of technology approaches that resemble those scoped by the MOLT definition. As quantitative methodologies have moved toward the randomized controlled trial model of evaluation, educational research appears to shy away from profoundly open technology tools such as MOLTs, the impact of which may often be tied up in broad and diffuse cultural factors, in preference for evaluating environments that themselves produce quantitative assessments of children's performance through test-giving and test-scoring mechanisms.

Around specific MOLT technologies or genres of technology, the research case may appear stronger. In something of a high-water mark for the rigorous evaluation of scaled-up deployments of MOLT-like technologies, Roschelle et al. (2010) describe several randomized control studies of the impact of SimCalc replacement units on student learning of advanced middle school mathematics in schools across Texas. These studies report statistically significant and large effect sizes, supporting conclusions of SimCalc's effectiveness in fostering mathematics learning in a diversity of settings. In the case of dynamic geometry software, for example, some studies demonstrate the achievability of a significant impact on broad student populations using only commoditized, scalably replicable, and

practically achievable instrumentations. For example, Arias Cabezas and Maza Sáez (2006) found a 30 percent increase in "mathematics performance" attributable to use of dynamic geometry software in a six-year study of some 15,000 high school students and 400 teachers. In randomized control trials, Jiang, White, and Rosenwasser (2011) found strongly significant improvement in middle and high school students' scores on standardized tests as a result of dynamic geometry use.

7.6 Implementing Mathematically Open Learning Technologies at Scale

Whatever the benefits to individual students and teachers, a policymaker is understandably concerned with the implications of scalable implementation and the structural costs and benefits of large-scale deployments. At this level of concern, drawing on the sociocultural work of Remillard (2005) and Bretscher (2014), Clark-Wilson, Robutti, and Sinclair (2014, 3) encourage an understanding of classroom technologies as having "institutional, contextual and historical dimensions, and not just cognitive ones." This section considers some of the more commonly identified cultural risks and rewards of systemic adoption of MOLTs and offers recommendations to policymakers attempting to balance them.

7.6.1 Risks

Mathematically Competent Teachers
The two examples both involve a mathematically competent teacher coordinating and facilitating student activity responsibly; indeed, this may be their most essential component. (Chapter 5 points out how an able, responsible adult is the most important ingredient of *any* classroom.) Since the MOLT provides an open doorway to diverse mathematical discovery and opportunity, the teacher is frequently called on to evaluate diverse mathematical claims, many of which he or she may never have encountered before. In addition to considering the viability of these claims and supporting students as mathematical practitioners, the teacher is also responsible for guiding open-ended inquiry ultimately toward specific curriculum objectives when it strays. In practice, this requires a teacher who has solid mathematical preparation and both emotional and pedagogical security in that preparation. Teachers need to draw from both to be able to skillfully extend mathematical authority to students while at the same time constraining it toward established classroom objectives. Risk here corresponds to the degree to which teacher populations are weak in these competencies. An implication

is that professional development in teachers' mathematical dexterity will almost always pay higher long-term benefits than corresponding investments focused only on their technological dexterity (not least because technologies change at a much faster rate than mathematics).

Mathematically Enfranchised Students

Supported by such a teacher, both examples also involve students pursuing mathematical meaning at least partially under their own direction, while engaging earnestly in both group and whole-class discussions to present and refine their work, and to engage with the (perhaps imperfect) work of others. While developing such a mathematically enfranchised student culture is possible at scale, in many contexts it is far from the norm. Many traditional teachers are more comfortable delivering noninteractive lessons and lectures than facilitating student exploration. Correspondingly, many students have been habituated to think of mathematics as a field of fixed truths and specific answers, delivered by teachers rather than developed by themselves. Moreover, institutional policies and traditions—ranging from attitudes and beliefs to how chairs are arranged in a classroom—can reinforce these orientations. Student-centric technologies like MOLTs can play an important role in developing and sustaining inquiry-based cultures, but such cultures emerge from willful practice and stakeholder engagement rather than miraculously as an effect of provisioning certain software packages. Considering implications of student-centric math technologies on teaching, Mason (2014, 21) writes "[a]rranging the energies of the classroom so that [a teacher] can dwell in mediating or in responding can be exhilarating as well as liberating for students. Provoking students into experiencing the desire to express promotes the maturation of their understanding and their appreciation of what they are integrating into their functioning, that is, the education of their awareness."

7.6.2 Rewards

Despite such risks, the definitional components of the MOLT—student-centric design, open-ended activity structure, and technologically specific mathematical contribution—correspond to clear cultural benefits beyond their impact on individual students' mathematical ability. This chapter has already argued that among competing technology options and possibilities, a policy emphasis on students learning mathematics is justified and eminently defendable (as the foundational concern of mathematics education). This section now considers collateral implications of MOLT use on institutional culture.

MOLTs Support Group Work and Collaboration

Because MOLTs support open-ended work, they tend to better accommo-
date multiple, diverse approaches to mathematical problems and practice
than do technologies offering fixed or more sequentially structured inter-
action experiences. While three different students working through a
multiplication drill e-worksheet have largely similar user trajectories, three
different students asked to construct isosceles triangles in a MOLT may
take up three completely different mathematical approaches. Thus the
MOLT naturally lends itself to the dynamics of a small group of students
working collaboratively, since different student voices can be additively
beneficial to the group's problem-solving, rather than simply redundant
with each other. The same dynamics apply across groups in situations
where students are organized into multiple small groups: open-ended
designs permit students in one group to take a problem in a different
direction than students in another.

Of course, small collaborative group work is alien to some school ecol-
ogies, and teachers new to it often fear the autonomy of groups and the
comparative lack of direction one teacher can provide to multiple groups
simultaneously. And yet, where teachers are willing to experiment with
such autonomy, the student-centric design of MOLTs often enables stu-
dent exploration without tremendous hands-on teacher mediation. And
because of the essential (rather than superficial) mathematical contribu-
tion of MOLTs, the process of self-guided student technology exploration in
such environments can often be, at least in part, one of student mathemat-
ical acculturation as well. (For example, research has found middle-school
students with low English language proficiency acquiring functional math-
ematical terminology directly from the menu structure of tools like The
Geometer's Sketchpad; see Dixon 1995.) More and more diverse mathe-
matical terrain can be covered per unit of class time by multiple groups
working toward common ends but through different trajectories than by
all students marching in lock step, and technology can often facilitate the
reconvergence of small groups into whole-class debriefings (by screen-
sharing, projecting work on an overhead, duplicating key performances on
an interactive whiteboard, etc.). Finally, from a cost perspective, deploying
technology at the group level rather than to individual students extends
the reach of limited resources.

MOLTs Amortize Technology Investment across Grade Levels

Because MOLTs are both mathematically focused and open-ended, rather
than narrowly focused on particular curricular moments, and since fun-
damental mathematical ideas often recur in different forms and different

contexts across the curriculum, a MOLT often has applicability across grade levels and across different mathematical content domains, with the same tool servicing different curricular instantiations of that tool's core mathematical contribution or domain. Thus Sketchpad, in the first example, is widely used in elementary schools—for example, when introducing basic properties of shapes and topics such as symmetry—while at the same time it is very popular at the secondary level, where geometry is construed as a formal course topic. TouchCounts, in the second example, begins its utility with learners not yet of school age, but finds application well into fourth and fifth grade as students work their way through arithmetic and fractions. This curricular "verticality" of the MOLT, in turn, allows technology investments to be amortized across multiple contexts, whether these investments can be considered in terms of technology acquisition or training in its use. The training payoff applies not only to teachers learning to teach with the technology (who may then take it into other teaching assignments beyond their initial course- or grade-level focus), but also directly to students. In large-scale, multi-grade-level MOLT adoptions, students take their growing expertise in a specific tool with them from grade to grade, and teachers at higher grade levels find their students already prepared to think, and productively work, with powerful digital technologies in the same way that they become increasingly proficient in their use of other learning infrastructure.

MOLTs Support Multiple Phases of Learning Activity

Finally, just as the pluralist design philosophy of MOLTs accommodates diverse students at diverse grade levels, so does it supports diverse forms of mathematical activity for each student in his or her specific grade and course context. Throughout this volume, effective mathematical activity has frequently been characterized by a three-phase model of student activity in which exploration of a mathematical concept or process leads to understanding, which is followed by the development of fluency (see Chapter 2). The role of a MOLT in supporting the first two of these phases is easy to see both by its definition and in practice, as in the two detailed examples in this chapter. Less obvious, though, is the contribution of MOLTs to students' mathematical or technical fluency, which is often conceived as the result of intentional repetitive practice. Critics inside and outside the educational system often invoke students' perceived lack of technical fluency in mathematics as a polemical indictment of approaches anchored in exploration.

Policymakers considering the issue of fluency in the mathematics classroom might articulate the difference between practice and drill.

MOLTs are decidedly not drill environments, in the sense that they offer no staged sequence of problems of leveled difficulty through which students advance linearly only when able to demonstrate "correct" answers according to a preestablished scoring metric. But drill environments, though frequent in the educational technology landscape, offer only a narrow and overly formulaic interpretation of practice, which largely robs students of the opportunity to perceive any intrinsic reward to accomplishing fluency. Drill environments often rely on external reward to compel students forward—for example, by placing the math drill in some sort of nonmathematical gaming context in which the student advances if he or she answers questions correctly, or, more bluntly, via the threat of external punishment, such as poor marks or test scores. In their exclusive emphasis on practice, drill environments only perpetuate the false dichotomy between practice and understanding raised in Chapter 5.

However, the research on learning most often finds functionally effective practice located in, and being initiated by, the pursuit of some other meaningful goal or activity, rather than as the goal of activity itself. In *Mathematical Fluency: The Nature of Practice and the Role of Subordination*, Hewitt (1996) offers an insightful analogy in considering how children come to walk, a learned skill (we are not born walking!) that requires substantial practice for fluency (having walked once, we do not necessarily walk the second time!). But practice in walking is not effectively accomplished by drill, that is, by being forced to repeat standing, tottering forward, and falling until we either collapse in frustration or become diagnosed as "fluent." Instead our ability to walk first emerges in a desire to go somewhere and achieving that goal through self-locomotion. We are impelled not (just) to walk more but instead to attain more *through* walking: "Children, having learned to walk, are not content to continue just walking. They want to walk on walls, walk on curbs, walk missing the cracks on the paving stones, walk up and down stairs, they want to run. The practice of walking is not just done by continually walking along a plain, flat area. The practice of walking is done by *subordinating* walking to some other task" (Hewitt 1996, 28).

From a mathematical perspective, practice is thus most authentically motivated by and developed in the pursuit of further or higher-order mathematical skills, concepts, and goals. If one considers an activity model moving from exploration to understanding to practice in isolation, practice is strictly terminal in that sequence, and thus appears unmotivated and, in turn, unmotivating. But if one considers the repetition of that model over the course of a student's work, practice of one set of skills can be motivated and reinforced by activity exploring and developing an understanding of the next set of skills. This recursive understanding implies that while

working toward meaningful purposes in MOLTs, students are constantly practicing concrete skills they need in order to pursue their own higher-order mathematical objectives. To the degree these practices can be aligned with curricular objectives, research suggests that MOLTs can be highly effective environments for achieving the mathematical fluencies stakeholders prize (Hewitt 2009).

7.6.3 Specific Recommendations

Considered collectively, these risks and rewards inform many if not all large-scale implementations of MOLTs. While local educational circumstances and system needs should actively filter the relevance of research recommendations and findings from elsewhere, the experience of large-scale MOLT implementations in managing risks while seeking to maximize rewards leads to several general policy recommendations outlined below.

Focus Professional Development on Mathematical as well as Technological Objectives

In 2003, as the Institute for the Promotion of Science and Technology Program began a multiyear process of adopting Sketchpad at the national level in public schools in Thailand, it imported a professional development curriculum from Sketchpad experts in the United States for Thai teachers. Half of the curriculum focused on curricular topics in middle school and high school, and the other half on much higher-level mathematics—well beyond the grade level and in many cases the prior mathematical exposure of participant teachers. Where teachers often rightly consider themselves already expert in their curricular domains, the experience of learning new mathematics through a MOLT-specific lens had immediate dividends not only in their technology training, but also in their fundamental mathematical preparedness. This in turn promoted an authentic—and not just modeled—understanding of Sketchpad's exploratory, discovery, and reasoning trajectory. The aim was for these teachers to become agents capable of supporting the cultural changes (in teaching practices and student enfranchisement) implied by adopting a MOLT. Today, 10 years later, professional development in Thailand focuses squarely on the Thai curriculum and the particular implementation challenges of Thailand's rural school populations. By vertically leveraging the mathematical openness of MOLTs to mathematical domains at the cusp of teachers' reach, professional development can put teachers temporarily in the position of students, allowing them to better partner with students in the implementation of new technology.

Stage Incremental Rather than Monolithic Rollouts

Just as people naturally seek to adopt new practices they perceive to be beneficial, so do they often resist cultural norms and practices imposed from outside, especially when these seem unprecedented and counter to norms already in the society. Given not only the vast heterogeneity of local school circumstances in any large-scale adoption—varying abilities of student populations, technological infrastructure, and teacher mathematical preparedness, but also, critically, the tremendous variability in teachers' prior experience and general disposition toward educational technologies—large-scale adoption of technology frequently encounters resistance that is less a function of the technology's suitability as an educational resource than a result of the very scale of the imposed new cultural practice. By supporting incremental adoption, perhaps on an opt-in school or district basis, rather than mandating an abrupt change in global practice, policymakers can help manage how new technologies are perceived and acquired in their communities.

In the mid-1990s, the Ministry of Education in Ontario, Canada provisioned dynamic geometry software for all ministry schools. Shortly thereafter, the Educational Quality and Accountability Office of Ontario—seeing an opportunity to leverage the new technology in assessing long-standing performance goals Ontario had for students' work on rich mathematical problems in sustained project-based learning—began developing a new component for its biannual mandatory provincial assessment that used Sketchpad in a multiday student project effort. When the office announced pilot testing of this new assessment less than a year after Sketchpad's adoption, as well as its plan to launch it at scale in the subsequent province-wide test, resistance from schools was extreme, and teachers voiced strong concerns about their (and their students') lack of preparedness both in using the software and working with the mathematically rich project contexts that—though a long-standing aim of curricular policy—had been traditionally underemphasized in most schools' teaching sequence. Protest was sufficiently vocal that the new assessment was ultimately cancelled. Instead, rather than forcing teachers to adopt the new technology "because it was on the test," teachers were put in the less stressful position of being encouraged to use the tool only where and as they saw fit, as a resource rather than a requirement. Yet rather than stand as a conclusive victory "against" technology, this development turned out to offer exactly the stimulus Ontario's teachers needed to begin to incrementally and constructively adopt the technology, as well as develop their own stakeholder voices to advocate and educate regarding the benefits of that technology. Two years later, based on a more consensually negotiated

perception of the technology's value, Ontario's teacher-led Steering Committee on Provincial Educational Technology Acquisition recommended that the ministry extend its Sketchpad license from schools to all students' home use as well.

While less politically dramatic than monolithic, countrywide adoption of technology, incremental rollouts better serve the purposes of changing classroom culture. Small- to medium-scale pilot projects help introduce a new technology nonconfrontationally. At the same time, they serve to identify and develop, from among their participants, local leadership and support that can be critical to managing a subsequent transition to larger-scale deployments successfully. The mathematical (as opposed to curricular) focus of MOLTs means they easily cross curricular borders, but in the case of adoption at a national level of MOLTs conceived and already widely deployed elsewhere, they often are first adopted only by teachers with sufficient mathematical preparedness to leverage their versatility. Thus, an immediately practical effort for the teacher-leaders identified in preliminary pilot studies is to translate, localize, or develop from scratch locality-specific supplemental curricular materials (print-based activities, professional development sequences, etc.) for use by the more general teacher population.

Yet even in the presence of new technologies, eager teachers, and fresh curricular materials, cultural change happens only slowly. While pilot initiatives can prepare the groundwork for such change, they rarely accomplish high-impact transformations themselves. Indeed, because they inherently disrupt the milieu in which they transpire, such initiatives may even backfire: technologies do not behave as advertised, exposing unanticipated limitations and frustrations; teachers, willing but ignorant, discover themselves less willing, once informed; or the fresh curricular materials turn out to be the wrong ones. The rate-limiting factor is often the mathematical enfranchisement of participating students themselves, who (as per the previous discussion), after having adapted to one set of cultural norms, expectations, and practices in learning math, are suddenly asked to perform under another. Berlinski and Busso (2013, 4) report a large-scale randomized control trial involving a dynamic geometry intervention among seventh graders in Costa Rica, at the end of which all of the treatment groups exposed to technology perform significantly worse than the technology-free control group. The authors find that the "best students"—that is, the students most successful at navigating the expectations of mathematics learning in a pretechnology environment—"were harmed the most by this intervention," and conclude that "[t]he evidence suggests that teachers went through the motions as prescribed but did not master

the innovation in a way that would have allowed students to get the most of it." By contrast, Jiang, White, and Rosenwasser (2011) report on a randomized control trial in a similarly aged and leveled student population in Texas using a similar technology over a longer acculturation period (both within the study, and also potentially preceding it, in terms of the general societal level of educational technology profusion in the United States as compared to Costa Rica), and find strongly significant improvement on standardized post-tests as a result of the use of dynamic geometry use. In sum, culture change is possible, but it requires time and iteration.

7.7 Conclusion

This chapter has shown the relevance of mathematically open learning technologies to the present needs of students in a broad range of mathematical environments. These technologies are defined by three characteristics: a student-centric design and user model; an open-ended disposition toward activity structure; and an innovative application of technology directly to mathematical representations and practices. A narrative woven through diverse research findings suggests such technologies are not only effective in their impact on student performance, but may also be more effective than many other varieties of educational technology. It is recommended that policymakers focus on teachers' professional development (in mathematics and pedagogy more than in technology) as well as on incrementally staged implementations as important ingredients of well-managed adoption of MOLTs at scale. Examples of such technologies have been highlighted across the history of educational technology, with particular attention drawn to two examples in use and relevant today: dynamic geometry manipulatives (specifically, The Geometer's Sketchpad), and mathematically embodied number environments (specifically, TouchCounts). By focusing critically not only on these examples, but on the role that their three essential characteristics play in constituting a coherent generalized approach to educational technology, policymakers can apply these findings and insights to tomorrow's technologies as well as today's.

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Orchestrating Instruction: Coordinating the Use of Technology with Traditional Math Activities to Improve Learning

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Today's classroom is an increasingly complex and demanding place. Within this context, teachers are not just responsible for preparing lesson plans, adapting the curriculum, and worrying about discipline and safety; they are also responsible for understanding and using a variety of resources to enrich the learning process (Sharples 2013). There is plenty of evidence to suggest that technology will not work on its own and that the digital learning environment must be linked to the learning experience (Luckin et al. 2012). Given this range of demands, this chapter proposes that technology be accompanied by orchestration—that is, personal guidance for teachers and students.

This chapter takes a closer look at the concept of orchestration, specifically within the context of teaching mathematics. It reviews the different elements of orchestration, its structure, and the conditions that it requires, and then analyzes the evidence of the impact that orchestration has had to date.

8.1 The Problem

Across Latin America and the Caribbean (LAC), as well as in other parts of the world, children are not learning what they ought to according to their stages of learning and development. For instance, a study in Chile based on a national standardized math test in fourth and eighth grades found that around two out of three children do not reach the minimum achievement level as determined by the Ministry of Education (Nussbaum et al. 2017).

There are innumerable policies throughout the world that provide schools with technology to improve the quality of education. However, international evidence shows that, at best, there is a weak or negative relationship between the use of educational technology at school and student performance. Evidence suggests that programs based on computer-assisted instruction produce limited improvements in student learning (Slavin and Lake 2008). In this sense, computer-assisted learning is a system in which a computer interactively presents instructional material, assesses the learning process, and provides feedback. It is therefore worth considering going beyond digital materials or computer-assisted instruction in order to guide the changes in teaching that are required by the introduction of technology.

Policies that provide schools with technological infrastructure, including the policy that has been in place in Chile for more than 20 years known as Enlaces (Donoso 2010), should start to include guidelines for teachers, such as orchestration (Nussbaum et al. 2013). Research has looked at how to support teachers in the task of adding value to learning experiences by using technology (Guzmán and Nussbaum 2009). This is achieved by successfully managing aspects of logistics and pedagogy, as well as encouraging social interaction within the classroom whenever the technology is available (Nussbaum and Díaz 2013).

Whereas a lesson plan details the teacher's actions from a logistical point of view, (i.e., aspects of space and time, as well as strategies for handing resources), and a pedagogical point of view (i.e., the elements that make up the process of teaching), orchestration explicitly details the social interactions that can take place within the classroom. Orchestration can therefore be understood as a cultural process for introducing new pedagogical practices into the classroom by guiding the work of teachers and students (Perrotta and Evans 2013). By coordinating the use of different resources and tools, orchestration provides the teacher with the flexibility to adapt activities to different structural and emerging needs (Prieto et al. 2011a).

8.2 What Are Orchestrated Models?

When thinking about introducing technology into the classroom, it is natural to think about the relationship between the elements and processes that converge within that process. The classroom is a systemic environment in which the proper functioning of each element affects the adjoining ones. Acknowledging this reveals interdependence between the logistical and pedagogical elements that underlie the teaching/learning process. Orchestration guides teachers to structure their classes so as to

successfully integrate conventional and digital resources (Nussbaum et al. 2013). In this case, the pedagogical decisions and learning experiences are always focused on the student (Chamberlain et al. 2001). An orchestration of classroom work using technology details the actions required for a teacher to implement new strategies. These can include novel tasks that are aimed at integrating a range of different resources.

This strategy does not draw on traditional teacher development processes. Instead, it involves a practical, step-by-step guide or set of guidelines that can be given to teachers digitally or as a small booklet. When designed well, this booklet encourages the teaching process to be focused on something other than the teacher (Goodyear and Dimitriadis 2013), and empowers students to participate and play a leading role in their own learning, with the teacher acting as a mediator (Nussbaum et al. 2013).

8.2.1 What Does the Strategy Guide Orchestrate, and Why?

Orchestration plans lessons associated with a standard curriculum by categorizing the teacher's actions from both a logistical and pedagogical point of view (Nussbaum and Díaz 2013). In this case, logistics refers to aspects of space and time, as well as strategies for handing out and collecting resources (both technological and conventional). Pedagogy, on the other hand, refers to all the elements that together make up the process of teaching. This includes the type of questions to ask students, examples, the type of monitoring needed, forms of interaction with the students, classroom dynamics (individual work, small groups), and so on.

Table 8.1 shows how an orchestration is built. The macro questions refer to the six dimensions of classroom work: subject/curriculum, time/frequency, purpose/objective, procedures/methodology, resources/organization, and monitoring/assessment. For each of these six dimensions, the micro questions in turn refer to the corresponding elements of the orchestration and go beyond the traditional lesson plan. The main difference between a lesson plan and an orchestration is that the orchestration specifies the social interactions within the classroom. It should be understood as a cultural process that shows how teachers in a particular context can adopt innovative practices by bringing technology into their teaching (Perrotta and Evans 2013).

An example of an orchestration can be found in Annex 8.1, where the micro questions are answered for an orchestration of a fifth grade class on fractions. This orchestration is for a single class taken from a series of eight classes that focus on the same learning objective. This example was taken from a set developed for a project in Colombia (Díaz et al. 2015b).

The framework presented in Annex 8.1 represents an orchestration that has been designed from a teaching perspective. However, it is also possible to design orchestrations from a learning perspective, that is, by

TABLE 8.1
A GUIDE TO PLANNING AN ORCHESTRATION

Macro Questions	Micro Questions
Which subject and grade level will the orchestration address?	For which grade level am I going to make the pedagogical decision? For which subject am I going to make the pedagogical decision? For which specific topic am I going to make the pedagogical decision?
How often and for how long will the orchestration be implemented?	How much time do I really have to teach this topic? Which aspect of the topic will I cover in the first week, the second week, the third week, etc.? How am I going to divide the time I have available for class into different learning experiences? For which moment of the class will I prepare these learning experiences?
What is the pedagogical purpose of the orchestration?	On which cognitive skill will I focus the learning experiences of this class? Which learning objective do I wish my students to achieve by the end of this unit? What is the specific learning objective that my students must achieve after class 1, 2, 3, etc. in the first week, second week, third week, etc.? What prior knowledge do my students need so that the learning experiences I have designed will be useful based on the objective of the class?
Which pedagogical procedures does the orchestration include?	Which methodologies will my students follow for these learning experiences? What sort of consistency is there between these? How should my students work so that the learning experiences progress accordingly? How should the space be organized in order to carry out the learning experiences? What specific instructions should I give my students regarding the use of resources (technological and conventional)? What instructions should I give my students so that they understand the learning experiences?
Which resources are used in the orchestration?	Which resources should I prepare so that the learning experiences can take place optimally? What sort of consistency is there between these resources?

(*continued on next page*)

TABLE 8.1 (*continued*)

A GUIDE TO PLANNING AN ORCHESTRATION

Macro Questions	Micro Questions
Which monitoring procedures does the orchestration include?	How will I interact with my students as the learning experiences take place? And, how can I promote interaction between my students as the learning experiences take place?
	Which resources will I use to monitor my students' progress, and how and when will I use them?
	Which questions will I ask my students to make sure they understand every explanation I give and every example I show?
	How will I make sure my students are meeting the learning objectives set for class 1, 2, 3, etc. in the first week, second week, third week, etc.?

Source: Prepared by the authors.

revisiting the phases and stages proposed in Chapter 2 of this book with regard to teaching mathematics.

Chapter 2 describes the three-phase balanced teaching model. In the first phase, the teacher asks students for preexisting models for ways to solve mathematical problems. In the second, the teacher presents accessible and suitable problem-solving methods by tackling preexisting models, typical errors, and newer, more suitable methods. In the final phase, students acquire fluency in applying the most suitable methods for solving mathematical problems by recognizing and avoiding common mistakes. These three phases can frame how an orchestration is viewed, either by including them as an additional element or by replacing existing elements with such elements as "Class Phases: Opening Phase, Instructional Phase, and Closing Phase;" "Cognitive Process: Recall, Understand, Apply, Analyze, Evaluate or Create," or "Class Objectives."

Furthermore, by going back to the five stages for acquiring mathematical skills proposed in Chapter 2, the way that an orchestration is read can also be framed. This can be done by specifying—using the "Cognitive Process: Recall, Understand, Apply, Analyze, Evaluate or Create" element—whether the pedagogical activities and orchestrated resources refer to the moment of conceptual understanding, adaptive reasoning, productive disposition, procedural fluency, or strategic competence.

This suggests that the flexibility of the orchestration and the use of the concepts that frame how it is read will depend on the position and pedagogical view of the context/reality in which it is going to be implemented. It is also worth noting that different guidelines will be provided depending on whether the orchestration is designed from a teaching or

learning perspective, i.e., from a teacher or student viewpoint. In either case, these guidelines will act as a structure to support the teaching/learning process.

8.2.2 Local Context and Reality

Innumerable successful international cases have been studied in order to suggest improvements to pedagogical practices and to design scaffolding to enable teachers to harmoniously coordinate learning experiences based on conventional and digital resources. However, nothing will be achieved through general policies that look to improve schools without acknowledging their local context and reality (Lupton 2005). For example, when designing and implementing an orchestrated model for schools in Uruguay (Díaz, Nussbaum, and Varela 2015), one of the project's weaknesses was identified as not having considered the teachers' local knowledge regarding how to implement the curriculum. In this case, the design of the orchestrations was based on the national curriculum at the time. However, by not considering the time frames that had already been adopted by the teachers, this sometimes translated into irregular use of the guidelines.

Based on the lessons learned in Uruguay, a subsequent project in Colombia invited teachers to participate in the initial planning process in order to aid the design of the orchestrations (Díaz et al. 2015b). This proved to be a turning point: taking local knowledge into account became fundamental in order for the proposal to make sense to the participants. It was also essential for identifying and addressing the needs of users.

Before initiating processes to support teachers, a needs analysis must first be conducted based on the teachers' knowledge of the local context. This analysis must be focused on reviewing the curriculum and looking at how it is applied in the classroom in terms of both depth and scope. This will help focus efforts on those aspects that teachers identify as being critical. If an orchestration is based on the reality of the users (in the case of Colombia, the teachers themselves), it is more likely that it will be adopted and used systematically. This is because it is no longer isolated or detached from the students' educational process. It is worth highlighting the importance of the systematic use of digital devices in a program that covers mathematics (Penuel, Singleton, and Roschelle 2011). The experience in Uruguay detailed above showed that positive results were obtained only when systematic use was made of the devices.

In this sense, taking into account local knowledge and detecting contextual needs are both essential when designing orchestrations. To set

educational policy and assess the impact of strategies such as orchestration, schools' social context must be taken into consideration (Thrupp and Lupton 2006). In the same way, an orchestration developed with one particular reality in mind will not necessarily be suitable for another (in terms of structure and content).

8.2.3 Training

Training courses that accompany orchestrated models are focused on a pedagogical discussion rather than on the technological devices to be used. Evidence suggests that teacher training is a basic requirement when introducing technology into schools (Falck, Kluttig, and Peirano 2013). Incorporating educational technology into the process of initial training and continuing professional development should therefore be a priority for any country interested in introducing these tools into its school system. The many government programs that focus solely on basic productivity tools such as email, Internet, and other administrative software (Kozma 2008) are neither sufficient for nor essential to the work of a teacher.

The training that accompanies orchestration looks to address a critical issue—the teacher's pedagogical view of technology—as revealed by the literature (Prieto et al. 2011b; Mishra and Koehler 2006; Shechtman and Knudsen 2009). Very few research projects have linked the development of technical skills (i.e., the way in which teachers understand and make use of information and communication technologies) with pedagogy. Instead, teacher training programs focus mainly on the potential of new tools, regardless of local teaching practices (Jung 2005). In the link between technical skills and pedagogy, there is a triangulation between pedagogical knowledge, technological knowledge, and (curricular) content knowledge (Koehler and Mishra 2009). Training, therefore, has to take this triangulation into consideration as it integrates logistical elements (technological knowledge) with pedagogical elements (pedagogical and content knowledge). The aim is to introduce different learning resources and dynamics into teaching practices (Beetham and Sharpe 2013). In this sense, when training teachers, it is necessary to consider elements of methodology, curriculum, technology, attitude, communication, and assessment (Guzmán and Nussbaum 2009).

The experience in Colombia described above featured a teacher training process that incorporated the characteristics detailed in the previous paragraph, as well as an element of coaching (Díaz et al. 2015b). The training process was designed based on the characteristics of the pedagogical strategy itself. This is in line with findings from previous

studies, such as Lawless and Pellegrino (2007), which place professional development at the center of effective access to digital resources and teaching strategies in order to improve the teaching/learning process. In Colombia, this process consisted of two 3-hour meetings (Training Sessions 1 and 2; see Figure 8.1) in which the teachers were encouraged to reflect on their appropriation of the different components of the teaching strategy.

The first training session marks the beginning of the implementation process and sets the foundation for transferring the orchestration based on the characteristics and pedagogical needs of each teacher. The second training session comes at a critical moment when the effort required by teachers has increased. This is because the orchestration demands changes in the teachers' practices, as guided by the scripted instructions. Here, the training is focused on the processes of change that have been implemented up until that point. Teachers are invited to identify the most effective elements of the orchestration so far, as well as those that need to be adapted. By verbalizing their experience, the teachers begin to cement their appropriation of the orchestration and thus the effort required of them starts to decrease. The four coaching sessions (see the following section) focus on how the orchestrations can adapt to teachers' local context and reality, thus leading to greater autonomy. The wrap-up meeting represents the end of the transfer period. Here, the stage is set to transfer this experience to other teachers, other grades, and/or areas of learning.

8.2.4 Coaching

Accompanying the teacher is particularly important because it allows for real transfer of the orchestration. By doing this, the use, flexibility, and adaptability of the orchestration can be modeled in situ for each context. This, therefore, empowers the teachers to successfully use the orchestration. Coaching is considered for orchestration training programs based on evidence of how it has been tried and tested in different fields (Díaz et al. 2015b). It has also been suggested that this type of instruction could be introduced to the field of education with positive results, given its practical focus (Knight and van Nieuwerburgh 2012).

Evidence suggests that after years of disappointing results from efforts to improve professional development, many programs now consider the use of coaches to improve the success of innovations in schools (Kretlow, Cooke, and Wood 2012). Successful coaches are those who emphasize showing teachers how and why certain strategies make a difference to

FIGURE 8.1
TRAINING AND COACHING MODEL BASED ON THE EXPERIENCE OF IMPLEMENTING ORCHESTRATIONS IN COLOMBIA
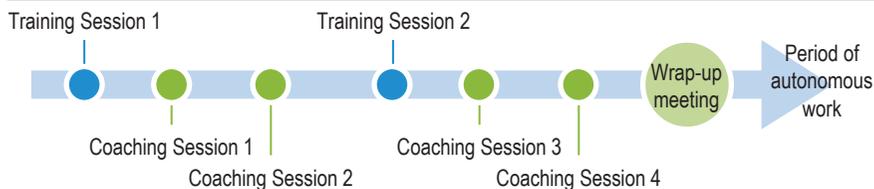
Source: Díaz et al. (2015b).

their students. The coaching is structured around the work by a coach with a small group of teachers with the aim of successfully using the instructions contained within the orchestrations. By doing this, the coaching improves the teachers' practices in the classroom and, as a result, their students' performance (Russo 2004).

The role of a trainer is to look to a model of professional development in order to widen the teachers' knowledge of effective classroom strategies. The role of the coach, on the other hand, is to put theoretical concepts into practice. The coach therefore contributes to generating change by encouraging pedagogical action (Knight and van Nieuwerburgh 2012).

Figure 8.1 shows the two sets of coaching sessions that teachers receive in the classroom. The objective of the first set of sessions (Coaching Sessions 1 and 2 in the figure) is to accompany the teachers in preparing their lessons, as well as in the classroom when they start to use orchestrations. A further objective is to model the use of the orchestration and show teachers how to follow the instructions suggested by the script. The second set of in-classroom coaching sessions (Coaching Sessions 3 and 4 in the figure) focuses on empowering the teachers and having them adopt the orchestration by helping them to see how adaptable, flexible, and relevant it can be. The objective of this second set of sessions is to promote autonomy in the management and use of the orchestration, again accompanying the teachers with the preparation of their orchestrated lessons.

In-class coaching takes into account the events that occur in the classroom, the social relationships between the actors, and the needs of the teachers and their students (Slavin 2006). For example, for the study in Colombia, Coaching Sessions 1, 2, 3, and 4 (Figure 8.1) were based on managing classroom logistics, allowing teachers to focus as much of their time as possible on pedagogy and teaching, rather than on the technology

itself. This was done by providing teachers with feedback on their work and looking to reinforce their appropriation of this new teaching strategy (Díaz et al. 2015b).

An orchestration specifies how the teacher should manage the time, space, resources and key interventions during the students' work. Each teacher therefore received two coaching sessions on effective lesson planning. This helped the teachers review an orchestration based on the opportunities and barriers that were present in their own context. Kennedy (2005) suggests that taxonomies in education have typically been based on idealized conceptualizations rather than on the reality in the classroom or on the teacher's needs. In this sense, the taxonomies fail to take into account the pace of a class or how to maintain the optimum environment for learning, which is possibly a teacher's greatest concern. However, good teaching is not only determined by school-level factors or by a teacher's knowledge, beliefs or attitudes. It is also determined by the students' needs, as well as class and student-level factors (OECD 2009). As a result, it is these elements that are reviewed and analyzed during the planning meetings.

The coaching sessions for lesson planning focused on how to manage time and space, as well as how to use resources and when to interact with students. Teachers received support based on their students' needs, such as in maintaining the pace of the class and a suitable learning environment.

Finally, there is an element of institutional coaching, where work is done with the senior management team (Leithwood et al. 2004). Involving the management team is fundamental. In the Colombia study, the researchers worked with a representative from each school's senior management team on scheduling the use of the technology, as well as reviewing the technological support that was available to the teachers (Díaz et al. 2015b).

8.2.5 Monitoring and Evaluation

As an extension of orchestration, it is vitally important to consider indicators for follow-up and monitoring that allow the implementation and use of the orchestrations to be tracked. The aim of this is to gather input that will allow the experience to be evaluated in terms of adapting, improving, or changing the orchestrations. By including a preplanned process of monitoring, pilot policies can be evaluated. Analyzing the criteria that are monitored can determine the most important elements in each context in terms of how they facilitate or hinder the implementation of strategies that look to enhance learning.

8.2.6 Requirements for the Different Elements of Orchestration

The reality of the school environment determines the needs of each orchestration. Table 8.2 details the requirements for each element of orchestration.

TABLE 8.2
REQUIREMENTS FOR THE DIFFERENT ELEMENTS OF ORCHESTRATION

Element	Requirement
Curricular content and timing	The teachers involved must define the relevant content and timing through a meeting, workshop, or seminar.
Technological and curricular abilities	The less capable the teacher, the more technical or curricular specifications the orchestration must contain.
Senior management team	At least one member of the senior management team must be involved.
In-school technical support	If there is no technical support, the teachers must receive technical training and be given time to prepare the logistics. The teachers must also receive a second round of training on the technical and logistical elements of the class.
Adults in the classroom	There must be at least one teacher in the classroom in order to implement an orchestrated model.
Lesson planning time	The orchestration must be reviewed before being implemented so that the teacher can adapt the strategies and prepare the necessary materials.
Printing materials	The worksheets that are to be completed individually by the children must be printed.
Technological devices for each child	When the work requires one computer per child but there are not enough computers in the classroom, simultaneous activities can be orchestrated for two groups. One of these groups works on the computers, while the other uses worksheets. The orchestration considers that a group will work with the computers in one session and with the worksheets in the next, so that all of the children end up completing the same activities.
Visual technology	If the orchestration includes visual presentations such as PowerPoint, there must be a projector and a computer to present these. If no projector is available, it is better not to use such presentations. In Uruguay, it was determined that having students sit around a standard-sized monitor was not very effective and could even be disruptive (Díaz, Nussbaum, and Varela 2015).
Internet connection	It is better to use activities that do not require an Internet connection, as this is not always available or does not always have the necessary bandwidth for the activity to run smoothly.

Source: Prepared by the authors.

8.3 How Are Orchestrated Models Structured and Why Might They Improve Learning in Mathematics?

8.3.1 Diversity of Learning Experiences

The more diverse the learning experience, the more the learning is consolidated. This is because a more diverse experience covers different points of view, learning domains, and sensory experiences. The study in Colombia described above involved orchestrations that featured diverse learning experiences based on different types of resources. The aim was to cater to different learning styles and paces within the classroom. In this sense, the orchestration provided a series of different opportunities to learn about a specific topic in mathematics. This allowed the students to consolidate their knowledge of this topic based on the range of the experiences.

The set of activities that can be included in an orchestration for learning mathematics is detailed in Table 8.3. All of these activities should be set within a narrative, as this allows abstract concepts to be expressed using everyday language that is familiar to the students (Burton 2002).

To analyze how learning in mathematics benefits from an orchestrated learning environment, the sections that follow will review the types of activities or learning experiences that can be included in an orchestrated model.

8.3.2 Digital Opportunities

Using digital devices has a positive effect on student learning (Cheung and Slavin 2013; Mo et al. 2013, 2104; Yang et al. 2013). Based on this, orchestrations include a digital component for classroom work. These technological resources are consistently integrated so as to provide the students with different experiences such as exploration, handling data, graphing different dimensions, and higher-level calculus. Furthermore, it is important to provide experiences that include graphical and symbolic representations in order to aid comprehension of different mathematical concepts (Nishizawa et al. 2012). In this sense, computer games and simulations are both tools that facilitate learning by encouraging students to manipulate, experiment, and visualize graphs of functions and equations (see Chapter 7; see also Recker, Sellers, and Ye 2013). At the same time, search engines and specific websites help children (Auzende, Giroire, and Le Calvez 2009) share and clarify doubts, as well as discuss content in virtual social environments (Ferguson and Buckingham 2012).

TABLE 8.3

A SAMPLE OF ORCHESTRATED LEARNING EXPERIENCES FOR STUDYING MATHEMATICS

Type of Activity	Materials	Objective	Interaction
Whole class	PowerPoint presentation	To explore and discover the content through a narrative that makes the new concepts significantly more accessible for students.	Mainly led by the teacher, with participation from students in questions and exercises.
Whole class	Conventional whiteboard	To share doubts and go over examples.	Mainly led by the teacher, with participation from students in questions and exercises.
Individual	Netbook, software with curricular sequence	To drill the contents of the core curriculum for mathematics.	Autonomy for the students, while the teacher walks around the classroom to answer questions and monitor the students' achievements and difficulties.
Individual	Worksheets	To practice reasoning and procedural skills in mathematics.	Autonomy for the students, while the teacher walks around the classroom to answer questions and monitor the students' achievements and difficulties.
Small group	Hands-on activities (cutouts, games, etc.)	To use group games in order to establish connections with the activities previously carried out using PowerPoint, software, and worksheets.	Students organized into small groups, mediated by the teacher.
Individual	Netbook, software with curricular sequence	To drill the contents of the core curriculum for mathematics according to each child's specific needs.	Autonomy for the students, while the teacher walks around the classroom to answer questions and monitor the students' achievements and difficulties.

Source: Prepared by the authors.

8.3.3 Oral Questions

Another element of orchestration is the interaction between the teacher and students, as well as the interaction among students that is encouraged by the teacher (Kiemer et al. 2015). The literature focuses on the importance of the quality and not the quantity of the interactions that take place in the classroom.

This includes the types of questions asked by the teacher, the type of feedback given, the types of responses, and so on. In order to maintain interaction throughout the activity, an orchestration provides the teachers with a series of suggested questions that can enhance the experience of learning mathematics (Díaz, Nussbaum, and Varela 2015). Oral questions in mathematics help students process information and lead them to solve problems with an appropriate comprehension of the meaning of each of its components and concepts (Huang, Liu, and Chang 2012). Furthermore, the orchestrated questions include examples that show how mathematics is present in everyday life.

8.3.4 Multiplatform Work

Definitions of what are called 21st century skills paint a picture of students who must face the challenge of processing innumerable stimuli, across different platforms, and simultaneously respond to them, just as they do outside school in daily life (Álvarez et al. 2013). Literacy in different media is supported by providing a multimodal space. This responds to the need to develop human capital to participate in a highly technologized world (Jenkins et al. 2006). In this sense, multiplatform orchestration fosters the development of collective intelligence, shared cognition, and navigation of different media across different knowledge domains (Álvarez et al. 2013). The definition of orchestration therefore also includes multiplatform work by detailing the integration of different areas of learning with different classroom dynamics (group and individual) and the use of conventional and digital resources (Dillenbourg, Järvelä, and Fischer 2009).

8.3.5 Exercises Based on the Individual Pace of Learning

One of the elements included in orchestration design is the digital content that comes with a learning experience based on technological resources. The value added by computer-assisted instruction strategies is mainly based on their ability to identify children's strengths and weaknesses (Slavin and Lake 2008). By doing so, they can provide students with exercises based on their specific learning needs, thus determining the pace at which students learn. It is important to note that students do not necessarily learn at the same pace at which the class is taught. Allowing students to learn at their own pace can therefore be particularly important in subjects such as mathematics. In this case, prior knowledge of basic concepts is required before tackling more complex topics. Here, the teacher mediates the learning process and promotes activities that help students build a solid foundation before moving on to more complex topics.

Orchestrating learning therefore supports diverse and different paces and learning needs (Chamberlain et al. 2001; Watts 2003). For example, the study in Colombia described above focused on catering to different learning styles and paces. This was achieved by using software that generated exercises for students based on their level of learning, as well as their strengths and weaknesses.

8.3.6 Learning Experiences with Formative Feedback

Formative feedback is critical to learning mathematics (Black and Wiliam 2009). It provides the opportunity to correct mistakes and detect faulty processes. It therefore avoids establishing incorrect practices for problem-solving or for learning mathematics in general. Detailed explanations provided by the teacher based on completed exercises have more of an impact than general advice given by teachers based purely on hypothetical solutions (Narciss et al. 2014). Furthermore, the constant feedback and support that a teacher can provide in the classroom is a determining factor in the success of students (Nussbaum, Alcoholado, and Buchi 2015). Feedback can be orchestrated through suggestions and guidelines for teacher interactions in the classroom.

8.4 What Evidence Is There of the Impact of Orchestration?

Technology-based instruction in elementary mathematics has been widely studied since the 1980s by such authors as Slavin and Lake (2008), whose study provides useful background information. It is now possible to find evidence from a series of small-scale studies regarding the use of orchestration in mathematics, science, and general classes, as well as orchestration in its varying formats (e.g., digital orchestration to guide student performance, action plans that guide the teacher's work, etc.). Four studies using orchestration are summarized in Table 8.4, highlighting the contribution of each study, as well as the entry barriers and lessons learned.

8.5 In Which Schools Might Orchestrated Models Work Best?

8.5.1 Ideal Conditions for an Orchestrated Model

Discussing what would be the ideal conditions for the optimum use of orchestrations is complicated by that fact that there is such huge diversity among schools in LAC. The elements suggested as minimum requirements for orchestrations may or may not be possible in all schools. However, the

TABLE 8.4
THE EFFECTS OF ORCHESTRATION: A SAMPLE OF STUDIES

Authors	Concept	Characteristics of the Sample	Attributes of the Study	Entry Barriers	Lessons Learned	Value
Sharples et al. (2015)	Orchestration for carrying out individual research projects.	Students between ages 11 and 14 from different schools.	• Integration of conventional and digital resources • Covers a range of pedagogical aims • Self-guided learning by the students	• Previous technological knowledge • Logistics of the technology in the classroom	The orchestration must take into consideration existing technical and pedagogical skills, without assuming a desired level of knowledge.	The effective combination of technology and conventional resources, covering a range of perspectives and pedagogical focuses among teachers. The teachers felt supported by the orchestration despite difficulties with managing the technology and using digital data to monitor student progress.
Díaz et al. (2015a)	Orchestration of a school year that takes into consideration the national curriculum and educational technology policy.	Twenty teachers from Montevideo (10 in the control group and 10 in the treatment group) with their 544 fourth grade students from 17 schools.	• Integration of conventional and digital resources	• Quantity and quality of available technology	Orchestrations must be designed by taking into account curricular restrictions, as well as restrictions of time and pedagogical ability.	The systematic use of orchestration promotes substantial improvement compared to technology used in isolation from the curriculum.

TABLE 8.4 *(continued)*
THE EFFECTS OF ORCHESTRATION: A SAMPLE OF STUDIES

Authors	Concept	Characteristics of the Sample	Attributes of the Study	Entry Barriers	Lessons Learned	Value
Díaz et al. (2015b)	As above, as well as taking into consideration individual learning needs.	Twenty teachers from Barranquilla (10 in the control group and 10 in the treatment group) with their 531 fifth grade students from 20 schools.	• Integration of conventional and digital resources • Learning at the student's own pace	• The cost of frequent and systematic coaching sessions	The involvement of the school's management team is essential when implementing any sort of pedagogical transformation.	The importance of not just orchestrating the curriculum but also adapting activities and the pace of learning in order to meet each individual student's needs.
Niramitranon, Sharples, and Greenhalgh (2010)	A system that defines the sequence of work in the classroom and promotes collaboration between students.	Four classes of between 20 and 30 students (11–14 years old) in a school where tablets were often used in the classroom.	• A group of editors to develop content in real time • Classroom management that allows activities using conventional and digital resources to be integrated, and that also allows for autonomy among students	• Lack of tools to allow the educational challenges in a virtual environment to be diversified	To be viable, teachers must have the tools and time to develop their own orchestrations.	The tools used have to empower the teachers so that they become the ones who design and orchestrate lessons for their students.

Source: Prepared by the authors.

study in Colombia analyzed above could be taken as a model from which certain ideal conditions for orchestration may be identified:

- Infrastructure within each school, including permanent access to electricity and a safe space to store technological equipment.
- Digital literacy among teachers and students in the use of the relevant devices.
- Promotion of the pedagogical use and care of relevant devices.
- Availability of time (within the working day) for the teachers to take part in training activities, review orchestrations, practice using the digital resources, and plan to include these orchestrated experiences in their lessons.
- Availability of someone with the necessary technical know-how within the school to provide technical support. This support will ensure the effectiveness and availability of the equipment, as well as help teachers manage this equipment by handing devices out, collecting them, and storing them.
- Support of the school principal (or academic head) throughout the innovation process. This requires that principals visit the classroom in order to understand the specific context, support the teachers in the challenges they face, and monitor the pedagogical use of these tools by the teachers.
- Planned use of the resources in each school to ensure that they are available when needed.
- Permanent contact with the local education authority so as to have institutional support that can guide the decision-making process within each school. By doing this, the work done by the teachers within the schools is also reflected in the administrative work by the relevant local authority.

It is worth highlighting that schools in LAC sometimes have issues that must be resolved before implementing orchestrated models. Examples of these issues include infrastructure problems, such as unstable electricity and water supplies; precarious systems of education management and administration; gaps in the teachers' knowledge of the curriculum, teaching strategies, and technology; and a fixed curriculum dictating what should be taught nationwide (Riqueleme 2017). In this sense, each school brings its own conditions based on its local context. It is therefore essential to conduct a diagnostic of the context in which an orchestrated model is to be implemented. This will lead to making more pertinent decisions regarding not only the provision of technology, but also the elements that

are included in the orchestration, as well as the coaching and training activities.

It is important to note that experience in several LAC countries has demonstrated that socioeconomic inequality cannot be overcome by simply introducing information and communications technology (ICT) into the classroom (Lamschtein 2016). It has even been shown that without suitable integration, the use of computers in the classroom can have a negative impact on student achievement (Patterson and Patterson 2017). The key to improving learning is therefore to integrate such resources into the children's everyday experiences at school (Ganimian and Murnane 2016). Orchestration can play a key role in this. However, the digital divide will not be solved through orchestration alone. Another key factor is the number of years that students have been working with computers (Jara et al. 2015). Therefore, it is important that orchestration is provided to teachers from kindergarten through grade 12.

8.5.2 Role of the Management Team

Changes and improvements in the student learning process must be considered in direct association with the management process in each school. It is estimated that 25 percent of student progress may be due to the work of the school's management team (Leithwood et al. 2004). It could therefore be claimed that having commitment from the teachers and empowering them in the pedagogical use of the technology is not enough. It is also essential to involve the management team in the project to help with the implementation of these new strategies.

Integrating technology in educational processes is not just a simple question of doing it (Area 2010; Valverde, Garrido, and Fernández 2010). It is a complex process in which the different actors and factors that are present in the classroom must be taken into consideration. The use of technology should not just be considered at a teacher level but also at a school level (Vanderlinde, Aesaert, and van Braak 2014). Around 14 percent of the variance in the use of technology by teachers is due to school-level characteristics. Furthermore, evidence reveals that student performance is greater in schools where there is a national policy for providing technology, coupled with high levels of management and support in terms of the teachers' pedagogical use of the resources (Salinas et al. 2017).

8.5.3 Teacher Profiles and Their View of Technology

In a study of three Latin American countries, it was found that most teachers rated themselves at the highest levels in terms of their adoption

of technology, regardless of their actual knowledge (Salinas et al. 2017). This helps in interpreting the data from Chile, which reveal that the use of ICT is not directly related to academic performance and that one of the main factors in the failure to use the available technology is a lack of pedagogical support (Claro et al. 2013). This, in turn, highlights the importance of the degree to which teachers have available orchestrations.

In this sense, a qualitative study revealed that the degree to which mathematics teachers used orchestrations was directly related to their opinions on technology (Drijvers et al. 2010). This again points to the importance of acknowledging the context in which the orchestrations are used. The orchestration must use familiar and accessible language in order to set the stage for successful use of the technology. Different perspectives can complement this process, i.e., the design of the orchestration need not be limited to a single view of how the subject (e.g., mathematics) should be taught.

8.5.4 Developing Skills

Orchestrations have to accommodate the reality of each classroom. This will affect how teachers receive and appropriate the orchestration. In schools where many students are from lower socioeconomic status families, or in less effective schools, the pressure of the social role played by schools often leads to inadequacies in the teaching/learning process. This is because in such cases the school's requirement to provide social services generally trumps its educational role (Auwarter and Aruguete 2008). In general, these contexts often see low levels of appropriation and systematic adoption of new strategies due to the range of different roles that the school community is asked to play. This brings us back to Section 8.2.2 of this chapter, "Local Context and Reality," It also reveals the inefficiencies of general policies that propose unsuitable models for making improvements in contexts that have different priorities in terms of needs and support.

When designing orchestrations, it is essential to take into account the diversity of contexts that exist. Orchestrations must consider the existence of contexts that require differentiated support given the characteristics of their geographical location and the socioeconomic of students' families (Lupton 2005).

A policy for implementing orchestrations that takes into account the curricular framework and local context, as well as teachers' experiences and knowledge, comes at a high cost. In this sense, it is increasingly important to look at developing skills within schools, an important issue when it comes to education policies and programs that look to be scaled. The aim

of involving in-service teachers in the development of orchestrations is to empower them in their planning of lessons. Orchestrations involve a series of small actions that together constitute suitable and effective teaching practices for using conventional and digital resources while focusing the teaching on the student. Detailing these actions ensures that implicit elements of the classroom are taken into consideration (such as interactions, types of questions, etc.), and become just as important as the explicit elements (time, resources, content, etc.).

In order to develop these skills among in-service teachers, a methodology is suggested based on a series of questions like those in Table 8.1. These questions lead teachers to reflect and make pedagogical decisions, the result of which is an orchestration. Using such questions and an example of an orchestration (such as the one in Annex 8.2), a school community can develop its own orchestrations.

8.6 How Can the Program's Expected Impact Be Maximized?

8.6.1 The Process of Appropriating Technology

The process of adopting technology for teachers must be taken into account when designing and implementing the orchestrations so as to maximize their impact. Appropriation can be seen as a process of dynamic transitions. Teachers who are early adopters of technology and spend a significant amount of their time in the classroom integrating educational technology in their teaching are more likely to adopt new technologies regardless of how complex they are. However, teachers who form part of the late majority of adopters and use little technology in their classes are less likely to adopt new technologies and are prone to abandoning adoption at certain identified points (Aldunate and Nussbaum 2013).

This process is shown in Figure 8.2, where three critical points can be identified. The first of these (point *A*) represents the initial state where users still have not managed to master the

FIGURE 8.2
PROCESS OF ADOPTING TECHNOLOGY
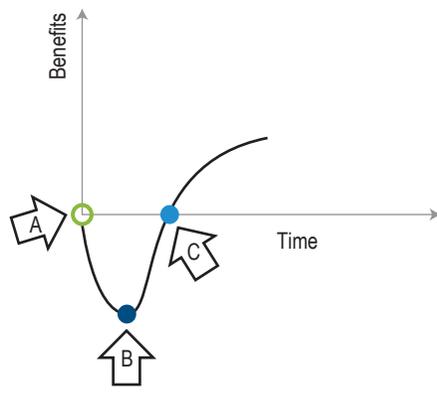
Source: Aldunate and Nussbaum (2013).

FIGURE 8.3
TRAINING AND COACHING MODEL BASED ON THE ORCHESTRATIONS IN COLOMBIA
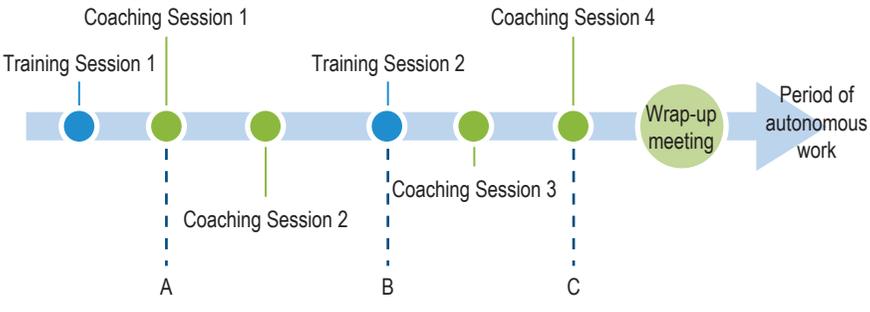
Source: Prepared by the authors.

technology and their experience gets gradually worse until point *B*. From here (*B*), the users start to master the technology, but until they cross the axis (point *C*), their experience is worse than before the technology was introduced. Orchestration must acknowledge these critical points and provide the necessary support through coaching to overcome them.

Figure 8.3 shows Figure 8.1 from the Colombia study, analyzed above, including the three critical points from Figure 8.2. Figure 8.3 therefore shows that point *A* in Figure 8.2 relates to the initial training that kicks off the project. Point *B* in Figure 8.2 is a critical moment: this is when the teacher sees the least value in using the orchestrations. This moment therefore requires coaching, not just to work through the teacher's doubts but also to reinforce all of the areas where weaknesses can be observed in the classroom work. Point *C* represents the moment when the teacher starts to see the benefits of using the orchestrations. Up until this point, the teacher is supported through in-classroom coaching. This point signals the beginning of the period of autonomy for the teacher and is marked by a wrap-up meeting for the project.

8.6.2 Pedagogical Support

The pedagogical support normally included in educational technology policies usually focuses on technical aspects of the technology, and rarely on aspects of the curriculum. Only in the minority of cases does this support link both aspects to the actual teaching practice.

Orchestration can be seen as a first step in providing teachers with pedagogical support. This is for three main reasons. First, teachers'

technological skills are related to their knowledge of and skills using different technological resources. Second, teachers' pedagogical skills allow them to use these resources when designing and developing a study plan, as well as in their lesson plans (Suárez et al. 2013). Third, teachers' constructivist beliefs have a positive effect on their use of technology, while traditional beliefs have a negative effect (Hermans et al. 2008).

Orchestration links the logistical and technical elements of a classroom with the pedagogical potential that can be provided by appropriate use of methodologies in the classroom. However, as stated previously, the gradual introduction of orchestrations within a school must be accompanied by a process of training and coaching (both in the classroom and while planning). This will provide opportunities for continuing professional development in terms of both the curriculum and methodology, and focused on the teacher's own practices. Pedagogical support that links both the training process as well as the process of teachers reflecting on their own practices leads to significant learning outcomes. This is because these outcomes draw on the teachers' own experiences, which in turn can be continually supported by orchestration.

8.6.3 Openness to Change

It is essential to take openness to change into account when designing support or improvement plans for schools. One theory imagines any change in schools as a process with set milestones (Murillo and Krichesky 2012). This cycle of change includes the following stages:

0. *Initiation phase*. When a group of people demonstrate an interest in initiating or promoting change in pedagogical practice in contexts where technology is available for classroom work. Meetings are held with the school community in order to identify their expectations and needs. This phase includes the diagnostic.
1. *Planning phase*. When the general direction of the project and next steps are defined. During this period, the group that is interested in initiating an improvement/support program for teachers works with the school community to plan the elements that will be included in the orchestration. At this stage, it is important that the process of change make sense to both the management team and the teachers. Orchestrations are developed during this stage based on the information gathered during the process of detecting needs and expectations.
2. *Implementation phase*. During this phase the strategies or actions are put into practice. Implementation of the orchestrations begins in

the classroom, with the teachers putting into practice the strategies and actions suggested by the pedagogical script (orchestration). This period begins with teacher training and includes in-classroom coaching sessions to accompany the teachers and advise them on how to effectively use the orchestrations in their classroom.

3. *Reflection or evaluation phase*. This is the period at the end of the pilot program for implementing orchestrations in an educational setting. It is based on reflection and evaluation of the processes of change that were experienced in terms of student learning, pedagogical practice, and management of the technological resources within the school.

4. *Dissemination phase*. This is when the most successful innovations are expanded to other grade levels or departments by institutionalizing the most effective strategies. Information on the changes that have been implemented and the results of these changes are shared throughout the school. Specific focus is placed on how these efforts have translated into strategies that have proven to be particularly effective.

8.7 Main Ideas to Take Away

When reviewing the main ideas from this chapter, six elements can be identified:

- Children are not learning and the computers that are being purchased by education systems are not being used. This is because there is a lack of pedagogical support for teachers in terms of guidelines that enable them to integrate the technology and the needs of their classroom into their teaching.

- Teachers need support, and orchestration provides them with scaffolding. It does so by coordinating pedagogical, curricular, and technological information. By joining these three elements together, orchestration establishes itself as the missing link for policies to provide school systems with educational technology.

- Orchestrations can either be provided to teachers or developed internally by schools. In this sense, a series of guiding questions and a diagnostic of the school's specific context can help school communities develop their own orchestrations. This clearly demonstrates the scalability of this proposal.

- Lessons can either be completely or partially orchestrated, depending on the teacher's tools, knowledge, and skills.

- Regardless of whether orchestrations have been provided to a school or developed internally, implementing them requires certain

social and infrastructural conditions that help teachers overcome the challenges they face.

- Orchestration has been shown to be an effective tool, not only when teaching mathematics, but also in other areas of the curriculum.

Table 8.5 summarizes the conclusions from this chapter, their implications for public policy, and recommendations going forward.

TABLE 8.5
CHAPTER CONCLUSIONS AND POLICY IMPLICATIONS AND RECOMMENDATIONS

Conclusion		Policy Implication or Recommendation
Learning in schools does not improve and computers being purchased by schools are not being used.	→	Change the focus from introducing technology into schools to increasing pedagogical support.
Lack of pedagogical support to integrate the technology into teaching.	→	Focus on pedagogical support, taking into account the cultural aspects of the classroom more than the technological aspects.
The need for orchestrations to structure and support the teaching/learning process.	→	Orchestration guidelines, training, and coaching for teachers must take into consideration the reality of the school where they are implemented.

Source: Prepared by the authors.

ANNEX 8.1
FRAMEWORK DETAILING THE ELEMENTS OF AN ORCHESTRATION[a]

Micro Questions	Element[b]	Category	Description	Example[c]
For which grade level am I going to make the pedagogical decision?	Grade or course targeted by the activity	Pedagogical	Grade or class at which the pedagogical activity is aimed.	*5th Grade*
For which subject am I going to make the pedagogical decision?	Area of the curriculum	Pedagogical	Area of the curriculum to which the pedagogical activity belongs.	*Mathematics*
For which specific topic am I going to make the pedagogical decision?	Name of the unit	Pedagogical	Specific topic to which the pedagogical activity belongs.	*Fractions*
How much time do I really have to teach this topic?	Time dedicated to the class	Logistical	Approximate amount of time required to carry out the pedagogical activity.	*45 minutes*
Which aspect of the topic will I cover in the first week, the second week, the third week, etc.?	Week in which the topic will be covered	Logistical	Week, month, etc. in which the pedagogical activity should be carried out, depending on the school's system.	*First week, second unit (depends on the school's system)*
How am I going to divide the time I have available for class into different learning experiences?	Class activities: Time assigned to each activity	Logistical	Time suggested for each activity, visualizing the distribution of the available time.	*10 minutes of opening phase* *30 minutes of instructional phase* *5 minutes of closing phase*
For which moment of the class will I prepare these learning experiences?	Class-time moment: opening phase, instructional phase and closing phase	Logistical	Identifies the ideal moment for carrying out the pedagogical activity (either the opening, instructional, or closing phase).	*Instructional phase*

FRAMEWORK DETAILING THE ELEMENTS OF AN ORCHESTRATION[a]

Micro Questions	Element[b]	Category	Description	Example[c]
On which cognitive skill will I focus the learning experiences of this class?	Cognitive process: recall, understand, apply, analyze, evaluate, or create	Pedagogical	Cognitive ability on which the pedagogical activity will be focused.	*Recall (depends on the curriculum)*
Which learning objective do I hope my students will achieve by the end of this unit?	Specific objective of the unit	Pedagogical	Learning objective associated with the unit. This can be taken literally from the curriculum or adapted to fit the school's plan.	*Read, write, and graphically represent various fractions, as well as identify situations where they are used in everyday life.*
What is the specific learning objective that my students must achieve after class 1, 2, 3, etc. in the first week, second week, third week, etc.?	Objective of the class	Pedagogical	Specific objective of the pedagogical activities for the particular class. Together with other classes, this will help achieve the unit objective.	*Work individually through trial and error on the contents covered previously by using technology and completing a worksheet.*
What prior knowledge do my students need so that the learning experiences I have designed will be useful based on the objective of the class?	Previous learning required for the development of the class	Pedagogical	Knowledge required in order for the students to be able to complete the pedagogical activity.	*Parts of a fraction; concept of whole and set; understand what cardinal and ordinal numbers are*
Which methodologies will my students follow for these learning experiences? What sort of consistency is there between these?	Skills and attitudes to develop	Logistical	Methodology proposed for the students during the development of the pedagogical activity.	*The class will be divided into two (equal) groups, where one of the groups will work with technology and the other with worksheets in order to work through the contents.*

FRAMEWORK DETAILING THE ELEMENTS OF AN ORCHESTRATION[a]

Micro Questions	Element[b]	Category	Description	Example[c]
How should my students work so that the learning experiences progress correctly?	Criterion for maintaining control of the learning environment	Pedagogical	Classroom dynamics when carrying out the pedagogical activity.	*The dynamic is based on individual work, with each student working on the math problems either digitally or on paper. Although the class is divided into two groups, this only determines whether the group uses technology or pen and paper. Other than during the closing phase, the students do not work together as a group.*
How should the space be organized in order to carry out the learning experiences?	Class activities: general indications about the organization of students	Logistical	Specification regarding classroom layout during the pedagogical activity.	*The room will be arranged so that the half of the class that will be working with pen and paper is in one place, while the students working with netbooks are in another. This will aid how the teacher monitors the classroom and provide the students with differentiated learning (personalization).*
What specific instructions should I give my students regarding the use of resources (technological and conventional)?	Class activities: guidance in the use of the resources for each activity	Logistical	Logistical guidelines regarding the use of resources during the pedagogical activity.	*Once all of the students have their netbook in place, ask them to choose their name from the available list and to select the "Group work" mode, Topic 1. (This work is complementary to and coherent with the work done on paper by the rest of the class.)*

Micro Questions	Element[b]	Category	Description	Example[c]
What instructions should I give my students so that they understand the learning experiences?	Class activities: description of the planned activities, referring to the specific actions of the teacher, and explicitly stating what is expected of the students	Logistical	Description of the steps that the students and/or teacher must follow when carrying out the pedagogical activity.	*For the digital activity, give the following instructions to the students:* • *When shown a graphical representation of a fraction, you must choose the fraction that it represents.* • *When shown a fraction, you must choose the drawing that represents it.* • *When shown a fraction in words, you must choose the image that represents it.*
Which resources should I prepare so that the learning experiences can take place optimally? What sort of consistency is there between these resources?	Resources	Logistical	Physical resources (worksheets, interactive activities on the computer, presentations with/without audiovisual support, complementary activities/homework, online resources).	• *Printed worksheet for each student* • *Printed worksheet for the teacher* • *Netbook with software (1–2–1)*
How will I interact with my students as the learning experiences take place? And how can I promote interaction between my students as the learning experiences take place?	Visibility: teacher to student, student to student, student to teacher	Pedagogical	Student monitoring and interaction suggested when carrying out the pedagogical activity.	*During the digital work, the teacher is expected to walk around the room and help the students with their work. It is recommended stopping the activity every 10 minutes in order to clear up any general doubts that may arise.*

ANNEX 8.1 (*continued*)
FRAMEWORK DETAILING THE ELEMENTS OF AN ORCHESTRATION[a]

Micro Questions	Element[b]	Category	Description	Example[c]
Which resources will I use to monitor my students' progress, and how and when will I use them?	Formative assessment guidelines: tools	Logistical	Resource that will be used for formative assessment. This can be digital or non-digital.	*Online website[d]*
Which questions will I ask my students to make sure they understand every explanation I give and every example I show? **How will I make sure my students are meeting the learning objectives set for class 1, 2, 3, etc. in the first week, second week, third week, etc.?**	Formative assessment guidelines: specific questions to be put to the students and the expected answers	Pedagogical	Suggested questions for the teacher to check whether the pedagogical activity is benefiting the students.	*Guide your students using questions such as:* • *What does a fraction's denominator tell us?* • *What does a fraction's numerator tell us?*

Source: Prepared by the authors.

[a] The elements that are included in this sort of orchestration were proposed by Nussbaum and Díaz (2013). These are classified as either logistical or pedagogical, with a brief description and example given for each.

[b] The presence of each element will depend on the school system in which the orchestration is used (e.g., characteristics of the curriculum). The arrangement of the elements is flexible. Sometimes instruction will contain more than one element, or an element could be further divided based on the specifications required by the range of resources that are to be integrated. The format of the orchestration is flexible.

[c] The examples have been taken from an orchestration that was part of a project carried out in Barranquilla, Colombia in 2013 and financed by the Inter-American Development Bank. The orchestration of the class featured in the examples can be found in Appendix 8.2.

[d] http://odas.educarchile.cl/objetos_digitales_NE/ODAS_Matematica/Ed_Matematica/fracciones_orden/player/start_fs.html?idx=0. (Only available in Spanish).

GUIDELINES FOR TEACHING TOPIC 2

Grade: 5th grade	**Area:** Mathematics	**Unit:** Fractions
Allocated time: 8 hours	**No. of classes:** 8 classes	

Topic: Types of fractions and ordering fractions.

Expected learning outcomes: Understand, represent, compare, and order proper and improper fractions, mixed numbers, and fractions equivalent to one whole.

Objectives: Read, write, represent, classify, and order proper and improper fractions, mixed numbers, and fractions equivalent to one whole.

Prior knowledge required for this activity:
1. The concept of fractions.
2. Parts of a fraction.
3. Reading and representing fractions.
4. The concept of wholes and sets.

CLASS 1: INTRODUCING THE CONTENT

TIME	TEACHER GUIDELINES	RESOURCES
10 minutes	**ACTIVITY 1** **OBJECTIVE:** Recall prior knowledge with students to help them understand the new content that will be covered in class (types of fractions and ordering fractions). **EXPECTED OUTCOME:** Students are expected to remember basic concepts related to fractions, with the aim of studying them in further detail later in the class. **CLASSROOM DYNAMICS:** Group activity (whole class) where the group will discuss the concepts, and the prior knowledge that emerges will be written on the board. **OPENING:** Start the class by explaining today's objective to your students: *"We will learn about different types of fractions, we will classify them and order them according to different criteria."* Then add that in order to meet said objective, students will start by recalling prior knowledge that will help them understand the new content. To recover the prior knowledge suggested at the beginning of the plan, the following questions are recommended: • *Prior knowledge 1: "What are fractions?" (A number that expresses distribution or a part of something).* • *Prior knowledge 2: "What are the parts of a fraction?" (Numerator and denominator).* • *Prior knowledge 3: "What does each part of a fraction represent?" (The denominator shows the number of parts into which a whole is divided and the numerator the number of parts that we take from it).* • *Prior knowledge 4: "What examples of fractions can we think of?" (Invite some students to write examples of fractions on the board).* • *Prior knowledge 5: "How can we represent these fractions using wholes and sets?" (Suggest representing some as wholes, clarifying that first we divide the whole into the number of parts indicated by the denominator and then we take the number indicated by the numerator. Then, follow the same procedure using sets).* As well as recalling this conceptual element of fractions, it is also important to emphasize the use of fractions and how we use them in everyday life. Once this information has been recalled, invite your students to be introduced to the content: "Types of fractions and ordering fractions."	• Whiteboard

CLASS 1: INTRODUCING THE CONTENT

TIME	TEACHER GUIDELINES	RESOURCES

30 minutes	**ACTIVITY 2**
	
	OBJECTIVE: Understand and classify types of fractions, as well as ordering them.
	EXPECTED OUTCOME: Students are expected to identify the characteristics, classify, and compare proper and improper fractions, mixed numbers, and fractions equivalent to a whole so that they can order them.
	CLASSROOM DYNAMICS: Group activity (whole class) where the content will be introduced. All students will be encouraged to participate by answering questions orally and in writing, making real examples using paper and comparing fractions.
	INSTRUCTION: Invite your students to take part in the presentation to be covered in class.
	  The material introduces the content through a contextualized situation, where two swimmers talk about how they have an important competition for which they must prepare. An important part of this preparation consists of eating healthily. With this in mind, they reveal a recipe for the type of food they eat, in this case a "fruit salad." When introducing the recipe, the following questions are recommended:
	
	• *What are the characteristics of this text? (It shows the ingredients and steps for making a fruit salad.)*
	• *What ingredients do you need to make this recipe? (Those included in the PowerPoint).*
	
	After introducing the recipe, a slide is shown that invites the students to recall previous classes. Give the students space to think about and recall what they are being asked.
	  Following this, show them the answers, which are written in red (this will be repeated throughout the whole presentation).
	  Once they have identified that the ingredients are expressed as fractions, the students are invited to think about how these could be represented. The presentation includes boxes with phrases such as "try it in your exercise book" and "let's write some ideas on the board." The idea is that when these appear, you give the students a few minutes to do what is asked of them.
	  All of the ingredients required for the recipe will be covered in this way, one by one. When it is time to represent the orange as a fraction, the students will realize that the numerator is greater than the denominator.
		• Computer for the teacher with PPT 01. – Introduction (types of fractions and ordering fractions)
		• Projector
		• Squares of paper
		• Exercise books

CLASS 1: INTRODUCING THE CONTENT

TIME	TEACHER GUIDELINES	RESOURCES
	At that moment, ask questions such as: • *"What can we do in this case?"* (*We need more oranges.*) • *"If the numerator says that I take seven pieces but the orange is divided into two, what can I do?"* (*Divide whole oranges in two until there are seven pieces to take.*) The idea is to guide the students to the conclusion that when the numerator is greater than the denominator, more than one whole is needed. Following this, a similar example is shown using the banana. Given that they have already seen what happens with the orange, encourage the students to apply what they have learned. Subsequently, a slide is shown explaining what the students have just observed. This time, it is revealed that the fractions that have been represented correspond to proper and improper fractions, as well as fractions equivalent to a whole. Read the definitions with your students, ask them to give other examples, and then have them write these definitions in their exercise books. It is then time to represent the yogurt as a fraction. This will be an ingredient that is expressed as a mixed fraction. When you show this, ask questions such as: • *"What characteristics does this number have?"* (*It has a whole number and a fraction.*) • *"What do you think this represents?"* (*Encourage them to share any ideas they have regarding this.*) Following this, the presentation shows how this fraction (the yogurt) can be represented. As with the previous cases, it is then explained that this type of fraction is called a "mixed fraction," and it is shown how mixed fractions are read and represented. Show other examples of a mixed fraction, ask the students to read and then represent them. Finally, have them write the definition in their exercise books. Once this way of classifying fractions has been explained, a series of fractions is revealed that the students must classify as proper, mixed, or fractions equivalent to a whole. Invite your students to participate, and comment on what their classmates are doing and whether they are right or wrong and why. When the recipe has been shown and the fractions have been represented, the swimmers explain that they are now ready to compete. It is in this part of the presentation where, based on the competition, it is explained how to compare and order fractions.	

(continued)

CLASS 1: INTRODUCING THE CONTENT

TIME	TEACHER GUIDELINES	RESOURCES
	The presenter of the competition will comment on how each swimmer is doing. In the beginning, the presenter says that one of them has covered "one fifth," while the other has covered "one sixth," asking which of them has swum further. In this part of the class, it is important for you to encourage your students to say what they think the answer is and to explain why. Then, an explanation is given of how to compare two fractions by representing them in picture form. In order to do so, ask the following question: • *"What characteristics must the wholes have in order to compare them?" (They must be equal.)* When the students answer this question, use the presentation to show how fractions are represented and compared using pictures. This exercise is repeated several times throughout the presentation as updates are given on the competitors' progress and the students must work out who is winning. At the same time, the students are shown how to represent numbers on a number line. In order to do so, ask the following questions: • *"What is a number line?" (It is a line that is divided into equal parts, with each of these parts a number.)* • *"What does each number on the number line represent?" (Each number represents a whole.)* Explain that, as with the previous case, in order to compare two fractions, the wholes on the number line must be the same size and divided into the number of parts indicated by the denominator. With this, we can then take the number of pieces indicated by the numerator. Show other examples of representing fractions on a number line, so as to ensure that the students have understood the procedure. Then, tell the students to write down how to compare two fractions in their exercise books. As the swimmers advance, the presenter indicates how much of the distance each one has covered. In one of his updates, explaining that one of them has covered "three sixths" and the other "one half." The students are asked to represent said fractions in order to work out which swimmer is in front. By doing this exercise, the students will notice that they are tied and that the fractions represent the same thing. The presenter explains that such fractions are called equivalent fractions. It is recommended that you stop for a moment during this part of the class in order to explain this type of fraction. Show some strategies for finding equivalent fractions, such as doubling both the numerator and the denominator, and if the numerators are the same, then the fractions are equivalent. Suggest some fractions and invite the students to apply what they have learned by determining which fractions are equivalent. Finally, the result of the competition is given and the students are asked to apply what they have learned in order to determine which of the swimmers has won.	

CLASS 1: INTRODUCING THE CONTENT

TIME	TEACHER GUIDELINES	RESOURCES
5 minutes	**ACTIVITY 3** **OBJECTIVE:** Show that the students have understood the main ideas of the class. **EXPECTED OUTCOME:** Students are expected to answer questions regarding the main ideas of the class, thereby showing they have understood the topic of "Types of fractions and ordering fractions." **CLASSROOM DYNAMICS:** Group activity (whole class), where questions will be asked that summarize and assess the work covered in today's class. Student participation must be encouraged so as to assess their learning. **CLOSING:** end your class by summarizing the main ideas of the class. In order to do so, it is recommended that you ask questions such as : • **"What did we do today?" (We watched a presentation that explained types of fractions and how to compare them before ordering them.)** • **"How useful is what we learned today?" (To understand the characteristics of a proper fraction, improper fraction, mixed number, and equivalent fractions so that we know how to identify, represent, compare, and order them.)** • **"For which of the fractions do I only need one whole in order to represent them?" (For proper fractions and fractions equivalent to one whole.)** • **"For which of the fractions do I need more than one whole in order to represent them?" (For improper fractions and mixed numbers.)** • **"What are the steps for comparing two or more fractions?" (Draw a whole or a number line that is the same size, divide it into the number of parts indicated by the denominator, and take the parts indicated in the numerator. Whichever covers more is the larger of the fractions.)** End the class by congratulating the students for the work they have done.	• Whiteboard

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The early twenty-first century has witnessed an explosion of technological changes that have revolutionized the way we travel, shop, interact and play. Technology can also transform education by boosting motivation, personalizing instruction, facilitating teamwork, enabling feedback, and allowing real-time monitoring. However, a gap exists between the potential impact of technology and the actual results of public initiatives. This book brings together leading regional and international experts in the field to shed light on how governments can take better advantage of the potential of technology to improve student learning.

Specifically, the book focuses on mathematics, a critical learning area in which most students in the region do not attain even basic levels of proficiency. The first part of the book presents a thorough diagnosis of the main challenges to mathematics learning in the region. The second part of the book describes a range of technological models and assesses their capacity to tackle these challenges and produce improvements in learning. By combining theoretical and empirical approaches, reviewing innovative initiatives, and drawing lessons from psychology, education, and economics, the book aims to become a reference for policymakers who want to make the promise of technology in education a reality for all students in the region.