P-12 PRACTICE

APRIL 2018

CHECKING IN: Are Math Assignments Measuring Up?

The Education Trust

EQUITY IN MOTION

AS OUR DATA SHOW, WE AS EDUCATORS MUST DO MORE TO PROVIDE STUDENTS WITH QUALITY MATH ASSIGNMENTS THAT PROMOTE COGNITIVE CHALLENGE, BALANCE PROCEDURAL SKILLS AND FLUENCY WITH CONCEPTUAL UNDERSTANDING, PROVIDE OPPORTUNITIES TO COMMUNICATE MATHEMATICAL UNDERSTANDING, AND ENGAGE STUDENTS WITH OPPORTUNITIES FOR CHOICE AND RELEVANCE IN THEIR MATH CONTENT.

BY KEITH DYSARZ

INTRODUCTION

Students can do no better than the assignments they're given. That simple idea has been a driving force for The Education Trust's practice work since its inception in the 1990s. And it animates a new generation of that work today, which involves analyzing classroom assignments in the context of more rigorous common standards and calling teachers to action. This kind of painstaking analysis of the daily academic experiences of students provides hugely important insights into what teachers know and understand about college- and career-ready standards — and what those teachers believe students can do independently as a result of their teaching.

Our experience shows that classroom assignments strongly reflect the expectations that educators hold for their learners, providing a lens into the day-to-day experiences of students and their interaction with curricula. So when assignments are not aligned with grade-level standards — as we found with roughly 6 in 10 middle-grades literacy assignments in previous review[1] — or tap only the lowest levels of cognitive demand, we worry that students will never meet the standards that state leaders adopted with such fanfare six-plus years ago. And as an equity-focused organization, we are always troubled when assignments in high-poverty schools are less rigorous than those in low-poverty schools. Yes, low expectations take many forms, but classroom assignments are perhaps the most concrete manifestation of them all.

Building on our previous analysis of ELA, science, and social studies assignments, we now turn our attention to math. Nationally, student performance in math has been flat, and achievement gaps persist. And with a growing economy driven by industries in the science, technology, engineering, and math (STEM) fields, educators are tasked with instilling interest in and preparing students for college studies and careers that will supply these in-demand jobs. Now, more than ever, it is critical that we as educators reflect on the daily experiences of our students, and consider how we are preparing them to meet the demands of more rigorous math standards.

For this analysis, we reviewed over 1,800 middle-grades assignments from over 90 math courses from 12 middle schools in six districts across the country *(see sidebar: A Deeper Look At What We Did)*. We used a framework comprising five key areas: alignment to the Common Core, cognitive challenge, aspects of rigor, communicating mathematical understanding, and the potential for motivation and engagement *(see sidebar: Math Assignment Analysis Framework)*.

SO WHAT DID WE FIND?

- Alignment with at least a part of a grade- or course-appropriate math content standard was high: roughly three-fourths of assignments. Furthermore, given the high rate at which multiple standards were addressed within a single assignment, it seemed that teachers were grasping the interconnected nature of the math standards, which is promising.
- But underneath what seemed to be good news, there was news of a different sort: Most of the assignments were low-level. Although generally aligned, at least in some part, with grade-appropriate standards, the assignments tended to have low cognitive demand, over-emphasize procedural skills and fluency, and provide little opportunity for students to communicate their mathematical thinking. Moreover, this tendency was often worse in higher poverty schools.
- These results were not just isolated to small districts or in districts implementing decentralized curriculum practices. A fair amount of the assignments we analyzed came from districts that have invested significant time and financial resources into aligning curriculum materials to the Common Core. Nonetheless, the majority of assignments that their students received on a daily basis — six-plus years into the adoption of new math standards — remained far below the college- and career-ready level.

As we have seen in past standards movements, rigorous content standards do not automatically lead to cognitively demanding tasks that promote mathematical reasoning and problem-solving. Rather, the implementation of the standards and resulting decisions we as educators make about how students experience content are critically important. If we are going to meet the true intent of the math standards and ensure mathematical proficiency for all students, it is imperative that we give attention to the quality of assignments that we are putting in front of students on a daily basis.

Why Assignments?

Historically, assignment analysis has been a powerful lens for viewing the day-to-day experiences of students.[2] Assignments:

- Are a clear window into classroom practice.
- Represent what teachers know and understand about the college- and career-ready standards.
- Give insight into the school leader's and/or district's expectations for what and how to teach.
- Reflect what teachers believe students can do independently as a result of their teaching.
- Show how students interact with the curriculum.

Keith Dysarz is director of P-12 practice at The Education Trust.

WHAT WE FOUND

1

More than 70 percent of math assignments we reviewed were at least partially aligned with one or more grade- or course-appropriate Common Core math content standards.

And over two-thirds of these aligned tasks addressed multiple standards, either within the same domain or across domains in the same grade level.

2

Only 9 percent of assignments pushed student thinking to higher levels.

The overwhelming majority required low cognitive demand, with more than 9 out of 10 assignments limiting students to recalling a fact, performing a simple procedure, or applying basic knowledge to a skill or concept. This was even more pronounced in high-poverty schools, where only 6 percent of assignments were classified as requiring strategic or extended thinking, compared with 12 percent in low-poverty schools.

3

Assignments were more than twice as likely to focus on procedural skills and fluency (87 percent) compared with conceptual understanding (38 percent) or application of a mathematical concept (39 percent).

This imbalance meant less frequent exposure to assignments containing multiple representations, a critical indicator for developing conceptual understanding in mathematics. Only 39 percent of assignments incorporated varied types of mathematical representations.

4

Less than one-third (32 percent) of math assignments provided an opportunity for students to communicate their thinking or justify their responses.

The majority of assignments were answer-focused and did not ask students to defend or explain their thinking at any point within the task. Only 36 percent required students to write anything besides an answer, and 95 percent of assignments showed no opportunity for discussion.

5

Students were rarely given opportunities for choice in their assignments (3 percent), and only 2 percent of tasks provided some aspect of relevancy using real-world experiences.

Both choice and relevancy are critical motivating factors in helping to engage adolescents in mathematical content. Despite this importance, very few assignments went beyond superficial attempts to connect with real-world events or students' own personal experiences.

A DEEPER LOOK AT WHAT WE DID

School Sites and Participants

12 middle schools from 6 school districts (urban, suburban, and rural) across three states

Ten of the 12 schools were traditional middle schools (grades 6-8), one was a junior high school (grades 7-9), and one was an intermediate (grades 5-6).

Free and reduced-price lunch (FRL) rates ranged from 16 percent to 82 percent across the schools. We classified six schools with FRL rates higher than 65 percent as high-poverty in our data analysis. Student racial/ethnic populations were also different; students of color (African American and Hispanic students) ranged from 8 percent to 84 percent of the total population. The percentage of English learners also varied across schools (from less than 5 percent to 22 percent).

63 teachers from 91 math courses

Sixth-, seventh-, and eighth-grade teachers teaching math courses, ranging from math 6 through geometry.

Average number of assignments submitted per course = 20. The median number of assignments submitted per course = 19.

Assignment Collection

Assignments were defined as any in-school or out-of-school task that a student completed independently or with a group of peers. Assignments completed during teacher-led practice or assignments given by substitute teachers were not counted for the purpose of this study.

We collected all classroom assignments meeting our definition over the course of a two-week period from each of our participating teachers. Collecting all assignments over a consecutive two-week period allowed us to see the full range of assignments students received (e.g., from brief tasks like exit tickets to longer-term math projects) and provided evidence of students' opportunities to learn and the competencies they are typically asked to demonstrate. Two-thirds of the assignments were collected between February and March 2016, with the remaining one-third collected during winter 2015.

All assignments were given a unique identification number to ensure teacher, school, and district confidentiality.

Assignments Scored by the Numbers

Total number of math assignments submitted: **2,176**

Total number of math assignments scored: **1,853** (85%)

Assignments were not scored if they were incomplete or illegible. Additionally, lesson plans or other curriculum documents were not scored.

Assignments by Math Course

Math 6 / **30%**

Math 7 / **19%**

Compacted 7/8 / **11%**

Math 8 / **14%**

Pre-Algebra/Algebra Prep / **9%**

Algebra I / **13%**

Algebra II / **1%**

Geometry / **3%**

Type of Assignments

Short/Brief **= 46%**

1-2 Class Periods **= 53%**

Extended **= <1%**

Assignments by Honors/ Advanced Designation

Non-honors courses **= 87%**

Honors courses **= 13%**

MATH ASSIGNMENT ANALYSIS FRAMEWORK

ALIGNMENT TO THE COMMON CORE

A Common Core-aligned math assignment should fully reflect the depth of the grade-level cluster(s), grade-level content standard(s), or part(s) thereof to be considered aligned. Additionally, an aligned assignment should clearly articulate the task so that students can fully understand what is expected of them as defined by the standard(s).

ANALYSIS INDICATOR	PERCENTAGE OF ASSIGNMENTS
The assignment aligns to at least part of one grade- or course-appropriate Common Core math content standard.	73%
The assignment clearly articulates the task.	98%

COGNITIVE CHALLENGE

Our analysis utilizes Norman L. Webb's Depth of Knowledge Levels to assess cognitive challenge. Assignments at the strategic level (level 3) or extended thinking level (level 4) are considered to have high levels of cognitive demand.

ANALYSIS INDICATOR	PERCENTAGE OF ASSIGNMENTS
The assignment requires high levels of cognitive demand.	9%

ASPECTS OF RIGOR

Mathematical rigor is defined in the Common Core as having a "deep, authentic command of mathematical concepts" pursued through three aspects of rigor: conceptual understanding, procedural skills and fluency, and application.[3] Connected to these aspects of rigor, particularly conceptual understanding, is the use of varied mathematical representations.

ANALYSIS INDICATOR	PERCENTAGE OF ASSIGNMENTS
The assignment incorporates conceptual understanding.	38%
The assignment incorporates procedural skills & fluency.	87%
The assignment incorporates application.	39%
The assignment provides multiple representations of a concept and/or equation.	39%

COMMUNICATING MATHEMATICAL UNDERSTANDING

A core principle of mathematical understanding is the ability to communicate one's thinking using the language of mathematics. This incorporates Standards for Mathematical Practice (SMP) 3 and 6, which note that mathematically proficient students construct and respond to arguments, justify their conclusions, and communicate to others using precise language.[4] Opportunities for writing and discussion provide insight into student thinking, and are useful indicators to measure when analyzing math tasks.

ANALYSIS INDICATOR	PERCENTAGE OF ASSIGNMENTS
The assignment requires students to communicate their understanding using the language of mathematics.	32%
The assignment requires students to write short phrases, sentences, or one or more paragraphs.	36%
The assignment provides opportunity for informal or formal discussion.	5%

MOTIVATION AND ENGAGEMENT

Both curriculum and the design of instruction impact student attention, interest, motivation, and cognitive effort and must be considered in the design of assignments. Specifically, two key areas hold priority: choice and relevancy. Students should be given opportunities for choice in their tasks, with rigor maintained across all options. And assignments should be relevant by focusing on poignant topics, using real-world materials and experiences, and giving students the opportunity to make connections with their goals, interests, and values.

ANALYSIS INDICATOR	PERCENTAGE OF ASSIGNMENTS
Students have choice in the assignment in one of the following areas: content, product, process, or mathematical tool.	3%
The task is relevant. It focuses on a poignant topic, uses real-world materials, and/or gives students the freedom to make connections to their experiences, goals, interests, and values.	2%

A DEEPER LOOK AT WHAT WE FOUND

The proceeding sections further explore each of our five key findings, with a handful of annotated example assignments selected from among the more than 1,800 tasks that we reviewed. Throughout each of these sections, we deepen our analysis with additional data and examine how factors like a specific math course, honors/advanced designation, and rates of free- or reduced-price lunch (FRL, a proxy for poverty) play out in the types of math assignments that we received. Courses classified as honors or advanced were self-reported by the school or district. In this analysis of middle school math tasks, all assignments from algebra II and geometry were reported as honors, in addition to select sections of math 6, math 7, math 8, and algebra I.

ALIGNMENT TO THE COMMON CORE

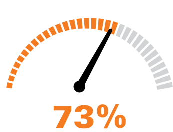
The number of middle-grades tasks aligned to the Common Core was high, with more than 70 percent of the assignments focusing on all or part of a grade- or course-appropriate math standard. In addition to overall alignment, we also looked at how frequently assignments addressed multiple standards within a grade or course. Over two-thirds of the aligned tasks addressed more than one standard, either within the same domain or across domains in the same grade level.

That these math assignments mostly aligned to the Common Core should not be overlooked, particularly given the substantial shift in focus that more rigorous standards call for in mathematics. Previous math standards typically required educators to cover lots of topics in a "mile-wide, inch-deep" curriculum approach, and we anticipated seeing remnants of this affecting the alignment in the current math assignments that students were receiving. But what we saw instead made us cautiously optimistic that teachers are embracing the deep and narrower philosophy called for in the Common Core — and truly using the standards to help focus instruction in the critical areas that have been identified at each grade level.

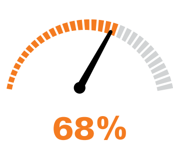
We would caution, however, that the aggregate rate of alignment for individual assignments can be somewhat misleading, as we discovered during our analysis. While assignments were considered aligned if they focused on grade-appropriate content of a grade-level cluster, standard, or part(s) thereof, we often saw teachers give two-week's worth of assignments that, when taken together, did not address all parts of the grade-level standard *(see Example 1)*.

We also observed a handful of instances in which a particular math standard was incorrectly executed across an entire school or district. In these assignments, our reviewers could see an attempt to address a particular standard, but in ways that clearly did not meet the standard's intent. In some cases, these assignments were replicated across multiple courses within a school, and even throughout the district, leading large numbers of students to experience content that entirely missed a standard's true target. This underscores the critical importance of thorough understanding by teachers and curriculum staff when it comes to the standards and the instructional shifts they demand.

Figure 1: Alignment to the Common Core in Math Assignments



Aligns to at least part of one grade- or course-appropriate Common Core math content standard



Addresses multiple standards within the same grade or course

EXAMPLE 1

MATH 6: PARTIAL, NEVER ALL

Grade 6 Math Standard:[5]

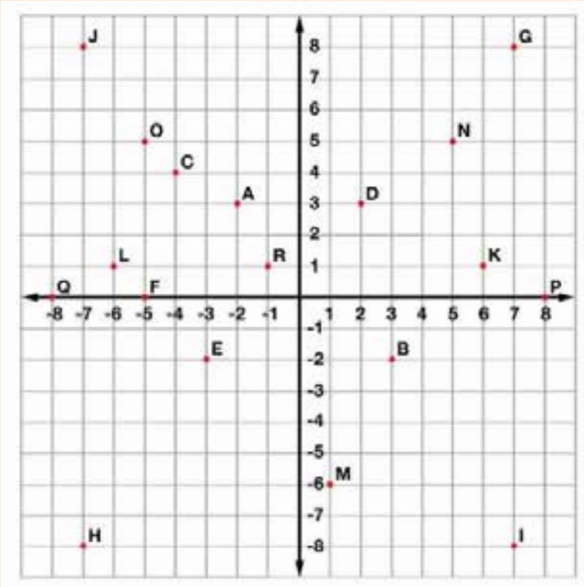
6.NS.C.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

ORDERED PAIRS



Write the ordered pair for each given point.

1. B ______ 2. A ______
3. L ______ 4. H ______

Tell what point is at each ordered pair.

5. (2, 3) ______ 6. (-7, 8) ______
7. (-5, 0) ______ 8. (6, 1) ______

Plot the following points on the coordinate grid.

9. S(3, 4) 10. T(3, -7)
11. U(-6, -2) 12. X(0, 4)

EXAMPLE 1
Math 6

In grade 6, students are expected to extend their knowledge of the coordinate plane by understanding the continuous nature of two-dimensional space using fractional increments. Specific to this standard, students should identify coordinates that include rational numbers as well as integers. Additionally, students should be able to identify points in all four quadrants and discuss the relationship between the signs of numbers in ordered pairs and their corresponding effect on the position in the coordinate plane.

In this example, and in all the subsequent assignments from this course, students practiced identifying the coordinate pairs for a given point and labeling points with a given set of coordinated pairs. This is representative of the tasks that were intended to meet standard 6.NS.C.6. However, no assignments – individually or taken as a group from this math 6 course, addressed all aspects of standard 6.NS.C.6 — parts a, b, and c — within our two-week collection period.

While this individual assignment did move beyond the grade 5 standard by incorporating all four quadrants, students were never asked to plot rational numbers, reason about the effects of signs on the position of the coordinates, or consider the relationship between coordinate signs and reflections across one or both axes during the two-weeks in which we gathered assignments.

COGNITIVE CHALLENGE

We found a very different story when we took a closer look at these assignments to analyze cognitive demand. Only 9 percent of tasks required high levels of cognitive demand that pushed student thinking to the strategic level. And when we disaggregated the data by school FRL rates, we found an even bigger disparity: In high-poverty schools, only 6 percent of assignments were classified as requiring strategic thinking, compared with 12 percent in more affluent schools. The majority of assignments demanded little of students, not moving them beyond recalling a fact, performing a simple procedure, or applying basic knowledge to a skill or concept. No assignments in the over 1,800 that we collected pushed students to extended thinking *(see Figure 2)*.

Given that we collected all tasks from teachers (including warm-ups, procedural practice, and exit tickets), we certainly would not expect *all* assignments to be cognitively demanding. We did, however, assume that the progression of a particular topic would unfold over a two-week period and allow us to see a rich distribution of tasks that promoted mathematical reasoning and problem-solving. Unfortunately, that was not the case. Teachers in more than three-fourths of the courses that we analyzed gave two or fewer cognitively demanding assignments within the two-week period, with students in 38 percent of courses never experiencing even a single task requiring strategic or extended thinking.

We were particularly alarmed by the extremely low number of challenging tasks in the pre-algebra/algebra-prep courses that eighth-graders took when they were not placed in algebra I. In these remedial courses, 97 percent of the assignments fell at the recall or basic application levels. While the students in pre-algebra/algebra-prep courses may have been unprepared for algebra I content, we question why they should be relegated to assignments any less rigorous. In these cases, the course content and instructional standards should change; opportunities to experience cognitively challenging math tasks should not. This becomes even more worrisome when you consider the intersection of student demographics and course access that we typically find in schools and districts across the country — specifically the disproportionate number of low-income students and students of color placed in lower level, remedial courses. Could this be yet another example of well-intentioned, but misguided, efforts to "catch students up" by slowing them down?

Not surprisingly, assignments with higher levels of cognitive demand were also much more likely to incorporate a number of other indicators on our framework, including opportunities to communicate mathematical understanding and develop conceptual understanding. And these high-demand assignments almost always took longer to complete, which makes us wonder about the impact of repetitive routines and formulaic structures that we often find in math classrooms. Could it be that we are seeing the results of such structures hinder our ability to get to more challenging math tasks that promote reasoning and problem-solving, and instead, implementing answer-focused tasks centered on the application of a formula or routine procedure?[6] Or should we be rethinking how we use warm-ups and exit tickets to promote higher levels of cognitive demand *(see Example 2)*? The warm-ups and exit tickets we saw in our analysis typically looked like Assignment A, though we questioned why individual prompts from Assignment B couldn't be used as stand-alone warm-ups or exit tickets in the same way, thereby creating brief tasks with high cognitive demand.

Figure 2: Cognitive Challenge of Assignments
Based on Norman L. Webb's Depth of Knowledge (DOK) Levels

32%

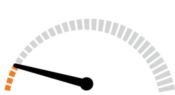
Recall and Reproduction

Recall a fact, term, principle, concept; perform a routine procedure or a simple algorithm; or apply a formula.

59%

Basic Application of Skills/Concepts

Use information, apply conceptual knowledge, select appropriate procedures for a task, complete two or more steps with decision points along the way, complete routine problems, organize/display data, or interpret/use sample data.



9%

Strategic Thinking

Requires reasoning or developing a plan or sequence of steps to approach the problem; requires some decision-making and justification; it's abstract, complex, or non-routine; and there is often more than one possible answer.

0%

Extended Thinking

An investigation or application to real world; requires time to research, problem-solve, and process multiple conditions of the problem or task; and requires non-routine manipulations across disciplines/content areas/multiple sources.

EXAMPLE 2

MATH 8 HONORS: A TALE OF TWO TASKS

Algebra Standard:[7]

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

ASSIGNMENT A

For each of the following problems, factor completely (you may have to use more than one type of factoring), and state for each step which type of factoring you are using. Label as "PRIME" if a polynomial cannot be factored.

1. $-x^{10} + 25$
2. $3x^4 - 12$
3. $x^2 - 8x + 15$
4. $10x^2 - 28x - 6$
5. $5x^3b - 3x^2b^2 + 15x^5b$
6. $x^4 + x^3 - 6x^2$
7. $x^{16} - 256$
8. $x^2 + 8x + 15$
9. $2x^2 + 12x - 32$
10. $25 - 10x + x^2$
11. $3x^2 + 3x - 90$
12. $x^8 - x^4$

ASSIGNMENT B

Create your own problems. Create expressions that can be factored according to the following criteria. Briefly explain the process you used to create your expression.

1. A quadratic trinomial that has a leading coefficient between 1 and 5. The trinomial should be factorable.
2. An expression that can be factored using the greatest common factoring first, then difference of squares factoring. The greatest common factor should be 5. Explain your process for determining your expression.
3. A quadratic trinomial with a leading coefficient of 1 that can first be factored using greatest common factoring. The greatest common factor should be 2x. Explain your process for determining your expression. (*Alternate wording*: An expression with the greatest common factor of 2x. When the greatest common factor is factored out, the remaining expression is a quadratic trinomial with a leading coefficient of 1. Explain your process for determining your expression.)
4. An expression that can first be factored using the greatest common factoring with a factor of $3x^2$. Explain why there are an infinite number of polynomial expressions that can satisfy the description.

EXAMPLE 2
Math 8 Honors

Assignments A and B are both aligned to the same algebra standard that focuses on seeing structure and producing equivalent forms of expressions (A.SSE.A.2). Both tasks also incorporate Math Practice Standard 7 — look for and make use of structure. And both assignments ask students to use their procedural knowledge of factoring, yet do so in distinct ways, ultimately leading to notably different levels of cognitive challenge.

Assignment A is a more conventional task found in algebra courses that asks students to select an appropriate procedure to factor different types of polynomial expressions – a routine algebraic practice that requires the basic application of a skill/concept (DOK Level 2).

Assignment B also requires students to use their knowledge of appropriate procedures, but does so in a way that requires strategic thinking (DOK Level 3) by having students create their own polynomials. To complete this task, students must develop a plan to approach the problems, make decisions about how to solve, and justify their solutions. Additionally, the prompts in Assignment B allow for more than one possible answer. This assignment also demonstrates how a procedural task can have high cognitive demand. Extension questions such as those posed in questions 2, 3, and 4 elevate the rigor from solely procedural to also include conceptual understanding about polynomial factoring by incorporating Math Practice Standard 3 — construct viable arguments and critique the reasoning of others.

Assignments A and B are both opportunities for students to meet the standards. However, only providing students with problems in Assignment A limits their opportunity to engage in cognitively demanding tasks.

ASPECTS OF RIGOR

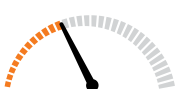
A central tenet of the Common Core math standards is the equal pursuit of rigor in three areas — conceptual understanding, procedural skills and fluency, and application — so that students can obtain an authentic command of mathematical concepts.[8] Taken together, these aspects of rigor allow students to develop a deep understanding of mathematical content, carry out procedures flexibly and accurately, and apply their knowledge in mathematical situations. Importantly, these three should be pursued with equal intensity.

However, in our analysis of math assignments, what we found was an over-emphasis on procedural skills and fluency compared with the other two aspects of rigor *(see Figure 3)*. Assignments were more than twice as likely to focus on procedural skills and fluency (87 percent) compared with conceptual understanding (38 percent) or application of a mathematical concept (39 percent). Though half of the tasks we reviewed contained two or more aspects of rigor within the same assignment, the other half focused solely on procedural fluency. And when multiple aspects of rigor were present in a single assignment, they were typically isolated as discrete sections in a particular order (e.g., a section of procedural problems at the beginning and problems involving application toward the end) rather than being integrated. Assignments like Example 3 were rare in our analysis.

Understandably, some of the middle-grades standards portrayed in assignments during our collection period may have lent themselves more clearly to a particular aspect of rigor. But even in the two-week window in which we collected assignments, it seemed extreme for the practice of procedural skills and fluency to be so disproportionately stressed. Given that the time students spend studying math is finite, this over-reliance on a particular aspect of rigor has the potential adverse effect of coming at the expense of others. If this same pattern is emphasized throughout the school year, how can students experience the balance in rigor called for in the standards? Does this not lead to a narrow, answer-focused perception of mathematics rather than the coherent body of knowledge it is intended to be?

Related to this, we also measured the use of multiple representations within each assignment, including contextual, visual, verbal, physical, and symbolic forms.[9] Thirty-nine percent of assignments prompted students to access, approach, or solve problems in more than one way using multiple representations. And logically, assignments incorporating multiple representations were more likely to focus on conceptual understanding and application of mathematical content rather than procedural skills and fluency. This opportunity to interact with varied representations in math tasks is critical for providing students multiple entry points into the same problem — a major implication in the equitable access of content in our math classrooms.

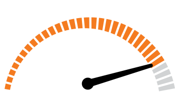
Figure 3: Aspects of Rigor in Assignments



38%

Conceptual Understanding

Students access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discreet procedures.



87%

Procedural Skills and Fluency

Students have speed and accuracy in calculation in order to have access to more complex concepts and procedures.

39%

Application

Students use math in situations that require mathematical knowledge.

EXAMPLE 3

MATH 8: A BALANCING ACT

Grade 8 Math Standard:[10]

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

SHHH … IT'S A SURPRISE!

Your family plans a surprise party for you. All your family and friends will be there. When you arrive, this is what you discover:

- 75 people are at the party.
- 12 are family.
- 23 are neither a friend nor family.
- 10 are both a friend and a family member.

Part A. Use the information above to answer the following questions:

- How many of your friends came to the party?
- How many of your family members came to the party?

Part B. Create a two-way table that displays the same data.

			Total
Total			

- If you were not given all of the data needed to complete a two-way table, can they be calculated? Explain how.
- Compare and contrast the information that is readily available in each representation.

Part C. Recreate your table below with the relative frequencies, based on the data given in Part B.

			Total
Total			

- How did you change from raw data to relative frequencies?
- If a guest is selected at random, what is the probability that the guest is neither a friend nor a relative?
- Give at least 3 statements that provide an interpretation of the association between variables based on your table.

EXAMPLE 3
Math 8

This example is an aligned assignment that integrates conceptual understanding, procedural skills, and application in a meaningful way. The task also shows how rigor can be enhanced by asking purposeful questions that can result in a deeper understanding of a mathematical concept.

Conceptual understanding can be found, as students:

- Translate between multiple representations of bivariate categorical data that include verbal descriptions with numbers and tables.
- Reason about which categories will make up the relative frequencies based on the data.
- Explain why and justify their thinking at multiple points throughout the task.

Procedural skills and fluency can be found, as students:

- Construct a two-way table to record frequencies of bivariate categorical data in part B.
- Calculate relative frequencies in order to complete the table in part C.
- Calculate probability in part C.

Application can be found, as students:

- Describe possible patterns of association for bivariate categorical data in a real-world context.
- Explain what the individual data summaries represent in terms of the context of the problem.
- Use the data to make generalizations or predictions of expected behavior.

It is important to remember that all three aspects of rigor do not always have to be presented together, just as they do not always have to be presented separately. Instructional balance among the three should be evident across a series of assignments and/or unit of study.

COMMUNICATING MATHEMATICAL UNDERSTANDING

The classroom tasks students receive directly impact their ability to cultivate the critical skills of reasoning, justification, and argumentation — all essential elements of learning to "do mathematics."[11] In the Common Core, we see these dominantly emphasized in at least two of the eight Standards of Mathematical Practice: SMP 3 (Construct viable arguments and critique the reasoning of others) and SMP 6 (Attend to precision).[12] Yet despite this attention within the standards, we found limited opportunity for students to engage in these important processes when completing their assignments. Just over one-third of the tasks we reviewed asked students to communicate their understanding using the language of mathematics. The majority of assignments were answer-focused and did not ask students to justify or explain their thinking at any point within the task.

Moreover, when it came to written explanations, only 36 percent required students to write *anything* besides an answer, with almost two-thirds requiring no writing or communication whatsoever *(see Figure 4)*. The 4 percent of assignments that asked students to write more than a few sentences almost always consisted of a single constructed response question at the end of an assignment. And while we saw a handful of assignments that stated the old math adage "show your work," this phrase often stood in isolation, as if to be obligatory, without further direction or indication of what was expected of students. Would an easy, but *intentional*, enhancement to that statement — asking students to *justify* or *explain* their answer — significantly improve the possibility for mathematical communication?

We also attempted to gauge the level of mathematical discourse by measuring opportunities for discussion evident within the task itself. Only 5 percent of assignments showed any opportunity for discussion, be it formal or informal *(see Figure 5)*. And though our analysis may not have captured the math conversations that took place within the lesson as a whole, we gave credit to assignments that referenced a previous or future discussion *(see Example 4)*. Still, the vast majority of tasks showed no sign of providing opportunities for students to discuss their thinking, create an argument, or critique the argument of others. Even if we assume that a greater number of discussions took place during the lesson that the assignment did not capture, it seems like a missed opportunity not to reinforce these, even if just in reference, within the classroom tasks that students received.

Like many of our other indicators, we also discovered interesting patterns when we looked at this data by math course and school poverty levels. Opportunities to communicate mathematical understanding were significantly higher in the advanced courses of compacted 7/8, algebra I, algebra II, and geometry compared with other non-advanced courses. And in schools with lower FRL rates, the number of assignments that required justification or argumentation was higher (38 percent) compared with tasks in high-poverty schools (26 percent).

Given these results, we wonder: Are math classrooms continuing to operate in a teacher-centered way, where teachers tell students the content they need to know and students rely on teachers to validate their thinking and responses?[13] And if so, what self-imposed barriers are we creating that may be preventing students from meeting the demands of college- and career-ready standards, particularly for low-income students and students of color for whom this type of teacher-centered instruction is more common?

Communicating Mathematical Understanding

- SMP 3: Construct Viable Arguments and Critique the Reasoning of Others
- SMP 6: Attend to Precision
- Mathematically proficient students construct and respond to arguments, justify their conclusions, and communicate to others using precise language.

Figure 4: Writing Demand of Assignments

63%	No writing or communication
32%	Short phrases up to 2 sentences
4%	Paragraph or more

Figure 5: Discussion Demand of Assignments

95%	No evidence of discussion
3%	Informal and/or brief discussion
2%	Formal and/or extended discussion

EXAMPLE 4

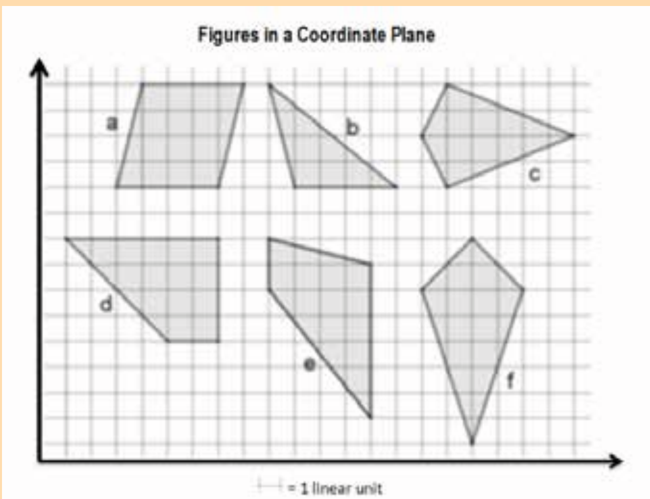
MATH 6: COMMUNICATING UNDERSTANDING

Grade 6 Math Standard:[14]

6.G.A.1 . Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

AREA OF QUADRILATERALS

Directions: Work on the exercises independently and then discuss your answers with your table. Be prepared to discuss the results as a class.



1. Find the area of figure **d** by using what you know about the area of triangles and rectangles.

2. Choose a quadrilateral from the figures above.

   a. Find the area of the quadrilateral using two different methods.

   b. Describe the methods used, and explain why they result in the same area.

3. Compare your methods and results with your partner. What is the same about your methods, and what is different?

4. Choose a different quadrilateral from the remaining figures. Find the area of the quadrilateral using two different methods. Discuss the advantages or disadvantages of each method.

EXAMPLE 4
Math 6

In this assignment, students find the areas of triangles and simple polygonal regions in the coordinate plane by composing into rectangles and decomposing into triangles and quadrilaterals (6.G.A.1). Students also use the coordinate plane as a tool to determine the area of figures with vertices at grid points.

This assignment demonstrates mathematical communication by asking students to think, talk, and write about their responses.

- The directions of this task convey the expectations of mathematical discourse.
- During the check-in with a partner and the whole class discussion, students have the opportunity to critique the reasoning of others (SMP 3) and communicate their understanding using the language of mathematics (SMP 6).
- In questions 3 and 4, students reflect on their learning, deepening their understanding of strategic approaches to finding area by writing to explain different mathematical processes that produce the same solution. The expected writing output is more than two sentences.

The expectations for mathematical communication in this assignment produce valuable information, allowing teachers to formatively assess learning and provide feedback based on student responses.

MOTIVATION AND ENGAGEMENT

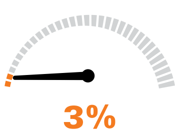
Much like our literacy analysis, a review of middle school math assignments showed little opportunity for choice and insufficient attempts at relevancy in the tasks that were given to students. Only 3 percent of assignments offered students choice in content, product, or process. And only 2 percent of assignments attempted to make the content relevant by focusing on a poignant topic, using real-world materials, or connecting with students' interests and values *(see Figure 6)*.

Many assignments attempted to connect with students using word problems that infused popular icons or familiar contexts (e.g., word problems about concert ticket sales). While real-life references and contexts can be helpful hooks, we do not consider them substitutes for meaningful connections with a student's experiences. Most often, the assignments that focused on a poignant topic or made relevant connections centered on data collection and analysis based on a student's personal interests *(see Example 5)*.

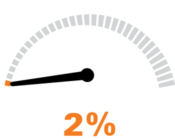
It is particularly troubling that opportunities for choice and relevancy in math tasks were so low, given what we know about best practices for engaging and motivating adolescents in mathematics. Undesirable math identities and attitudes often start at a young age and are reinforced in multiple ways, promoting a fixed mindset that certain students are innately better at math than others. Students come to see math as something that is beyond them, particularly when they are not given opportunities to connect it to their interests or experiences.[15] Providing engaging and relevant tasks that connect mathematics to students' experiences and backgrounds can help students see themselves as a "doer of math," rather than a passive spectator — something particularly important as we work to address access and equity issues in mathematics for historically underserved populations.[16]

The need to engage students in mathematics is even more critical, given frequently proclaimed phrases — "I'm not a math person" or "I'm never going to use <insert math topic>" — that all math teachers have heard from their students at one point or another in a given school year. The Common Core math standards provide a unique opportunity for educators to address these issues of math identity, inclusion, motivation, and engagement; so that all students experience mathematics in a way that prepares them for success in college and their careers. High-quality, relevant classroom assignments should be a critical tool for accomplishing this.

Figure 6: Choice and Relevancy



Students have choice in the assignment in one of the following areas: content, product, process, or mathematical tool.



The task is relevant. It focuses on a poignant topic, uses real-world materials, and/or gives students the freedom to make connections to their experiences, goals, interests, and values.

EXAMPLE 5

MATH 8: SEEING ONESELF IN MATHEMATICS CONTENT THROUGH RELEVANCY AND CHOICE

Grade 8 Math Standard:[17]

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

CREATING A CHANGE STORY

Directions: During our unit on linear and nonlinear functions, we have been analyzing graphs to describe relationships between two variables. In this culminating project, you will work as a group to create a story about a changing quantity over time. You will record your story and create a graph that models it. Your story should be about the change in a quantity (e.g., speed, height, length, volume, size, weight, distance to OR away from something) over time.

SETUP

1. Choose roles for each group member: director, lead actor, director of props, recorder, final graph creator, and presenter.
2. As a group, decide which quantity you want to measure over time. Choose a variable that can be shown on video changing over a 15-second time frame.

VIDEO

1. Create a video script for your lead actor to use (with props!) that portrays the variable's change in quantity over time. There must be at least two different actions (increasing, decreasing, constant, etc.).
2. Using your script, props, and an iPad, the recorder should record your 15-second change story, starring your group's lead actor.
3. Once the graphing story begins, the director should count 15 seconds out loud. (This will help once you start sketching your graph!)

GRAPH

4. When your video is complete, each member of your group should watch the video and graph the story on their own. Be sure to:
   a) decide on a graph title that accurately describes what is happening, and
   b) label your y-axis (you can choose the exact scale of your y-axis later).
5. Each group member should share their graph. Once all group members have presented, discuss the parts of each group member's graph that most accurately reflect the change story your group recorded. Be sure to ask your peers to justify and explain their thinking!
6. Based on this discussion, the final graph creator should sketch the group's final graph that will be presented.

PRESENT

7. The presenter, with support from the group, should present the team's graphing story to the class. Be sure to show your recorded video and describe what is happening as the video plays.
8. Ask classmates to sketch their own graph based on the video and description provided by the presenter.
9. Reveal the group's graph and have a brief discussion on similarities and differences between what the group created and what your classmates sketched.

EXAMPLE 5
Math 8

Assignments like this one were rare. This task not only highlights relevancy and choice, but is also aligned to a grade-appropriate standard, requires strategic thinking, and encourages collaboration and communication through discussion with classmates. Students work in groups to select a variable of interest (distance, speed, etc.) that they can measure over time, and then create a video and graph displaying the relationship between the two quantities. Given the connection to real-world experiences and the choices in product (via varying contexts) and process, this assignment has the potential to engage students with a wide range of interests and abilities in deep mathematical thinking.

Relevance

This assignment meaningfully connects mathematical topics (describing qualitatively the functional relationship between two quantities by analyzing a graph and sketching a graph that exhibits the qualitative features of a function that has been described verbally) to experiences that are relevant to students' lives. Students are prompted to choose a real-life experience that is relevant to them to record and then translate into a graphical representation. This assignment provides students with freedom to make connections to their experiences and interests.

Choice

In this assignment, students can choose what graphing story they want to tell and how it will be depicted in their video. They are also asked to select which role they will assume in producing the video, providing an opportunity for individuals to highlight their identified strengths. Additionally, students are provided a choice in how they will graph their story.

WHERE DO WE GO NEXT?

This analysis of middle-grades math assignments show that schools and districts across the country are falling short when it comes to providing their students with high-quality math tasks that meet the demands of college- and career-ready standards. The high percentage of aligned assignments demonstrates that teachers are adjusting from the "mile-wide" philosophy of previous standards movements and embracing the focused prioritization of content that the math standards provide. These high rates of alignment should be celebrated and strengthened. However, alignment on its own is not enough to meet the high bar set by rigorous college- and career-ready math standards. As our data show, we as educators must do more to provide students with quality math assignments that promote cognitive challenge, balance procedural skills and fluency with conceptual understanding, provide opportunities to communicate mathematical understanding, and engage students with opportunities for choice and relevance in their math content. As with our literacy analysis, we recommend two starting points for this work:

1. **Dig deeper through questions.** This analysis has cued for us important questions that all stakeholders should be asking about math tasks in middle schools in the era of college- and career-ready learning standards. Now more than ever, we wonder:

   - What level of cognitive demand are we asking of our students in mathematics? Are we pushing students, particularly low-income students and students of color, to think strategically in math? When and how frequently?
   - Do we have different expectations for what cognitive demand levels can be met based on accelerated or remedial math course identification? How can we provide all students in all courses cognitively challenging math tasks?
   - How are we ensuring a balance between procedural skills and fluency, conceptual understanding, and application within and across the classroom assignments we provide for our students?
   - How are we utilizing multiple representations as an opportunity to build conceptual understanding through multiple points of entry?
   - How frequently are we asking students to communicate their mathematical understanding by asking them to explain or justify their responses, or critique the reasoning of others?
   - What role does student choice play in our math classrooms?
   - Do we offer opportunities for students to bring their own ideas, experiences, and opinions into the work they do?

2. **Begin with assignments.** As we suggested in our literacy analysis, teachers and leaders need to track what their students are being asked to do on a daily basis in their classrooms. Analyzing the math tasks that students experience provides the necessary insight to gauge the quality of college- and career-ready standards implementation. It illuminates how the standards have been actualized in classrooms. And it prompts us to question the frequency with which we are providing students with high-quality math tasks that promote mathematical reasoning and problem-solving.

Based on our analysis, we have created a Math Assignment Analysis Guide that practitioners can use to engage in their own analysis of math assignments in their school or district. And we look forward to diving deeper into policy questions and implications that schools, districts, and states might want to consider (e.g., the role of assessment expectations, instructional time, and curriculum decisions) as they work to ensure their students are college- and career-ready. As we explore these topics in further detail, it is already clear that important work lies ahead for those committed and determined to strengthen the implementation of these demanding standards. Our nation's students deserve no less.

NOTES:

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5. Common Core State Standards.
6. Cathy L. Seeley, Smarter Than We Think: More messages about math, teaching, and learning in the 21st century (Sausalito, Calif.: Math Solutions, 2014): 264.
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9. National Council of Teachers of Mathematics, *Principles to Actions: Ensuring Mathematical Success for All* (2014): 24-25.
10. Common Core State Standards.
11. Courtney Koestler et al., *Connecting the NCTM Process Standards & the CCSSM Practices* (National Council of Teachers of Mathematics, 2013): 29-30.
12. These practice standards build on the prior recommendations of the National Council of Teachers of Mathematics' Reasoning and Proof, Communications, and Representation Process Standards, as well as the National Research Council's adaptive reasoning strand from *Adding It Up*.
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14. Common Core State Standards.
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16. National Council of Teachers of Mathematics, *Principles to Actions: Ensuring Mathematical Success for All* (2014): 60.
17. Common Core State Standards.

ABOUT THE EDUCATION TRUST

The Education Trust is a nonprofit organization that promotes closing opportunity gaps by expanding excellence and equity in education for students of color and those from low-income families from pre-kindergarten through college. Through research and advocacy, the organization builds and engages diverse communities that care about education equity, increases political and public will to act on equity issues, and increases college access and completion for historically underserved students.

EQUITY IN MOTION

ABOUT THIS SERIES

In this series, we will take a close look at how issues of equity are playing out in the daily activities of schools and educators. We aim to advance the work of practitioners and connect district, state, and federal actions aimed at improving education for low-income students with meaningful teaching and learning in schools. Most importantly, however, work in this series will continue to ask how we can adjust our practices, systems, and policies so that low-income students and students of color are actually benefitting from these efforts.

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1250 H STREET, NW, SUITE 700, WASHINGTON, D.C. 20005

P 202-293-1217 F 202-293-2605 WWW.EDTRUST.ORG