



# The Mathematics of Opportunity:

Rethinking the Role of Math in Educational Equity

By Pamela Burdman | November 2018





# Acknowledgments

The author wishes to thank the many people who made possible the development and publication of this report: Jenn Bevard, Just Equations project manager, assisted in countless ways, including coordinating meetings, conducting library research, and overseeing design and production. Christopher Edley, Jr., Molly Mauer, and Sean Simplicio of the Opportunity Institute provided inspired leadership, a venue for our team to work, and a terrific set of colleagues.

Our partners and advisors provided invaluable input on the report by participating in a strategy discussion and/or providing comments on an early draft: Lauren Asher, Kyndall Brown, Linda Collins, Phil Daro, Christopher Edley Jr., Rebecca Hartzler, Juana Hernandez, Hal Huntsman, Hayin Kimner, Mayra Lara, Molly Mauer, Christopher Nellum, Ravin Pan, James Ryan, Kimberly Samaniego, Kimberly Seashore, Katherine Stevenson, Bob Shireman, Bruce Simon, Myra Snell, Leticia Tomas-Bustillos, and Ian Walton.

Thanks also go to Gretchen Kell, copy editor, and Anna Clark, graphic designer.

# About Just Equations

Just Equations re-conceptualizes the role of mathematics in ensuring equal opportunities for students. An independent resource on the role of math in education equity, Just Equations works across educational segments and advances evidence-based strategies to ensure that math policies give all students the quantitative foundation they need to succeed in college and beyond. Just Equations is a project of the Opportunity Institute, in partnership with LearningWorks, Policy Analysis for California Education, the Education Trust—West, and the Campaign for College Opportunity.

The James Irvine Foundation and College Futures Foundation provided funding to support this report and other work of Just Equations.

# About the Author

Pamela Burdman—a policy analyst and strategist on college access, readiness, and success—is the founder of Just Equations. She works at the intersection of education research, policy, and practice, to synthesize knowledge from the field to define problems and advance strategies that support student success. She began her career as a reporter for the San Francisco Chronicle more than 20 years ago, and first focused on math readiness as a program officer at the William and Flora Hewlett Foundation.

**B**eginning in kindergarten and continuing into college, mathematics is not just an academic subject: It's a key mechanism in the distribution of opportunity. Solid quantitative reasoning skills are an important underpinning for achievement in school, in a profession, and in many aspects of life. Whether studying physics, managing inventory, buying a car, reading polls, or following sports scores, a facility in mathematics is essential.

But, perhaps because it plays such a primary role, math can also be wielded in ways that arbitrarily close doors to educational advancement. Even as math expectations can serve as a foundation for academic success by supporting quantitative literacy, they can also operate as a filter that literally stops many students in their educational tracks. While such claims may be made of educational requirements generally, math is unrivaled in its use as a marker of intelligence that can limit access to future opportunity.

This role creates an imperative for the field of mathematics—all who teach it, test it, train teachers, and write textbooks about it—as well as for education leaders who determine math-related policies: They must ensure fairness and equity in the pursuit of achievement. But doing so is complicated by the fact that responsibility is spread across K-12 and postsecondary institutions and diffused among instructors, administrators, and policy leaders, all whose efforts are needed to effect change.

Plus, there are questions about what equity in math entails. A useful definition, proposed by math education scholar Rochelle Gutierrez, is that equity is attained when it's not possible "to predict mathematics achievement and participation based solely on student characteristics such as race,

class, ethnicity, sex, beliefs, and proficiency in the dominant language."<sup>1</sup> Despite the best intentions of many, there is no evidence that the educational establishment is meeting this responsibility:

- Only about one-quarter of high school seniors are proficient in mathematics, as measured by the National Assessment of Educational Progress (NAEP), vs. more than one-third who are proficient in reading.<sup>2</sup>
- Large proportions of college students are placed into remedial mathematics courses: In California, until the adoption of recent policy changes, as many as three-quarters of community college students and one-third of California State University (CSU) students were required to take remedial math in their first semester or the summer before.
- Mathematics practices and policies contribute to educational equity gaps, with African American and Latino/Latina students disproportionately judged below proficient or in need of remedial math coursework.<sup>3</sup>

Rather than the actual capacities of students, such data underscore the inequitable conditions under which students learn, the often questionable ways in which their learning is assessed, and the influence of racism and other forms of discrimination.<sup>4</sup> And

<sup>1</sup> Gutierrez, 2012, p. 19. For a current review of other perspectives on equity in mathematics, also see Marshall & Leahy, 2018a.

<sup>2</sup> National Center for Education Statistics

<sup>3</sup> At California State University (CSU), not including students assigned to a summer brush-up course, 14% of white students and 16% of Asian American students required remedial math in Fall 2016 vs. 50% of African American students and 37% of Latino/Latina students (Blanchard, 2017). In California's community colleges, nearly 85% of African American and Latino/Latina students take remedial math courses, compared to 72% of white students and 52% of Asian American students (Cal-PASS Plus, 2018).

<sup>4</sup> Martin, 2003



they highlight the stakes of traditional practices: Across the country, millions of students each year<sup>5</sup> are hindered in reaching their educational goals because of their math achievement in high school or college.

Those who don't perform well in high school math and/or don't take advanced courses face limited access to college. And many attending less selective colleges, especially community colleges, wind up in remedial math sequences, which severely curtail their chances of ever completing a degree. This predicament casts serious doubt on math's gatekeeping role.

Traditional practices fall short of achieving equity and, if anything, have reinforced inequities. In addition, they may have subverted the purpose and promise of mathematical literacy, as defined by the National Council of Teachers of Mathematics (NCTM): to help students “expand professional opportunity, understand and critique the world, and experience wonder, joy, and beauty.”<sup>6</sup>

The way mathematics is typically taught and tested, as well as the very requirements students are expected to meet, appear designed to winnow students out, effectively surrendering to the notion that only a few students are “math worthy.” For example:

- Mathematics is the only area of the high school curriculum where an accelerated pace is needed to reach an Advanced Placement (AP) course.
- Though pre-calculus is considered a college-level course almost everywhere, a high school pre-calculus course rarely confers college credit. (At best, it signals readiness for a college-level course; but often, it doesn't even do that.)<sup>7</sup>
- In some states—including California, until this year—math is the only discipline in which public universities have specified a prerequisite (intermediate algebra or Algebra 2) for a college-level course. This policy effectively resulted in tens of thousands of community college students taking remedial courses in order to transfer to universities, regardless of whether the content was needed for success in their general education math courses. (In the case of popular college courses like statistics, it was not needed.)<sup>8</sup>
- Though calculus is rarely (if ever) used by doctors, medical schools have traditionally used it as an admissions screen.<sup>9</sup>
- Standardized tests, like the SAT, used for college admission, are scored on a bell curve, effectively designed to pick “winners and losers.” Research shows that demographic characteristics are the

<sup>5</sup> Based on National Assessment of Educational Progress (NAEP) data, each year more than 2 million high school students across the country are not proficient in math. In California alone, an estimated 200,000 students require remedial mathematics each year, and the majority of them don't go on to earn degrees.

<sup>6</sup> National Council of Teachers of Mathematics (NCTM), 2018, p. 9

<sup>7</sup> According to Melguizo & Ngo, 2018, 36% of students who passed trigonometry or pre-calculus with a B or better and subsequently attended community college were placed into remedial math. For years, CSU has accepted a passing grade in an approved pre-calculus course to exempt a student from remedial math, but only a score of 3 on an AP Calculus exam meets the college math requirement.

<sup>8</sup> Burdman, 2015b

<sup>9</sup> Burdman, 2015a

strongest predictors of how students perform on these tests, and that the tests are only modestly predictive of students' performance in college.<sup>10</sup>

These uses of mathematics exemplify a critique leveled two decades ago by UCLA professor Sandy Astin in an address to the American Council on Education:

We value being smart much more than we value developing smartness. In our relentless and largely unconscious preoccupation with being smart, we forget that our institutions' primary mission is to develop students' intellectual capacities, not merely to select and certify those students whose intellectual talents are already well developed...We continue to support a grading system and standardized testing industry that are geared to ranking and rating students, rather than to reflecting how much they are actually learning.<sup>11</sup>

Fairness and equity demand policies and practices that aim to foster quantitative literacy that helps students advance in life, not to ration opportunity. For too long, the latter has been the case, with damaging aftereffects. As leading mathematician and math reformer Uri Treisman summed up his speech to an assembly of the NCTM back in 2013:

There are two factors that shape inequality in this country....The big one is poverty. But a really big one is opportunity to learn. As citizens, we need to work on poverty and income inequality or our democracy is threatened. As mathematics educators...we need to work on opportunity to learn. It cannot be that the accident of where a child lives or the particulars of their birth determine their mathematics education.<sup>12</sup>

Numerous strategies and initiatives are being implemented by educators and policymakers toward

that end, and some will be described later in this report. However, to ensure the success of those strategies, it is essential that they are grounded in evidence and aligned across educational segments, and monitored for their impact on equity.

Just Equations is releasing this report, *The Mathematics of Opportunity*, to help envision a new role for mathematics in education equity. The report is intended to advance the work of Just Equations and others working to advance math opportunity by:

- Informing and stimulating dialogue among educators, policymakers, and equity advocates about the role of mathematics in educational equity;
- providing a cross-segmental framework for adoption and implementation of practices and policies that are coherent and aligned across educational systems; and
- inspiring a research agenda (to be published in the coming months) to inform those policies and practices to ensure that they add to, rather than subtract from, educational equity.

## In the “Aftermath”: Benefits & Costs

Education is intended to improve students' futures. The costs—both psychic and economic—to individuals and society of not effectively and equitably educating students are great, and traditional math practices bear a fair share of the blame: There are many students who survive, and even thrive, in math courses, but—despite considerable efforts of math instructors and those who train them—a vast number of students have negative experiences. In fact, surveys have shown that a majority of the U.S. population dislikes and

<sup>10</sup> Geiser, 2017

<sup>11</sup> Astin, 2008

<sup>12</sup> Treisman, 2013

fears mathematics.<sup>13</sup> It is not surprising, then, that proficiency rates are low.

Learning math involves developing a “mathematics identity,” which math scholar Danny B. Martin says “encompasses a person’s self-understanding and how they are seen by others in the context of doing mathematics.”<sup>14</sup> Too many students receive the message in school and beyond that they are incompetent and unable to do mathematics. The resulting outlook can last a lifetime, limiting individuals’ horizons.

Such a message, often implicit, undermines students’ confidence as math learners. Research has found that mathematics anxiety is quite prevalent, and its effects can linger even long after students finish school.<sup>15</sup> Possible causes include teacher emphasis on correctness and the resulting fear of public embarrassment, working memory overload, and even math-anxious teachers.<sup>16</sup> Effects may be more pronounced among students who already face disadvantages, including racial and gender stereotypes about their educational competence.<sup>17</sup>

Also helping to convey the message are ability grouping and tracking, common practices in math. As decades of research confirm, tracking runs counter to the best interests of most students. It does not support increased academic achievement or positive behaviors. And it inflicts the greatest harm on low-income students and students of color. “If schooling is intended to provide access to economic, political, and social opportunity for those who are so often denied such access, school tracking appears to interfere seriously with this goal,” according to Jeannie Oakes’ landmark study on the topic.<sup>18</sup>

The message is also perpetuated by parents and teachers who themselves have internalized the

## In Their Own Words: Student Expressions of Math Anxiety<sup>20</sup>

*“I hate mathematics, and I would rather die.”*

*“It SUCKS, and I wouldn’t want to spend any more of my time looking at algebra and other crap.”*

*“I despise the way it is taught.”*

*“Who needs to know trigonometry in everyday life?”*

*“I have no confidence, I will get stressed, and it will get on top of me.”*

*“I don’t think it will take me where I want to be in life.”*

*“During the [high stakes test] last year, I thought I was going to throw up when we did the math part.”*

<sup>13</sup> Ramirez, Gunderson, Levine, & Beilock, 2013

<sup>14</sup> Martin, 2007, p. 150

<sup>15</sup> Bustillos, forthcoming

<sup>16</sup> Dowker, Sarkar, & Looi, C.Y., 2016

<sup>17</sup> Maloney & Beilock, 2012

<sup>18</sup> Oakes, 2005, p. 101. (Quote was from earlier chapters of book, published in 1985).

notion that they are not “math people.” This phenomenon is so common that university psychology departments across the country have researchers specializing in areas like math anxiety, math identity, and math belonging. “Although mathematics is not the only subject that elicits anxiety, anxiety may indeed be more severe, and possibly affect performance more, for mathematics than for other subjects,” according to a 2016 study.<sup>19</sup>

“Mathematics, more than any other subject, has the power to crush students’ spirits, and many adults do not move on from mathematics experiences in school if they are negative,”<sup>21</sup> notes Jo Boaler, a Stanford University expert on math education,

Students tend to underestimate their competence in math,<sup>22</sup> and those who are already marginalized in educational settings may be more prone to doing so. In just one example, a study of community college students in California found that African American and Latino/Latina students taking placement tests in California tended to choose a lower-level test than their high school records suggested they were prepared to take.<sup>23</sup> This phenomenon is especially troubling because it contributes to poor performance in math, even though all students are capable of learning math.<sup>24</sup> Students’ confidence in their math competence contributes to stronger performance.<sup>25</sup>

Further damage occurs when even those students who do successfully navigate the high school math gauntlet are told that they lack the skills they most need. Large numbers—up to 80 percent of

community college students, for example—have been placed into remedial courses. And even for those who aren’t, the math they learned in high school is often poorly aligned with the needs of their academic disciplines. According to a 2013 study,

A substantial part of the high school mathematics we teach is mathematics that most students do not need, some of what is needed in the first year of community college is not taught in our schools, and the mathematics that is most needed by our community college students is actually elementary and middle school mathematics.<sup>26</sup>

This combination of limited success and misalignment would be easier to tackle if all students were harmed, but students who do perform well under current practices often reap benefits in the form of future academic success. Given that performance in math is commonly (if wrongly) associated with intelligence, students who do benefit may see little reason to change practices that confer a valued pedigree and access to other opportunities.

That contrast presents unique challenges for changing the status quo. However, math education leaders and major math associations increasingly have concluded that change is needed—for the benefit of students, for the sake of equity, and for the future of the math discipline.<sup>27</sup>

<sup>19</sup> Dowker et al., 2016

<sup>20</sup> Brown, Brown, & Bibby, 2008; Maloney & Beilock, 2012

<sup>21</sup> Boaler, 2016, p. x

<sup>22</sup> California Acceleration Project, 2018, p. 3

<sup>23</sup> Fong & Melguizo, 2016

<sup>24</sup> See, for example, Devlin, 2000, and Berry, n.d.

<sup>25</sup> Chen et al., 2018. Much of the psychology research in math has focused on gender, not race.

<sup>26</sup> National Center on Education and the Economy, 2013, p. 24

<sup>27</sup> The most recent example is the NCTM’s Catalyzing Change in High School Mathematics report. For a summary of other statements from the math discipline, see chapter 2 of Burdman et al., 2018.



# The Aftertaste of Math

Javier Cabral will never forget his first taste of educational failure.

“It was after a parent-teacher conference in seventh grade,” he recalled. “I remember my mom being really disappointed, because that algebra class was the first F that I ever brought home. I was always a pretty good kid when it came to academics.”

Instead of the usual post-conference trip to the Southern California pizza parlor, Cabral and his mom drove home in silence—and thus began his experience with math anxiety.

It’s hard to know today what went wrong in that math class, but most experts say it is premature for schools to start teaching algebra in seventh grade. In all, Cabral believes he failed algebra 11 times—including in seven college remedial courses. His high school diploma came, thanks to a policy that allowed him to take, in lieu of algebra, an accounting class, in which he earned a B+.

Having discovered his passion for writing at a young age, Cabral had planned to transfer from community college to the University of Southern California as a journalism major. But after he couldn’t complete the math sequence at Pasadena City College, he finally gave up on his dream of earning a college degree.

It is a fate that has befallen tens of thousands of California students who leave community college each year without completing their remedial math sequences. Cabral is unique in reflecting on it publicly. In a 2013 article, he wrote, “I don’t think



my inability to solve quadratic equations should be a deal-breaker for any further education.”<sup>28</sup>

Cabral made the most of his situation: Through his way with words, he managed to land work as a Los Angeles food writer. Recently, he has been busy writing his first cookbook, about Oaxacan food. “I’m converting all the recipes from metric to American. That’s the most math I’ve used since college,” he said.

“Conversions and simple multiplication and division and subtraction, that’s the only math that I use nowadays. It’s food. You’re making a mole, or you’re making arroz con leche, or a really delicious cocktail, and you have to count off your ounces to milliliters or to grams. There’s an instant reward for that math, and it usually tastes very good.”

But that article he wrote five years ago keeps resurfacing, because he continues to hear from students who haven’t been as lucky. “I’ve had some pretty heartfelt letters from students, saying how much they can relate,” he said. He’s not sure of the solution. But with so many casualties, it seems that either math instruction or math requirements aren’t working, so at least one of them needs to change.

“If you’re a solid student, and if you do well in the rest of your classes, and there’s one class that keeps you back, at a certain point it starts to affect your self-esteem. It’s a universal problem.”

<sup>28</sup> Cabral, 2013

# The Architecture of Math Opportunity

To be sure, mathematics is not the only gatekeeper that determines access to advancement, or the sole source of inequity in the education system. So how does it structure opportunity in such a significant way? Examining the literature on math achievement and equity and analyzing existing policies suggest that the architecture of math opportunity is:

- undergirded by **misconceptions** about math ability,
- scaffolded by existing educational **inequities** and stereotypes, and
- reinforced by math's use as a marker or **pedigree** that confers access to opportunities.

## Popular Misconceptions About Math Ability

Misconceptions about mathematics ability proliferate, not only among students and families, but within the U.S. education system itself. Consider these common myths:

- Math ability is innate: Only some people are good at math.
- There is a single right way to do math: It lacks creativity or expression.
- In math, speed and acceleration matter: Process and depth are secondary.<sup>29</sup>

Mathematician Keith Devlin has demonstrated that the ability to do math is rooted in the ability

to use language.<sup>30</sup> While learning differences, such as working memory and slow processing, present challenges for some students, math anxiety constitutes a greater barrier to learning.<sup>31</sup> Researchers such as Carol Dweck and Jo Boaler have demonstrated that the notion that only some people have math skills reflects societal and self-perceptions, not underlying abilities. Experiments have shown that changing those beliefs enhances students' performance.<sup>32</sup>

Other research has revealed that productive struggle is essential to math learning, and that making mistakes is a necessary part of the learning process. Even students who haven't answered a problem correctly can contribute to the classroom by sharing their thinking during in-class discussion. As NCTM President Robert Q. Berry III notes:

*Correct answers matter, but not as indicators of who is able to do mathematics. Engaging in mathematical discourse is essential for developing mathematical identity and should be recognized as a better indicator of mathematical competence.<sup>33</sup>*

Nevertheless, common U.S. teaching practices can reinforce these damaging misconceptions about mathematical competence. Teachers in this country tend to view the wide range of abilities as a barrier to effective teaching, and to see confusion as thwarting—rather than aiding—learning. They approach math as a set of procedures to learn, rather than a system for thinking.<sup>34</sup> An emphasis on speed and correct answers contributes to math anxiety and interferes with students' ability to develop positive mathematical mindsets and become proficient in math.<sup>35</sup>

<sup>29</sup> Boaler, 2016; Gutierrez, 2018

<sup>30</sup> Devlin, 2000

<sup>31</sup> Maloney & Beilock, 2012

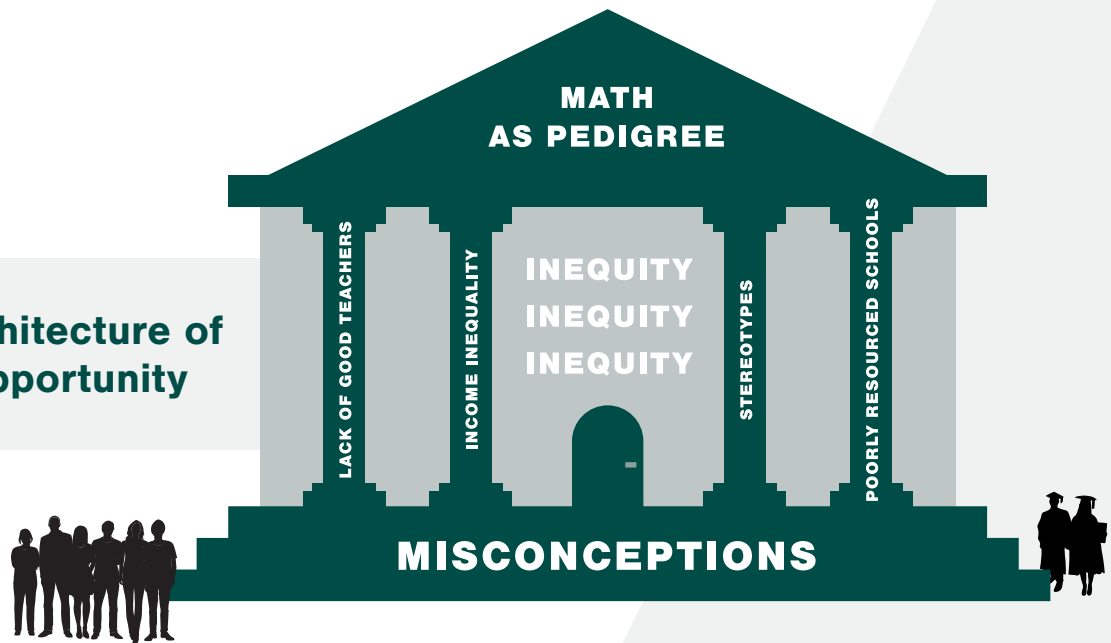
<sup>32</sup> Kimball & Smith, 2013; Boaler, 2016

<sup>33</sup> Berry, 2018

<sup>34</sup> Stigler & Hiebert, 1998

<sup>35</sup> Boaler, 2016

## The Architecture of Math Opportunity



## Inequities & Bias

While these misconceptions present challenges to learning mathematics in general, they are especially potent in reinforcing the ways some students are already disadvantaged in the education system: those who face bias based on race, ethnicity, gender, English learner status, or disability status. These students, in particular, may experience math class as a dull and “dehumanizing” environment, in that it seems they are asked to leave their identities, values, and creative impulses at the door.<sup>36</sup>

In addition, students who belong to groups that are subject to a stereotype—say, of being less smart—can face implicit bias, as well as stereotype threat. Bias occurs when educators underestimate students’ abilities based on race, gender, or other characteristics. For example, researchers have documented cases of black and Latino/Latina ninth graders being placed lower than white and Asian American students with similar prior math

achievement.<sup>37</sup> Under stereotype threat, students underperform relative to their own abilities because they are vulnerable to conforming with a stereotype.<sup>38</sup>

Educational inequity is maintained by a series of systemic factors,<sup>39</sup> including:

- poorly-resourced schools,
- differential access to strong curriculum and good teaching,
- income inequality and insufficient support for students’ needs,
- bias (implicit or explicit) on the part of educators, and
- the psychic effects, such as negative math identity and stereotype threat, that can result from these inequities.

These conditions present challenges for learning in general, and all the more so in math.

<sup>36</sup> Gutierrez, 2018

<sup>37</sup> Gao & Adan, 2016

<sup>38</sup> Steele & Aronson, 1995

<sup>39</sup> For a more extensive discussion of factors in educational inequity, see, for example, U.S. Department of Education, 2013.

## Pedigree vs. Preparation

While preparation is supposed to provide what people need in order to advance to the next level, pedigree functions to preserve the position of individuals and groups that already enjoy privilege. “Definitions of merit at any given time and place tend to reflect the values and qualities of elites,” notes Lauren Rivera in *Pedigree: How Elite Students Get Elite Jobs*. “It is not surprising that, in whatever manner merit is defined and measured in society’s gatekeeping institutions, elites seem to have more of it.”<sup>40</sup>

Math prerequisites—in the form of courses taken, grades in those courses, and test scores—often function without regard for what math content is needed for a given degree or major, or whether the required tests are valid predictors of success. Such arbitrary uses further cement the way math policies operate to ration opportunity and reinforce

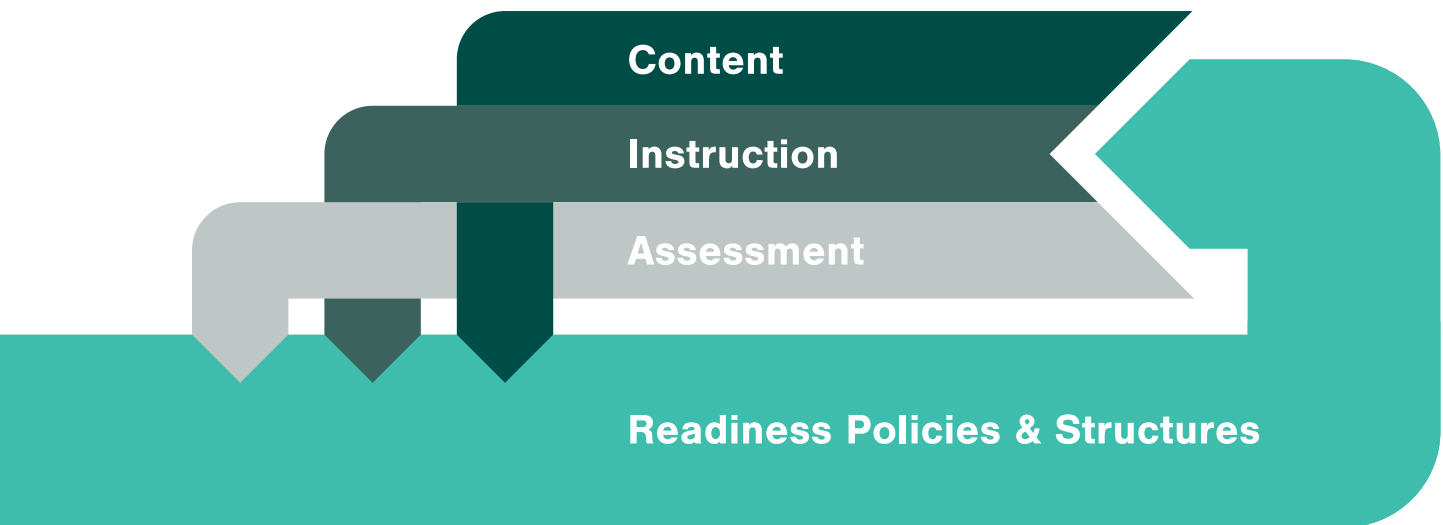
inequitable structures, disproportionately influencing students’ ability to advance in their education. Such policies determine whether students can:

- take advanced courses in high school,
- earn a high school diploma,
- earn acceptance to a selective college (or, in some places, any college),
- take non-remedial courses in college,
- enter desired programs, and
- ultimately complete a college degree or credential.

Given the challenges presented by many math learning environments, attempting to run this gauntlet is an intimidating task for many students. The process can hinder students from nurturing their interests and aspirations while developing positive math identities. Yet changing the status quo is no simple task: It can entail shifting practices in more than one sector, across multiple dimensions.



<sup>40</sup> Rivera, 2015, pp. 9-10



## Equity Dimensions of Math Education

Opening up opportunity requires rethinking existing math practices and policies in K-12 and postsecondary education. Inequity is reflected within four interconnected dimensions of math education, and emerging practices within each suggest avenues for advancing equity:

- Content
- Instruction
- Assessment
- Readiness Policies & Structures

### Content

Content expectations reside in places like high school and college graduation requirements, state standards, and college admissions and readiness requirements. A three-year sequence—first delineated in the late 19th century—that combines two years of algebra and one year of geometry continues to comprise the core high school curriculum in most of the country.<sup>41</sup> It is also reflected in many states'

content standards, college admissions requirements, and college readiness definitions.

The historical emphasis on preparation in algebra is tied to the fact that advanced algebra courses like Algebra 2 and college algebra prepare students for calculus. Calculus is needed to pursue some STEM fields, and often is viewed as a hurdle for admission

<sup>41</sup> NCTM, 2018

to selective colleges. In the late 20th and early 21st centuries, studies showing a correlation between eighth grade algebra coursework and admission to competitive universities led to efforts to ensure that all students could take Algebra 2 as well as a push to start Algebra 1 during middle school so students could reach calculus by their senior year.<sup>42</sup>

This priority can come at the expense of material that is more relevant for most students. Even the former chair of the committee that develops the AP Calculus test has acknowledged this:

Topics like statistics, probability, matrices, mathematical induction, graph theory, linear programming, and even financial topics like amortization and mortgages that will affect almost every student someday, are given short shrift in the core curriculum precisely because they are not necessary for studying calculus.<sup>43</sup>

Public two- and four-year institutions traditionally have replicated high schools' emphasis on algebra by expecting students to pass math placement tests that emphasize algebra—or take one or more remedial courses. Nationwide, assignment to remedial math sequences reduces the chances of college completion for as many as hundreds of thousands of students a year.<sup>44</sup> Many colleges and universities also have required students to take algebra-intensive general education courses, such as college algebra or pre-calculus, as a condition of graduation.

Such practices increasingly are coming into question because of low success rates and the fact that most majors do not require students to use advanced algebra skills.

## New Directions in Math Content

Equity objectives require content decisions that are designed to prepare students for future success, rather than to select who gets the opportunity to succeed. In recent years, math requirements for college readiness and completion have begun to shift and broaden. Increasingly, the math education community is recognizing that there are rigorous branches of math outside of the traditional pathway. Leading math associations have pressed for new options to ensure that students remain engaged in math and become quantitatively literate in ways that are relevant in their lives and careers.<sup>45</sup>

**K12.** The Common Core State Standards endorsed a move toward an **“integrated” curriculum** that emphasizes connections between algebra and geometry content and is similar to that used in countries with strong math performance. The standards also support efforts to modernize the curriculum with the addition of content, including **statistics and probability, and mathematics modeling**. Perhaps most importantly, the standards also prioritize a set of **“mathematical practices”** that detail the math literacy skills that students ideally should learn, regardless of specific content.<sup>46</sup> (SEE BOX, *Common Core State Standards - Standards for Mathematical Practice*, p. 14) However, these innovations have yet to be implemented widely.<sup>47</sup>

**Higher Ed.** Spearheaded by Treisman's Charles A. Dana Center and other organizations, at least 20 state higher education systems have moved to a policy of offering **diversified math pathways**. They are expanding offerings such as statistics,

<sup>42</sup> Loveless, 2013

<sup>43</sup> Dan Kennedy, as cited by Dan Teague, 2016, p. 30

<sup>44</sup> Hodara, 2011

<sup>45</sup> Burdman, Booth, Thorn, Bahr, McNaughton, & Jackson, 2018

<sup>46</sup> California Department of Education, 2015

<sup>47</sup> NCTM, 2018

quantitative reasoning, and mathematics modeling as general education courses. Rather than expect all students to take the traditional algebra-to-calculus sequence, this approach supports aligning students' math requirements with their intended areas of study.<sup>48</sup> Another rapidly expanding pathway is data science: When UC Berkeley started a course in 2015, it became the fastest growing class in recorded campus history.<sup>49</sup> Similarly, some medical schools, including Stanford's, have begun to stress statistics over calculus in the admissions process.<sup>50</sup> (SEE BOX, *Positive Results of Diversified Math Pathways*, on this page)

**Algebra 2.** The thorniest policy issue is the role of Algebra 2, a course that includes topics such as linear equations, polynomials, quadratic equations, and functions. A stepping stone to the calculus content needed in many STEM fields, it has become the “default eye of the needle” because of its frequent use as a gatekeeper to signal whether a student can pass a grueling course or merits admission to a selective college.<sup>51</sup> However, even math teachers acknowledge that much of the content is outdated, incoherent, and not terribly relevant, even for students going into STEM fields.<sup>52</sup>

While retaining much Algebra 2 content for high school students, the Common Core standards shift the emphasis to understanding algebraic concepts and relationships through modeling and reasoning rather than performing routine procedures.<sup>53</sup>

The move to diversified pathways has many postsecondary institutions abandoning policies that required Algebra 2 remedial courses as a prerequisite for any college-level math course. In 2014, the American Mathematical Association of

Two-Year Colleges (AMATYC) adopted a position that “courses other than intermediate algebra can adequately prepare students for courses of study that do not lead to calculus.”<sup>54</sup> Many state higher education systems, including CSU, have followed suit when it comes to remedial prerequisites, though Algebra 2 remains an admissions requirement for many public universities.

## Positive Results of Diversified Math Pathways

Research on initiatives such as the **Carnegie Math Pathways** and the **California Acceleration Project** has shown that under diversified pathways, students are three to four times as likely to complete a college-level math course, usually in less time, as in traditional remedial sequences.<sup>55</sup>

An experiment at the **City University of New York** showed that students who skipped remedial algebra courses and went directly into introductory statistics outperformed similarly-prepared students who took remedial algebra. Within two years, the statistics students had completed more credits and expressed more positive attitudes about math. Plus, they were more likely to pass advanced mathematics courses, directly challenging the assumption that completion of remedial algebra is necessary to ensure success in math and STEM courses.<sup>56</sup>

<sup>48</sup> Burdman et al., 2018

<sup>49</sup> Burdman, 2018

<sup>50</sup> Burdman, 2015a

<sup>51</sup> Daro, 2008, p. 3; Rosenstein & Ahluwalia, 2016

<sup>52</sup> Daro, 2008; NCTM, 2018

<sup>53</sup> California Department of Education, 2015

<sup>54</sup> American Mathematical Association of Two-Year Colleges, 2014

<sup>55</sup> Burdman et al., 2018; Marshall & Leahy, 2018

<sup>56</sup> Logue, Watanabe-Rose, & Douglas, 2017; Logue, 2018

## Instruction

If content is instrumental in determining which math skills students ultimately develop, instruction is equally central in helping students, especially at early ages, to develop quantitative literacy, whatever the specific content.

Math classroom experiences can dramatically affect students' ability to succeed. Observers of U.S. classrooms have noted a misplaced emphasis on speed, performance, giftedness, test scores, rules, and procedures, as opposed to learning, depth, context, thought, values, and creativity.<sup>57</sup> Rooted in misconceptions about math learning, such approaches can have the perverse effect of undermining students' self-efficacy—likely exacerbating the phenomenon of math anxiety—without enhancing achievement. Such an emphasis, notes Berry,

creates an environment where students' mathematical reasoning goes unexamined and unvalued; consequently, little is known about how they make sense of mathematics, how they use their mathematical understanding in developing solutions, and why their solutions do or do not make sense.<sup>58</sup>

For students who are already marginalized within the education system, such approaches only compound the barriers they face. And the fact that the math teaching force itself is predominantly white intensifies the difficulty for students of color, in particular, to see themselves as math learners.<sup>59</sup> “The gatekeeping role of mathematics contributes to the lack of diversity in the mathematics education workforce,” according to leading math associations.<sup>60</sup> Current shortages of math teachers exacerbate the problem.

## Common Core State Standards – Standards for Mathematical Practice<sup>61</sup>

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Attend to precision.
6. Use appropriate tools strategically.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## New Directions in Math Instruction

Changing the assumptions held by teachers is central to rethinking math instruction. Berry advocates positioning students as mathematically competent by “promoting and valuing students' participation in mathematical discourse—sharing their reasoning; creating, critiquing, and revising arguments; and engaging in collaborations aimed at making sense of and using mathematical ideas.”<sup>62</sup>

The Common Core “math practices” encourage teaching of the type Berry describes and illustrate that mathematics is more than memorizing math facts, running procedures, and getting the right answer. For students to have a “growth mindset” and believe in themselves requires instruction and curriculum grounded in the assumption that all students can learn advanced mathematics.<sup>63</sup>

<sup>57</sup> Boaler, 2016; Gutierrez, 2018

<sup>58</sup> Berry, n.d.

<sup>59</sup> See Martin, 2009, and Sterns et al., 2016

<sup>60</sup> National Council of Supervisors of Mathematics & TODOS: Mathematics for All, 2016, p. 3

<sup>61</sup> Common Core State Standards Initiative, n.d.

<sup>62</sup> Berry, n.d.

<sup>63</sup> Singh, July 2018; Gutierrez, 2012



Instructional practices that are equitable, culturally responsive, and provide opportunities for students to see themselves in the curriculum and engage in collaborative, inquiry-based work are key elements in what Boaler calls “mathematical democratization,” Gutierrez promotes as “re-humanizing mathematics,” and Bustillos describes as a “pedagogy of care.”<sup>64</sup>

Using mathematics to model real-life problems, for example, has been promoted as a strategy for improving math learning: It helps develop mathematical processes, promotes interdisciplinary thinking, and builds teamwork and communication skills.<sup>65</sup> As a 2016 report explains,

Mathematical modeling should be taught at every stage of a student’s mathematical education. After all, why does society give us so much time to teach mathematics? In part, it is because mathematics is important for its own sake, but mostly because mathematics is important in

dealing with the rest of the world. Certainly mathematics will help students as they move on through school and into the world of work. But it can and should help them in their daily lives and as informed citizens.<sup>66</sup>

Teacher recruitment, training, and professional learning with an eye to social justice are also essential to ensuring math learning.<sup>67</sup> Leading professional associations recommend increasing “recruitment and retention of mathematics teachers and leaders from historically marginalized groups,” and requiring “professional development opportunities that focus on social, cultural, linguistic, contextual, and cognitive facets” of math learning. Who teaches math to whom also matters—in addition to growing and diversifying the math teaching force in general, the NCTM has advocated ensuring that teachers are not tracked (so that, for example, the most experienced teachers aren’t concentrated in upper-level math courses).<sup>68</sup>

## Examples of Equity-Focused Pedagogy

The **Algebra Project**, founded by civil rights leader Bob Moses, recognized more than three decades ago the disparities in math opportunities. Moses created a culturally-sensitive professional development program for teachers to help more underserved students become mathematically prepared for college.<sup>69</sup>

**Mathematics for social justice** courses, as conceptualized by math educators including Eric Gutstein, approach mathematics as a tool for understanding and critiquing social issues, with

an aim to provide a relevant context in which students can build math skills.<sup>70</sup>

**Social-emotional learning strategies** are a feature of many community college math pathways initiatives. These approaches have emphasized using recommendations from learning science to address students’ social-emotional needs and to support them in becoming effective learners. Such strategies go by different names: Carnegie Math Pathways calls its approach “productive persistence.”<sup>71</sup> The California Acceleration Project talks about “intentional support for students’ affective needs.”<sup>72</sup>

<sup>64</sup> Boaler, 2013; Gutierrez, 2018; Bustillos, forthcoming

<sup>65</sup> Consortium for Mathematics and its Applications (COMAP) & Society for Industrial and Applied Mathematics (SIAM), 2016

<sup>66</sup> COMAP & SIAM, 2016

<sup>67</sup> NCSM & TODOS, 2016, p. 4

<sup>68</sup> NCTM, 2018

<sup>69</sup> Moses & Cobb, 2002

<sup>70</sup> Martin, 2003

<sup>71</sup> Silva & White, 2013

<sup>72</sup> Henson, Huntsman, Herr, & Shell, 2017

# Assessment

Test content and type play a key role in the high-stakes nature of math education. Tests are routinely used to rank and sort students, whether for course placement, scholarships, or admission to colleges or “gifted and talented” schools. Timed tests also contribute to the overemphasis on speed in mathematics. Lastly, many tests, especially multiple-choice tests, provide exposure to mathematics through the myopic lens of right and wrong, neglecting the ways in which math can be generative, creative, or interpretive. Noted Uri Treisman in his NCTM address:

When you visit most math classrooms, it's like you're in a Kafkaesque universe of these degraded social worlds where children are filling in bubbles rather than connecting the dots. It's driven by a compliance mentality on tests that are worthy neither of our children nor of the discipline they purport to reflect.<sup>73</sup>

In addition, high-stakes tests, such as the SAT, have been shown to have a disparate impact on underrepresented students, low-income students, and students whose parents did not attend college. A recent study of the SAT in California found that this impact has only been increasing.<sup>74</sup> A disadvantage has also been shown for women in math.<sup>75</sup>

The SAT and ACT also have been recruited for purposes they weren't designed to fulfill, such as postsecondary placement and high school accountability. Because these tests are norm-referenced, which means they are scored on a bell curve, they are not designed for measuring proficiency. Rather, small differences in test scores with marginal validity can determine students'

opportunities in ways that disadvantage low-income and underrepresented applicants, even when their achievement is improving. Criterion-referenced tests (of which the Smarter Balanced assessment is an example) are more appropriate for these purposes, according to testing experts.<sup>76</sup>

But even tests that are designed specifically for remedial placement purposes have been found to be weak predictors of students' ability to pass a college-level math course.<sup>77</sup> Research also has revealed that such tests may not be well-aligned with students' high school curriculum. In addition, they tend to emphasize algebra content, even though many students take non-algebra-based general education courses, such as statistics.<sup>78</sup>

## New Directions in Math Assessment

Based on research showing the numerous drawbacks of standardized tests, some higher education institutions are eschewing their use when it comes to placement and admissions. Many public colleges and universities are **de-emphasizing placement tests** to determine students' remedial needs, placing greater reliance on high school records, which are more valid predictors of college performance. Some universities, including elite ones like the University of Chicago, have **eliminated the SAT and ACT in admissions**.

Besides summative assessments, which are used for evaluating students for admissions and placement (or for state accountability purposes), there is a need for lower-stakes ways of measuring

<sup>73</sup> Treisman, 2013, 30:10

<sup>74</sup> Geiser, 2017

<sup>75</sup> Niederle & Vesterlund, 2010

<sup>76</sup> Atkinson & Geiser, 2009

<sup>77</sup> Scott-Clayton, 2012

<sup>78</sup> Burdman, 2015c

student progress to advance learning. **Formative assessment** is used to inform students and teachers of students' needs in order to improve mastery of the material, rather than to penalize or rank students. While it could be a test or quiz, formative assessment might be any activity teachers undertake to probe students' understanding of the material with a goal of using it to tailor their teaching.

The NCSM notes that such a feedback cycle helps teachers adjust their instruction and “can result in students reflecting and rethinking their mathematics, while increasing their effort and motivation.”<sup>79</sup> It has also been said that, because formative assessment has the “flexibility to incorporate attention to context, it can more easily address issues of cultural validity.”<sup>80</sup>

There is also growing support in education for performance assessments, which aim to offer more authentic evaluation of student skills through, for example, a research investigation, capstone project, or open-ended real-world problem. Such assessments are said to “have promise for better reflecting the achievements and potential of historically underserved students, responding to concerns raised by many stakeholders about racial and socioeconomic gaps in standardized test scores,” as well as to be “more valid measures of higher order thinking and performance abilities” than multiple-choice tests.<sup>81</sup> Such assessments can be used for summative or formative purposes.

## The Limits of Traditional Summative Assessments

Summative assessments are typically used to rank or evaluate students, such as for college admissions and placement. However, research has highlighted the limits of the tests traditionally used for these purposes:

**Admissions Tests.** Research on admissions tests such as the SAT and ACT has documented a growing correlation between race and test scores over the past 25 years, according to a recent study. “Rather than declining in salience, race has become more influential than either family income or parental education in accounting for test score differences,” the author wrote.<sup>82</sup> One explanation is that these admissions tests are “norm-referenced,” which

means they are scored on a bell curve and thus designed to differentiate among students for selection purposes, rather than to measure proficiency.

**Placement Tests.** Over the last six years, numerous studies on remedial placement testing have made clear that traditional tests under-place a significant proportion of students, and that high school grades are a much better predictor of students' ability to succeed in college math classes. Furthermore, placement tests explain just 6 percent of the variation in freshman math grades.<sup>83</sup> Furthermore, studies have shown that placement into remedial courses does not improve students' chances of success in college and, in some cases, students are worse off.<sup>84</sup>

<sup>79</sup> National Council of Supervisors of Mathematics & Association of Mathematics Teacher Educators, 2013, p. 2

<sup>80</sup> Trumbull & Lash, 2013

<sup>81</sup> Guha, Wagner, Darling-Hammond, Taylor, & Curtis, 2018, p. v, p. 4

<sup>82</sup> Geiser, 2017, p. 15

<sup>83</sup> Scott-Clayton, 2012

<sup>84</sup> Community College Research Center, 2014

## New Approaches to Assessment

**TODOS:** Mathematics for All (an organization focused on social justice in mathematics) and the National Council of Supervisors of Mathematics have called for “fair and holistic assessment systems for students and teachers of mathematics that provide productive and timely information on learning, and are free from high stakes pressure, static labeling of students and schools, and arbitrary sanctions.”<sup>85</sup>

**Assessment for Learning**, developed in Britain, is a research-based system for providing feedback to students about their mathematics learning and a prototype for formative assessment in math. The vast majority of studies of this and other forms of formative assessment have found positive, though generally modest, results. Formative assessment is as much about instruction as it is about testing: One of

the greatest challenges in effective formative assessment is that even teachers who correctly analyze evidence of student learning may not know which instructional steps may be needed to eliminate learning gaps.<sup>86</sup>

The **California Performance Assessment Collaborative** works in over 40 schools across the state to advance the use of competency-based approaches for high school graduation to support deeper learning. The system is designed to “take into account the needs of the most underserved students.”<sup>87</sup> Pasadena Unified School District, for example, has adopted a “graduate portfolio and defense requirement” in which students demonstrate their proficiency in four Cs (critical thinking, creativity, communication, and collaboration). Through this process, students develop an online portfolio and then present their work to a panel of judges.

## Readiness Policies & Structures

Math coursework and tests are part of the currency that determines students’ readiness for future opportunities. While test scores are just a snapshot in time, coursework involves *how many* and *which* math courses students take—and *when* they take them—as well as how they perform. Together, these factors can be used to signal readiness for taking the next class, earning a degree, or transitioning to the next institution. Readiness policies are salient anywhere students’ paths may diverge. And they are often most opaque at points of transition—including between middle school and high school, high school and college, and community colleges and

four-year universities: Here, they involve numerous policies across multiple education sectors that don’t necessarily align with each other. These include:

- high school graduation requirements, as well as policies on course placement, tracking, and acceleration; and
- college policies on admissions, remedial placement, general education requirements, math requirements for specific majors, as well as requirements for transferring from two-year to four-year institutions.

<sup>85</sup> NCSM & TODOS, p. 4

<sup>86</sup> Trumbull & Lash, 2013

<sup>87</sup> Guha et al., 2018, p. 12

Interactions among these policies determine what pathways are available to which students, and the pathways needed for students to progress toward various educational goals. The effects of these policies can also be mediated by other factors, such as teaching practices, popular perceptions about math pathways, and student choice.

For high school graduation, 17 states require four years of math. Most others require three. The Common Core State Standards include three years of high school math, while recommending four. A few states, including California, require just two years of math for high school graduation, although about two-thirds of California school districts have additional requirements.<sup>88</sup> However, as in most states, California public universities require three or more years, including Algebra 2, for admission.

This wide variation in policies reflects competing priorities in determining math requirements. Fewer requirements in high school may be conducive to higher graduation rates, while more requirements could help increase admission to selective colleges.<sup>89</sup> There has been particular ambivalence around Algebra 2, with some states recently adding it as a high school graduation requirement, even as others have eliminated it.<sup>90</sup>

Further tension exists around questions of tracking and acceleration. Despite the widespread impression that tracking is necessary because of divergent student abilities, Boaler notes that the countries with the strongest math performance “are those that group by ability the *latest and the least*.”<sup>91</sup> Yet, strong pressure exists for schools to allow some students to advance in mathematics by accelerating through the curriculum. The reasons range from a demand for greater challenge to the drive to make

students’ college applications more competitive. This pressure has resulted in a “race to calculus” that is prevalent in U.S. high schools.<sup>92</sup>

Determining students’ pathways through mathematics is a high-stakes practice, and middle school is a high-stakes time. Middle school math courses can influence college access (in terms of positioning students to reach the advanced high school math that may open doors to selective colleges), as well as college readiness (in terms of whether students can avoid remedial math in college).

In recent years, evidence has shown that the benefits of starting algebra early are less clear than originally assumed: Pressure for students to start Algebra 1 in seventh or eighth grade to ensure they can reach calculus during high school can diminish overall learning, with key concepts glossed over.<sup>93</sup> This drive to accelerate students in middle school can exacerbate inequities and contribute to tracking.<sup>94</sup> Either some students have the opportunity to accelerate, leaving others behind, or all students take



<sup>88</sup> Gao, 2016

<sup>89</sup> Gao, 2016

<sup>90</sup> Weissert, 2014

<sup>91</sup> Boaler, 2016, p. 112

<sup>92</sup> Rosenstein & Ahluwalia, 2016

<sup>93</sup> Loveless, 2013,

<sup>94</sup> Champion & Mesa, 2016; Loveless, 2013, part 2

algebra prematurely, and large numbers of students fail.<sup>95</sup> Javier Cabral (SEE BOX, *The Aftertaste of Math*, on p. 6) appears to be an example.

The emphasis on calculus pathway courses is also reflected in higher education's traditional approach to readiness. However, as mentioned earlier, many public postsecondary systems have been moving away from readiness policies that required all students to demonstrate proficiency in Algebra 2, regardless of their intended programs of study. Colleges and even professional schools are now placing more emphasis on statistics and other types of quantitative reasoning, especially for students not pursuing STEM fields. Yet, because of high demand for admission to selective institutions, as well as pressure to maintain traditional notions of rigor, it is not yet clear whether changes in admissions policies will ensue.

## New Directions in Readiness Policies & Structures

In recent years, education leaders and policy makers have increasingly been rethinking high school math pathways. Attention to ensuring appropriate course-taking has centered on the transitions into and out of high school:

1. middle school acceleration and ninth grade placement, which typically determine students' high school math trajectories, and
2. senior year math courses, especially "transition" courses designed to ensure readiness for college-level math courses and to prevent students from needing remedial math.

**Ninth grade placement and acceleration.** In California, the discovery that African American and Latino/Latina ninth graders were more likely than their

white and Asian American classmates with similar performance to be required to repeat Algebra 1 resulted in 2015 **placement legislation**. Intended to make the placement process fairer and more transparent, the law requires districts to use multiple measures of student performance and to improve the consistency of placement policies for students transitioning to high school.

But research had shown that increasing Algebra 1 enrollment in 8th grade was tied to higher failure rates.<sup>96</sup> To address this, some school districts have reversed course, delaying acceleration until high school, when more students are ready for Algebra 1. Plus, the Common Core eighth grade math course is considered essential preparation for high school, thus harder to skip.<sup>97</sup> San Francisco's school district pursued a **delayed-acceleration policy** and experienced a dramatic reduction in the proportion of students repeating Algebra 1.<sup>98</sup> As part of the policy, the district offered several routes for students to accelerate during high school—including doubling up on courses in 10th grade and taking summer classes. As a result, the proportion of students ready for AP Statistics or AP Calculus has also grown.<sup>99</sup> Still, the change has been controversial among parents who expect their students to start algebra before high school, as peers in many other districts still do.

While not going as far as endorsing such delays, the NCTM has advised caution around acceleration. In its recent report, it assailed the race to calculus as "often misguided," advocated that "critical concepts" not be "rushed or skipped," and urged that acceleration "be open to a wide range of students...not just to those who are identified through traditional assessment instruments."<sup>100</sup>

Higher education institutions also have begun sending signals to high schools that, while math skills are important, getting ahead in the traditional algebra-to-

<sup>95</sup> Loveless, 2013

<sup>96</sup> Loveless, 2013

<sup>97</sup> Daro, 2014

<sup>98</sup> Sawchuk, 2018; Ryan, Barnes, & Torres, 2018

<sup>99</sup> Sawchuk, 2018, Ryan et al., 2018

<sup>100</sup> NCTM, 2018, pp. 19-21

calculus sequence is not necessary for all students. In 2016, for example, the UC system's admissions board issued a statement noting that calculus is not required for admission.<sup>101</sup> It followed an earlier statement from the NCTM and MAA that also frowned on high school calculus.<sup>102</sup>

**Senior year math courses.** Twelfth grade has been seen as an opportunity to ensure students are mathematically prepared for college. New offerings are also a vehicle for high schools to align with the diversified pathways movement, as well as to provide off-ramps from the race to calculus for students

## Emerging Readiness Strategies

In very different ways, K-12 districts and higher education institutions are pursuing strategies to advance equitable outcomes in mathematics by eliminating traditional tracking and gate-keeping structures.

**Delayed Acceleration.** In 2014, San Francisco Unified School District took a hard look at its policy to place all eighth graders into algebra. Though the policy was intended to level the playing field and give more students access to higher-level math, it wasn't, in fact, achieving that goal: Fewer than half of eighth graders were testing proficient in Algebra 1 and less than one third of sophomores were proficient in Algebra 2.

The district adopted a policy to delay acceleration until high school, ensure all students take Common Core eighth grade math (which includes some elementary algebra), and allow all students to accelerate after taking Algebra 1 in ninth grade. Four years later, the district has found that only 8 percent of students need to repeat Algebra 1, compared to 40 percent under the prior policy. At the same time, acceleration opportunities in students' sophomore and junior years led to a 30 percent increase in the number of students on track to take an AP course in their senior year.<sup>103</sup>

## Postsecondary Corequisite Strategies.

The poor track record of developmental math sequences led community college educators to develop corequisite strategies for students considered unready for college-level math courses. Rather than assign these students to remedial math, this approach involves placing the students in college-level courses with concurrent support.<sup>104</sup> While initially designed for students just below the cut-off on placement tests, it has more recently been used for a broader set of students, with success.

Tennessee's higher education system pioneered the statewide use of the corequisite model for all students. During the first full year of implementation, students there experienced a fourfold improvement in math completion, with underrepresented minority students seeing a sixfold improvement.<sup>105</sup>

California's community colleges are just beginning to implement corequisite strategies statewide, but a recent study of two early adopter colleges found promising results: The average proportion of underrepresented students who completed a general education math course in their first year was more than double the statewide average. These schools also showed dramatically lower equity gaps in math completion.<sup>106</sup>

<sup>101</sup> University of California Board of Admissions and Relations with Schools

<sup>102</sup> Teague, 2016

<sup>103</sup> Ryan et al., 2018

<sup>104</sup> Rodriguez, Cuellar Mejia, & Johnson, 2018

<sup>105</sup> Tennessee Board of Regents, n.d.

<sup>106</sup> Rodriguez et al., 2018. Also see Henson, Huntsman, Hern, & Snell, 2017

not pursuing algebra-intensive fields. This strategy builds on CSU's pioneering initiative, in which students' 11th grade test results could indicate their expected readiness for college-level math courses, with successful completion of an optional **senior year readiness course** to guarantee placement in college-level math.<sup>107</sup>

As part of that initiative, CSU began working with high schools almost two decades ago to design a successful senior year English course, but only recently have the state's education systems focused on math in the senior year:

- In 2016, CSU faculty proposed requiring a **fourth year of math for admission**, but stipulated that non-algebraically intensive courses such as statistics, computer science, and financial literacy could be acceptable.<sup>108</sup>
- Also that year, the state of California allocated \$6.4 million for professional development efforts to expand the **range of senior year math courses** designed to ensure college readiness.

Amid ambivalence around Algebra 2, pressure has arisen for some of these new, more innovative

courses to be offered to juniors in lieu of Algebra 2. The UC system has even allowed a new high school data science course to count as an Algebra 2 course for admission purposes.<sup>109</sup> Such trends are in sync an NCTM position: As part of its recommendation that all students take four years of high school math, NCTM suggests eliminating some "obsolete legacy content, especially from Algebra 2."<sup>110</sup> The result would be a streamlined sequence of two or two-and-a-half years (vs. the current three), followed by divergent paths of students' choosing.<sup>111</sup>

**Promising postsecondary pathways.** At the college level, one approach to **eliminating arbitrary prerequisites** has been to offer multiple general education math courses, place most or all students into college-level courses, and provide the support they need to succeed there. The support may take various forms—including tutoring and other supports scaffolded into the courses. An increasingly common strategy is **corequisite courses**, in which students are supported to succeed in college-level courses rather than required to take remedial classes.<sup>112</sup> Evidence shows that this combination of approaches has the potential to reduce equity gaps.<sup>113</sup> (SEE BOX, *Emerging Readiness Strategies*, on p. 20)



<sup>107</sup> Burdman, 2015c

<sup>108</sup> Academic Senate of the California State University Quantitative Reasoning Task Force, 2016

<sup>109</sup> Burdman, 2018

<sup>110</sup> NCTM, 2018, p. 5

<sup>111</sup> NCTM, 2018

<sup>112</sup> California Acceleration Project, 2018

<sup>113</sup> Burdman et al., 2018; Marshall & Leahy, 2018b



# Toward a New Mathematics of Opportunity

“All students deserve the chance to prepare for college,” wrote Phil Daro, one of the authors of the Common Core State Standards in Mathematics, a decade ago. “Furthermore, as children they deserve more than the chance; they deserve our determination to overcome any damage done to what they expect for themselves. They deserve pathways to college designed as preparation, not as obstacle courses more appropriate for selection than preparation.”<sup>114</sup>

Policies that emphasize pedigree over preparation reinforce inequities and subtly, but decisively, alter the purpose of mathematics. Education equity calls for new ways of preparing students and assessing learning so students don’t just pass courses, but actually cultivate mathematical literacy for success in college and in life. Achieving this goal requires re-conceptualizing the role of math in opportunity structures and can entail change across the four dimensions discussed above.

New postsecondary math pathways and placement mechanisms have shown great promise for increasing equity in college attainment. Rather than face arbitrary requirements, students should be able to pursue pathways relevant to their interests, once they have learned core content.<sup>115</sup> However, given historically unequal access to advanced math courses, new non-algebra-based pathways mustn’t be used to divert women and students of color from STEM fields. And all students must be able to take rigorous courses that teach higher-order skills and align with college success.

Diversified pathways at the college level call for renewed scrutiny of high school math pathways. Vigilance is required to ensure that reforms don’t unwittingly stifle opportunity: Despite well-founded reservations about the race to calculus, to the extent that the course remains a gatekeeper, limiting access to it runs counter to equity. It is true that forcing all students to take advantage of an “opportunity” that many may not want or need is an ineffective way of removing obstacles for those students who do seek the opportunity. Yet, simply lowering expectations for all students doesn’t ensure that students can access the opportunities they seek.

New college pathways will have the greatest impact if high schools minimize tracking in math so that all students have the chance to prepare for college. Without appropriate design, accelerated high school pathways can undermine equity by promoting arbitrary sorting and tracking at the expense of alignment, relevance, and support for students. Policies that keep most students on a common trajectory, at least through the beginning of high school, are consistent with supporting positive outcomes across population groups.<sup>116</sup> The same is true of discouraging the “race to calculus” and broadening access to new, rigorous math pathways during high school and college.<sup>117</sup> But making

<sup>114</sup> Daro, 2008, p. 2

<sup>115</sup> NCTM (2018) analyzes this content, which it calls “essential concepts.”

<sup>116</sup> Daro, 2014; NCTM, 2018

<sup>117</sup> Daro, 2008; Marshall & Leahy, 2018b

such leaps can be risky for individual students or individual school districts unless others also disarm.

Math policies have the potential to support equity when:

- all math pathways are rigorous, rich, and relevant to college and/or career success;
- students' pathways are based on their goals and aspirations, rather than on perceived skill or outmoded placement practices;
- prerequisites are not arbitrary gatekeepers; and
- students receive the instruction and other support they need to be successful in their chosen pathway (including bridges between pathways for students whose goals change).<sup>118</sup>

As this report has illustrated, the new architects of math opportunity are re-building structures across all dimensions of math education—structures designed to support learning, enhance equity, and ensure students have the opportunity to encounter meaningful math while reaching their educational objectives. These new architects are working in schools and colleges, research institutes and professional development programs, advocacy organizations, and legislative offices.

Momentum is growing, but there's much to be done: It's critical to continuously evaluate new policies and practices to ensure they have a positive impact on student success and equity. In addition to grounding the disparate efforts in evidence, aligning them in a common vision is necessary in order to eliminate systemic impediments in educational pathways. A shared vision is also needed in order to win the support of faculty, students, and families around implementation of new, more effective strategies.

*The Mathematics of Opportunity* is intended to help fill this need for a common vision to ensure that math policies and practices enhance educational equity. The goals are to stimulate innovation, spark dialogue among math opportunity architects, and lay a cross-segmental foundation for future work. In the coming months, Just Equations will also publish a set of design principles and a research agenda for enhancing math equity.

Stunning progress has been made in recent years as a result of some of the initiatives mentioned within this report. That progress has greatly increased the potential for a new mathematics of opportunity to design more equitable routes through education systems. Growing that potential will allow mathematics to fulfill its intended purpose, while allowing more students a chance at college success.

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<sup>118</sup> Burdman et al., 2018; Getz, Ortiz, Hartzler, & Leahy, 2016



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