TODOS
Research Monograph 3

Embracing Resources of Children, Families, Communities and Cultures in Mathematics Learning

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Foreword

We are proud to present the Third TODOS Research Monograph. Following the example of the first two monographs, this monograph focuses on issues related to diversity and equity in mathematics education, and contributes to the main goal of TODOS to advocate for an equitable and high quality mathematics education for all students, particularly Hispanic/Latino students.

The specific focus of this monograph is about sharing research-based ways to help teachers recognize, embrace, cultivate, and build upon resources of children, families, communities, and cultures for teaching mathematics to all students. The monograph focuses on ways of identifying and embracing the myriad of resources that can be drawn upon or co-constructed in school with students from all groups, but especially Latinos, African Americans, students from families with low income, and other groups whose resources traditionally are not recognized and used to support students' mathematics learning in schools. The chapters in the monograph aim to inform researchers and practitioners on ways to support future and in-service teachers, as well as other practitioners, to enhance the learning of mathematics by embracing, eliciting, drawing upon, and co-constructing students' resources that go beyond curriculum and school mathematics materials.

The first chapter by Judit Moschkovich focuses on positioning and using students' language as a resource for mathematical communication, providing two examples of the language resources that students use during mathematical discussions as well as suggestions for how teachers can build on these resources. Moschkovich's work pushes mathematics education researchers and practitioners to move beyond general notions of "using language as a resource" to instead consider language as a resource *with respect to specific mathematical practices*.

The second chapter by Anita Wager and Kate Delaney also illustrates how teachers can build on students' strengths and interests to support their mathematics learning. Situated in the context of early kindergarten for four-year-old students, Wager and Delaney describe the action research projects of two teachers that link early mathematics, funds of knowledge, and developmentally and culturally responsive teaching. In each case, the authors illustrate how the teacher (a) understood multiple perspectives on funds of knowledge, (b) connected students' multiple mathematical resources to classroom practice, and (c) extended what she learned from one child to rethink pedagogical practices more broadly. This paper adds important examples to the research literature on the ways in which teachers can elicit and build on children's funds of knowledge to support their learning of mathematics.

The third chapter, written by Erin Turner, Julia Aguirre, Tonya Bartell, Corey Drake, Mary Foote and Amy Roth-McDuffie, gives examples of how prospective teachers make substantive connections to children's cultural funds of knowledge in mathematics lesson plans that also attend to children's mathematical thinking. They illustrate how prospective teachers identify and connect to mathematical practices in children's homes and communities, and how they make such connections across different parts of a lesson. They also discuss how the prospective teachers position children's families and communities, in some cases in opposition to a school culture fraught with stereotypical views about the community they serve.

The fourth chapter by Rebecca Neal and Dan Battey draws attention to the kinds of relational interactions (moment-to-moment communicative actions between teachers and students) that can support mathematics teachers in drawing on students' myriad of resources through opening multiple ways of being in the mathematics classroom. Specifically, it showcases how two mathematics teachers acknowledge student contributions, frame students' abilities, and access culture and language in an urban school setting. Illuminating the kinds of relational interactions reflective of culturally relevant mathematics instruction that exist in classrooms helps to operationalize ideas of what caring teacher-student relationships are that explicitly consider issues of culture, race, and power.

Higinio Dominguez's chapter also focuses on moment-to-moment teacher-student interactions. Dominguez argues for a conception of resources not as something pre-existing that students "bring" to the classroom, but rather argues for recognizing resources as jointly generated by student and teacher during instructional interactions. He argues that recognizing resources in this way assigns more responsibility to the teacher to notice, explore and eventually inventory the resources recognized in these interactions. Similar in some ways to the work of Cognitively Guided Instruction (e.g., Carpenter, Fennema, Franke, Levi & Empson, 1999), this perspective emphasizes the mathematical ideas, conceptions or insights that students might connect to during instructional interactions and build on in the joint generation of additional resources that can enhance teaching and learning simultaneously.

Across these papers, readers will find that by looking beyond "typical" conceptions of school-based mathematics, authors describe ways in which teachers value the mathematical resources children bring to the classroom as meaningful and important. At the same time, the authors illustrate how these resources are also different from, intersecting with, and providing access for school mathematics. With this resource perspective, the authors (and the teachers) explicitly counteract prevailing deficit views about children who have been traditionally marginalized because of the color of their skin, the low income of their families, or because they are immigrant children or second language learners[1]. In this and other ways, the authors of the monograph articles pay attention to issues of power, identity and status in relation to effective mathematics teaching and learning for all students.

[1] We use the phrase "second language learners" here because unlike "English language learners" the phrase "second language learners" makes explicit that the child brings to school something that a monolingual teacher does not, namely fluency in another language. At the same time, we recognize that the term "second language" is not always accurate because in reality many students are fluent in multiple languages.

Acknowledgements

We would like to thank the scholars who submitted an article to review for this third TODOS Monograph. We believe that the five papers published here best represented the goals of the monograph. With respect to review, all manuscripts were peer-reviewed by at least two people. Four of the manuscripts were reviewed independently by each of the two editors. Authors revised their articles, sometimes substantially, sometimes more than once, based on our comments. The authors' perseverance and willingness to meet our high expectations is quite laudable. For the fifth article, because the first editor participated as co-author, she did not participate in the review or decision making process; rather, the second editor was completely in charge of the process of the review and decision making. The invaluable advice, comments and constructive criticism from two outside reviewers, Anthony Fernandes and Kathryn Chval, helped the authors improve the article and were crucial for the second editor to make the decision of accepting the paper. We would also like to thank Marta Civil, chair of the Research and Publications Committee, for her patience and her support and encouragement of this work. Finally, we want to thank the Board of Directors of TODOS for their decision to publish a third monograph and assigning the monetary resources to print it.

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Language resources for communicating mathematically: Treating home and everyday language as resources

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INTRODUCTION

English language learners (ELL) are a large and growing population in United States schools. In 2007-2008, 10.7 % of the students enrolled in pre-K to 12th grade (more than 5.3 million) were labeled as English learners (Batalova & McHugh, 2010). In some states the percentages are even greater. For example, in California, 25% (about 1.5 million) of the children in public schools in 2007-2008 were labeled English learners (Batalova & McHugh, 2010). The number of ELL in United States schools between 1997-1998 and 2007-2008 increased by 53.2 percent (from 3.5 million to 5.3 million), in the same period that the number of all pre-K-12 students increased by 8.5 percent, from 46.0 million in 1997-1998 to 49.9 million in 2007-2008 (Batalova & McHugh, 2010). As the population of English language learners increases in the U.S. public school system, so do concerns with the needs of these students in mathematics classrooms.

Linguistic Resources of English Learners

It is difficult to make generalizations about the linguistics resources that *all* students who are learning English might bring to the classroom. Specific information about students' previous instructional experiences in mathematics is crucial for knowing what resources bilingual learners bring to mathematics classrooms. Classroom instruction should be informed by knowledge of students' experiences with mathematics instruction, their language history, and their educational background (Moschkovich, 1999b). In addition to knowing the details of students' experiences, research suggests that high-quality instruction for English learners that supports student achievement has two general characteristics: a view of language as a resource, rather than a deficiency; and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009).

General Guidelines for Teaching English Learners

Research provides general guidelines for instruction for this student population. The general characteristics of environments proven to be effective in supporting academic success for students from non-dominant communities, including English learners, are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (Garcia & Gonzalez, 1995, p. 424). Teachers with documented success with students from non-dominant communities share some characteristics: (a) a high commitment to students' academic success and to student-home communication, (b) high expectations for all students, (c) the autonomy to change curriculum and instruction to meet the specific needs of students, and (d) a rejection of models of their students as intellectually disadvantaged.

Guidelines for Teaching Mathematics to English Learners

Research specific to mathematics instruction for this student population provides several guidelines for instructional practices for teaching English learners mathematics. Mathematics instruction for English learners should: 1) treat language as a resource, not a deficit (Gándara & Contreras, 2009; Moschkovich, 2000); 2) address much more than vocabulary and support English learners' participation in mathematical discussions as they learn English (Moschkovich, 1999a, 2002, 2007a, 2007b, 2007d); and 3) draw on multiple resources available in classrooms—such as objects, drawings, graphs, and gestures—as well as home languages and experiences outside of school. This research shows that English learners, even as they are learning English, can participate in discussions where they grapple with important mathematical content (for examples of lessons where English learners participate in mathematical discussions, see Moschkovich, 1999a and Khisty, 1995).

Academic Language for Mathematics

The phrases "mathematical discourse" and "academic language" have many meanings. We could imagine that mathematical discourse and academic language are mostly about teaching vocabulary. And we might wonder whether English learners can participate in mathematical discussions before they learn English.

Although learning the multiple meanings of words is important, mathematical discourse involves much more than using individual words, phrases, or technical vocabulary. In general, students are learning to participate in valued mathematical *practices* (Moschkovich, 2007c). They are learning to communicate mathematically by making conjectures, presenting explanations, constructing arguments, and so on, and these arguments involve mathematical objects, with mathematical content, and towards a mathematical point (Brenner, 1994).

In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued in mathematics classrooms (Forman, 1996). Abstracting, generalizing and searching for certainty are also highly valued mathematical discourse practices. The mathematical practice of generalizing is reflected in common mathematical statements, such as "The angles of any triangle add up to 180 degrees," "Parallel lines never meet," or "$a + b$ will always equal $b + a$." Making claims is another important mathematical discourse practice. Mathematical claims apply only to a precisely and explicitly defined set of situations, as in the statement "multiplication makes a positive number bigger, except when multiplying by zero, one, or a number smaller than one." Claims are often tied to representations, such as graphs, tables, or diagrams. And last but not least, mathematical communication often involves talking and writing about imagined things—such as infinity, zero, infinite lines, or lines that never meet—as well as visualizing imaginary shapes, objects, and relationships.

When describing mathematical discourse, we should not confuse "mathematical" with "formal" or "textbook." Textbook definitions and formal ways of talking are only one aspect of school mathematical discourse. It is also important to avoid construing everyday and academic mathematical discourse as opposites (Moschkovich, 2007c). When communicating mathematically in the classroom, students use multiple resources from their experiences both in and out of school. We cannot say whether something a student says originated in their everyday or their school experiences. Everyday meanings or ways of talking should not be seen as obstacles to learning academic language or mathematical discourse, because some everyday experiences may provide resources for communicating mathematically.

What about vocabulary? While vocabulary is necessary, it is not sufficient for supporting mathematical communication (Moschkovich, 2002, 2007a). Learning to communicate mathematically is not primarily a matter of learning vocabulary. Students need to learn to participate in mathematical practices such as describing patterns, making generalizations, and using representations to support their claims. Studies of vocabulary learning report that students learn vocabulary most successfully through instructional environments that are language rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000; Pressley, 2000). To develop written and oral communication skills, students need to participate in negotiating meaning (Savignon, 1991) and in tasks that require output from students (Swain, 2001). Because mathematical discourse is central to success in mathematics, teachers need to balance vocabulary instruction with modeling of and opportunities for student participation in mathematical discussions.

Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas (Moschkovich, 2007a). One of the goals of mathematics instruction for English learners should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical concepts and reasoning, rather than on pronunciation, vocabulary, or low-level linguistic skills. By learning to recognize how English learners' express their mathematical ideas as they are learning English, teachers can maintain a focus on mathematical reasoning as well as on language development.

Using Everyday Language for Mathematics

Instruction needs to shift from monolithic views of mathematical discourse as only one kind of talk (i.e. what we read in textbooks or how mathematicians talk). For example, definitions are not a single (monolithic) mathematical practice. Although in lower-level textbooks they are presented as static and absolute facts to be accepted, in journal articles they are presented as dynamic, evolving, and open to revisions by the mathematician. Neither should textbooks be seen as homogeneous. Higher-level textbooks are more like journal articles in allowing for more uncertainty and evolving meaning than lower-level textbooks (Morgan, 2004), evidence that there are multiple types of mathematical texts.

We also need to shift away from dichotomized views of discourse practices as being *either* everyday *or* mathematical, and move to seeing everyday and mathematical discourse practices not in opposition, but as interdependent, dialectical, and related. Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics. It is not useful to dichotomize everyday and academic language (Moschkovich, 2007c; 2010a).

Instead, instruction needs to consider how to support students in connecting the two ways of communicating, building on everyday communication, and contrasting the two when necessary. In looking for mathematical practices, we need to consider the spectrum of mathematical activity as a continuum, rather than creating a rigid separation between practices in out-of-school settings and the practices in school. Rather than debating whether an utterance, lesson, or discussion is or is not mathematical discourse, teachers should instead explore the resources that students use to communicate mathematical ideas. In particular, students' uses of everyday language should be accepted and treated not as a failure to be mathematically precise but as fundamental to making sense of mathematical meanings and to learning mathematics with understanding.

Resources for Mathematical Discussions

The National Council of Teachers' of Mathematics Standards (NCTM, 1989, 1991, 2000) and the Common Core State Standards (CCSS 2010a, 2010b), recommend that all students have opportunities to participate in mathematical discussions and engage in mathematical practices. Research supports such recommendations (Moschkovich, 1999a, 1999b, 2002, 2010a, 2010b, 2010c, 2011). Teachers may wonder, however, can English learners participate in mathematical discussions before they learn English? Yes, they can. Research has provided many examples of classrooms where students who are learning English or are emergent bilinguals are participating in mathematical discussions (for some examples see Khisty, 2001; Khisty & Chval, 2002; Moschkovich, 1999a, 2002, 2007a, and 2011). These examples can serve as useful cases for further discussions among teachers for how to support the participation of English learners in mathematical discussions.

For classroom instruction that includes mathematical discussions, what *language resources* do English learners bring to the mathematics classroom? How can English learners express mathematical ideas in emerging (and sometimes imperfect) language? How can teachers build on the resources that students bring? The next section presents two examples of the language resources that students use during mathematical discussions and suggests how teachers can build on these resources. The examples point to a variety of language resources that English learners use to communicate mathematical ideas: their first language, everyday language, mathematical practices, gestures, objects, and drawings (Moschkovich, 2007a).

Example 1: Describing parallel lines using everyday language

The lesson excerpt presented below (Moschkovich, 1999a) comes from a third-grade bilingual classroom in an urban California school[2]. In this classroom, there were 33 students identified as Limited English Proficient. In general, this teacher introduced students to topics in Spanish and then later conducted lessons in English. The students had been working on a unit on two-dimensional geometric figures. For several weeks, instruction had included vocabulary such as "radius," "diameter," "congruent," "hypotenuse," and the names of different quadrilaterals in both Spanish and English. Students had been talking about shapes and the teacher had asked them to point, touch, and identify different shapes. The teacher identified this lesson as an English as a Second Language mathematics lesson, one where students would be using English in the context of folding and cutting to make Tangram pieces (see Figure 1).

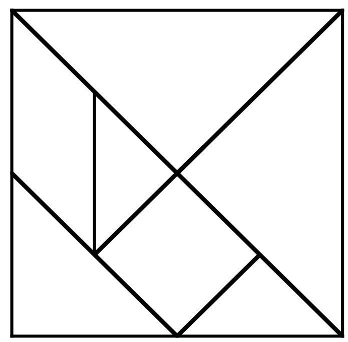


Figure 1: A tangram puzzle

1. Teacher: Today we are going to have a very special lesson in which you're really gonna have to listen. You're going to put on your best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles . . . high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?

2. Eric: A rectangle has . . . two . . . short sides, and two . . . long sides.

[2] This work was supported by Grants #REC-9896129 and #ROLE-0096065 from the National Science Foundation (NSF). The Math Discourse Project at Arizona State University videotaped this lesson with support by an NSF grant.

3. Teacher: Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn't know what it looked like, what, what . . . how would you say it.

4. Julian: Paralela [holding up a rectangle, voice trails off].

5. Teacher: It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?

6. Julian: Never get together. They never get together [runs his finger over the top side of the rectangle].

7. Teacher: What never gets together?

8. Julian: The parallela . . . they . . . when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines], they never get together.

9. Antonio: Yeah!

10. Teacher: Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel. Good work. Excellent work.

The vignette shows that English learners can, and do, participate in discussions where they grapple with important mathematical content. Students were discussing not only definitions for quadrilaterals but also describing the concept of parallelism. Students were using mathematical practices because they were making claims, generalizing, imagining, hypothesizing, and predicting what will happen to two lines segments if they are extended indefinitely. To communicate about these mathematical ideas, students used everyday expressions, objects, gestures, and other students' utterances as resources.

It is important to notice that this teacher did *not* focus directly on vocabulary development but instead on mathematical ideas and arguments as he interpreted, clarified, and rephrased what students were saying. This teacher provided opportunities for discussion by moving past student grammar or vocabulary errors, listening to students, and trying to understand the mathematics in what students said. He kept the discussion mathematical by focusing on the mathematical content of what students said and did.

Hearing the mathematical content and the language resources in Julian's contributions are certainly complex endeavors. It may take close attention and some work to hear the mathematical content in Julian's utterances in turns 4, 6, and 8. He uttered the word "paralela" in a halting manner and his voice trailed off, so it is difficult to tell whether he said "paralelo" or "paralela." His contribution includes a mixture of English and Spanish. Most of the sentence is pronounced in English, while the word "paralelo" or "paralela" sounded like he pronounced it in Spanish. Also, the grammatical structure of the utterance in line 8 is imperfect. The apparently singular "paralela" is preceded by the word "the," which can be either plural or singular, and then followed with a plural "they… when they go, when they go higher." However, if we move beyond the imperfect utterances and look more closely, we can uncover Julian's competencies in both mathematical practices and use of language resources.

What competencies in using mathematical practices did Julian display? Julian was participating in three central mathematical practices: abstracting, generalizing, and imagining. He was describing an abstract property of parallel lines, the fact that they do not meet. He was making a generalization, saying that parallel lines will *never* meet. He was also imagining what happens when the parallel sides of a rectangle are extended.

What language resources did Julian use to communicate this mathematical idea? He used colloquial expressions such as "go higher" and "get together" rather than the formal terms "extended" or "meet." He also used gestures and objects in his description, running his fingers along the parallel sides of a paper rectangle.

And lastly, how did the teacher respond to Julian's contributions? The teacher moved past Julian's confusing uses of the word "paralela" and focused on the mathematical content of Julian's contribution. He did not correct Julian's English, but instead asked questions to probe what the student meant. This response is significant in that it shows the teacher's stance towards student contributions during mathematical discussion: listen to students and try to figure out what they are saying. When teaching English learners, this means moving beyond vocabulary, pronunciation, or grammatical errors to listen for the mathematical content in student contributions (for a discussion of the tensions between these two, see Adler, 1998.)

How did the teacher build on these language resources? This vignette illustrates several instructional strategies that can be useful in supporting student participation in mathematical discussions. These strategies include using gestures and objects, building on what students say, asking for clarification, and re-phrasing student statements. I turn to each of these strategies in the following paragraphs.

The teacher used gestures and objects, such as the cardboard geometric shapes, to clarify what he meant. For example, he pointed to vertices and sides when speaking about these parts of a figure. Although using objects to clarify meanings is an important instructional strategy for supporting English learners, it is crucial to understand that these objects do not have meaning that is *separate* from language. Objects acquire meaning as students have opportunities to talk about the objects. These meanings are not predetermined, since students negotiate the meanings of mathematical objects through their talk (Moschkovich, 1996). In this example, although the teacher and the students had the geometric figures in front of them, and it seemed helpful to use the objects and gestures for clarification, students still needed to sort out what 'parallelogram' and 'parallel' meant by using language and negotiating common meanings for these words.

The teacher supported students' participation in mathematical arguments by using three instructional strategies that focus on mathematical practices:

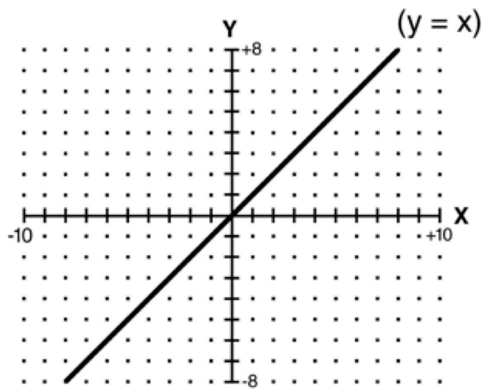
- The teacher accepted and built on student responses. For example in turns 4-5, the teacher accepted Julian's response and probed what he meant by "parallel."
- The teacher prompted the students for clarification. For example, in turn 7 the teacher asked Julian to clarify what he meant by "they."
- The teacher re-phrased (or re-voiced) student statements, by interpreting and rephrasing what students said. For example, in turn 10 the teacher rephrased what Julian had said in turn 8. Julian's "the parallela, they" became the teacher's "sides" and Julian's "they never get together" became "will never meet." The teacher thus built on Julian's everyday language as he re-voiced Julian's contributions using more academic language.

Another example of how a teacher can build on students' use of everyday language appears in Moschkovich (2011) during a discussion of the scales on two graphs that took place between a teacher and two students. In that longer example, the teacher focused on students' sense-making and reasoning. She built on student reasoning, in part, by first using the students' own language and ways of talking and only later describing student reasoning using a mathematical concept. The teacher did not supply the correct interpretation of the scales or make an explicit contrast between the student reasoning and the right answer. Instead, the teacher clarified and connected different ways of reasoning. She described her own reasoning to the students—how she interpreted the scales on both graphs. Overall, the teacher used several strategies to support student reasoning: she used student-generated products, she used gestures and objects to clarify meaning, she accepted and built on students' responses, and she connected student reasoning to an important mathematical concept, in this case unitizing. Student reasoning was taken seriously, time was available for describing and taking different points of view, and room for clarification was evident. The teacher supported this mathematical discussion by describing how she understood each student's descriptions, rather than evaluating student work. Discussions that make multiple ways of reasoning explicit and compare different meanings can afford important opportunities for students to participate in sense making and develop mathematical reasoning.

Example 2: Describing lines using two languages

The second example illustrates how both home and school languages—the language of the home and the academic language of instruction learned through previous schooling—can offer resources for mathematical reasoning. In the following discussion, students used both everyday and academic language to clarify the mathematical meaning of a description.

8a. If you change the equation y = x to y = -0.6x how would the line change?



A. The steepness would change. Why or why not?

___ NO	___ YES	____ STEEPER
		____ LESS STEEP

Figure 2. Problem for Example 2.

The example is from an interview with two ninth-grade students, Giselda and Marcela, after school. The students had been in mainstream English-only mathematics classrooms for several years. One student, Marcela, also had some previous mathematics instruction in Spanish. These two students were working on the problem in Figure 2.

They had graphed the line $y = -0.6x$ on paper (Figure 3) and were discussing whether this line was steeper than the line $y = x$.

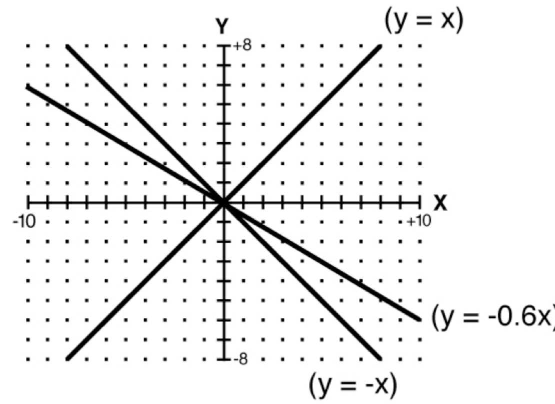


Figure 3. Lines drawn by students for Example 2

Giselda first proposed that the line was steeper and then decided it was less steep. Marcela repeatedly asked Giselda if she was sure. After Marcela proposed that the line was less steep, she explained her reasoning to Giselda. (Transcript annotations are in brackets. Translations are in italics beneath the phrases in question.)

1 *Marcela:* No, it's less steeper . . .

2 *Giselda:* Why?

3 *Marcela:* See, it's closer to the *x*-axis . . . [looks at Giselda] . . . isn't it?

4 *Giselda:* Oh, so if it's right here . . . it's steeper, right?

5 *Marcela:* Porque fíjate, digamos que este es el suelo.

[*Because look, let's say that this is the ground.*]

Entonces, si se acerca más, pues es menos steep.

[*Then, if it gets closer, then it's less steep.*]

. . . 'cause see this one [referring to the line $y = x$] . . . is . . .

está entre el medio de la x y de la y. Right?

[is between the x *and the* y*]*

6 *Giselda:* [Nods in agreement.]

7 *Marcela:* This one [referring to the line $y = -0.6x$] is closer to the x than to the y, so this one [referring to the line $y = -0.6x$] is less steep.

In this discussion, the two students were negotiating and clarifying the meanings of "steeper" and "less steep." Marcela used two languages (English and Spanish), mathematical representations (the graph, the line $y = x$, and the axes), and everyday experiences as resources. The premise that meanings from everyday experiences are obstacles for mathematical reasoning does not hold for this example. In fact, Marcela used her everyday experiences and the metaphor that the x-axis is the ground ("Porque fíjate, digamos que este es el suelo" [*Because look, let's say that this is the ground*]) as resources for making sense of this problem. Rather than finding everyday meanings as obstacles, she used an everyday situation to clarify her reasoning. The everyday experience of climbing hills thus provided a resource for describing the steepness of lines (Moschkovich, 1996).

What mathematical practices can we see in Marcela's mathematical reasoning? Marcela explicitly stated an assumption when she said, "Porque fíjate, digamos que este es el suelo" [*Because look, let's say that this is the ground*]. She supported her claim by making a connection to mathematical representations. She used the graph, in particular the line $y = x$ (line 5) and the axes (lines 5 and 7), as a reference to support her claim about the steepness of the line. Marcela was using two important mathematical practices: stating assumptions explicitly and connecting claims to mathematical representations.

The practice of using two languages during one conversation or within one sentence that we see in the preceding discussion is called *code switching*. Research does not support a view of code switching as a linguistic deficit (Valdés-Fallis, 1978; Zentella, 1981). In fact, the opposite is true. Although code switching has an improvised quality, it is a complex, rule-governed, and systematic language practice reflecting speakers' understanding of their community's linguistic norms. The most significant reason for a bilingual student's language choice is the language choice of the person addressing the student. We should not assume that bilingual students switch into their first language because they are missing English vocabulary or cannot recall a word. Neither should we assume that code switching is evidence of a deficiency in a student's mathematical reasoning. Code switching can offer resources for communicating mathematically (Moschkovich, 2007b, 2009). For example, students sometimes code switch as they describe a mathematical situation, explain a concept, justify an answer, elaborate an explanation, or repeat a statement.

SUMMARY

If classroom instruction only focuses on the obstacles English learners face, for example, what vocabulary English learners know or don't know, they will always seem deficient because they are, in fact, learning a second language. If teachers perceive English learners as deficient and see formal mathematical vocabulary as the only linguistic resource, there is little room for addressing these students' mathematical ideas, building on them, and connecting these ideas to the discipline. English learners thus run the risk of being caught in a repeated cycle of remedial instruction that does not focus on mathematical content.

The two examples in this chapter show that English learners can and do participate in discussions where they grapple with important mathematical content and that both home and everyday language can serve as resources for communicating mathematically. The examples also show that students who are learning English can use important mathematical practices. One of the goals of mathematical discussions for English learners should be for students to have opportunities to express mathematical ideas and participate in mathematical practices, regardless of their proficiency or fluency in English. Teachers can move towards this goal by learning to recognize the multiple language resources that students use to express mathematical ideas. English learners may know how to make comparisons, describe patterns, abstract, generalize, explain, and use mathematical representations. They may show these competencies using everyday or colloquial language as a resource. Colloquial expressions are legitimate resources for communicating mathematical ideas. Teachers can support students in both displaying their competencies as well as in learning to communicate in more formal mathematical language.

The two examples above also show that seeing the language resources in student contributions is a complex task. A crucial question that is useful for uncovering students' mathematical competencies when these are expressed through language is: "What competencies in using mathematical practices (describing patterns, abstracting, generalizing, etc.) do students display?" Building on students' linguistics resources is certainly a complex task, perhaps especially when working with students who are learning English. It may not be possible to decide whether a student's utterance reflects the student's conceptual understanding or the student's proficiency in expressing their ideas in English. However, if the goal is to assess students' mathematical content knowledge and build on their competencies, it is important to listen past English fluency to hear students' mathematical ideas and their use of mathematical practices.

FUTURE RESEARCH

To design instruction that builds on student resources, research needs to examine in more detail the resources that students who are emergent bilingual or learning English use for mathematical reasoning. Many more studies are needed that describe how students who speak more than one language use multiple resources such as two languages, gestures, objects, and mathematical representations to communicate mathematically. Studies will need to distinguish among multiple modalities (written and oral) as well as between receptive (i.e. listening, comprehending) and productive (i.e., expressing orally or in writing) language skills. Other important distinctions are between listening and oral comprehension, comprehending and producing oral contributions, and comprehending and producing written text.

It is crucial for both research and instruction to move away from construing everyday and school mathematical practices as dichotomous. During mathematical discussions, students use multiple resources from their experiences across multiple settings, both in and out of school. Everyday practices should not be seen only as obstacles to participation in academic mathematical practices. The origin of some mathematical practices may be everyday practices and some aspects of everyday experiences can provide resources in the mathematics classroom. Everyday experiences with natural phenomena can be resources for communicating mathematically. In the second example, climbing hills was an experience that provided a resource for describing the steepness of lines. Other everyday experiences with natural phenomena may also provide resources for communicating mathematically.

In addition to experiences with natural phenomena, O'Connor (1999) proposes that students' mathematical arguments can be at least partly based on what she calls argument protoforms.

> Experiential precursors (arguments outside of school, the provision of justification to parents and siblings, the struggle to name roles or objects in play) may provide the discourse "protoforms" that students could potentially

build upon in the mathematical domain. (p. 27)

These precursors are related to academic mathematical practices such as arguing, making and evaluating a claim, providing justification, or co-constructing a definition. Research should consider what aspects of everyday discourse could serve as resources for mathematical arguments.

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Exploring Young Children's Multiple Mathematical Resources through Action Research[3]

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INTRODUCTION

After several years of debate, a local school district decided to implement a new public pre-kindergarten program for four-year-old children referred to in the state as four-year-old kindergarten or 4K. This move was, in part, a response to local pressure to improve student achievement, specifically of historically marginalized groups, and a national move to include public pre-kindergarten in the K-12 education system. The local school district's decisions were grounded in research that suggests that children have differing educational experiences prior to entering kindergarten, which exacerbates differential achievement later (Arnold & Doctoroff, 2003) and that the benefits of quality pre-school education has lasting academic gains for children from historically marginalized backgrounds (Magnuson, Ruhm, & Waldfogel, 2007). Given that landscape, and research demonstrating that young children's success in early mathematics is a greater predictor of future academic success than early literacy skills (Duncan et al., 2007; Romano, Kohen, Babchishin, & Pagini, 2010), we partnered with the local district to offer professional development to all teachers in the new 4K program. The professional development was designed to weave together work in early mathematics, funds of knowledge, and culturally and developmentally responsive teaching. The overall project included three cohorts of 4K teachers who participated in four graduate courses over a two-year period, which culminated with an action research project.

This paper examines the experiences of two of the teachers from the first cohort—Marie and Enid—who conducted action research projects over one school year. We selected Marie and Enid because they taught in ethnically and economically diverse classrooms. Their cases offered insight into the ways in which White, middle class teachers might take up theories about teaching children from historically marginalized backgrounds, including those learned through the professional development. To understand the phenomena we drew on interviews with the teachers, discussions with their action research group, and most heavily on the action research projects themselves. In the spirit of action research, we have analyzed these data not for generalizability but to offer authentic findings from real teachers in real classrooms (Cooper, 2012). That being said, we also believe that these two cases offer models for understanding how teachers can merge theoretical learning and practice into anecdotal understanding of culture and cultural practices. Understanding how teachers may move from theoretical knowledge to anecdotal knowledge through practice are important considerations for researchers providing professional development experiences.

In our stories of both Marie's and Enid's projects, the contexts in which they work, and their own experiences, we provide narratives to follow the processes they went through as they incorporated theory into practice. Following Marie's and Enid's stories, we look across the narratives to identify (a) the multiple ways funds of knowledge can be taken up, (b) how they drew on the multiple mathematical

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resources of their focal child to support his learning to count, and (c) how they extended what they learned from one child to rethink their practices more broadly.

Children's Multiple Mathematical Resources

There is broad evidence suggesting that teachers need to draw on the multiple resources children bring in order to provide culturally responsive mathematics teaching (see for example Strutchens, et al., 2012; Foote, 2010a). For young children, these resources include children's mathematical thinking, play experiences that provide naturalistic engagement with mathematics concepts, and experiences in their homes and communities. Research on children's mathematical thinking suggests that teachers' understanding of the ways children construct problems, the strategies they use to solve them, and their common misconceptions leads to greater learning (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Although this research was focused on problem solving, these ideas are applicable to supporting younger children as they are learning to count (National Research Council, 2009). Moreover, the informal mathematical experiences that children engage in as they play are a resource for teachers (Ginsburg, 2006). In observing young children during play in preschool, Seo and Ginsburg (2004) found that 88% engaged in some form of mathematical activity. In addition to understanding how children think mathematically and the ways in which play provides a site for mathematical engagement, teachers also need to recognize the skills, practices, linguistic knowledge and experiences that children bring from home – their *funds of knowledge* (González, Andrade, Civil, & Moll, 2001). By accessing all of these resources, teachers are better equipped to support children in ways that are culturally (Ladson-Billings, 2006) and developmentally (Copple & Bredekamp, 2009) responsive.

The research on funds of knowledge has been used in multiple contexts to identify the resources available in children's homes. From the original studies with predominately Latino/a communities in the Southwest (Civil, 2002; 2007; Civil & Andrade, 2002; González et al., 2001), the practice has been successfully adapted to examine out-of-school mathematical practices of young immigrant children in the UK (Andrews & Yee, 2006); teachers accessing family histories through story telling (Marshall & Toohey, 2010); and funds of knowledge as both exchanges of knowledge from school-to-home as well as from home-to-school (Hughes & Greenhough, 2006). One extension of the definition of funds of knowledge is Hedges' (2011) inclusion of popular culture. In pointing out Moll's (2005) contention that the concept of funds of knowledge is not static, but adapts with changes in contexts and cultures, Hedges suggests that popular culture can be a form of funds of knowledge. This is particularly evident for young children who often incorporate their interest in media culture (an aspect of popular culture) in their play—an important site for learning. We are not suggesting that the act of watching television is play or funds of knowledge but rather that the ways in which children bring these ideas into play is a potential resource for teachers. Hedges takes this further, arguing that more than just an interest, popular culture "represented something that influenced children's language, play, relationships and behaviour in ways consistent with the concept of funds of knowledge" (p. 26). Therefore, teachers need to be aware of children's interests and popular culture.

Although we consider both interests and funds of knowledge as potential resources, we articulate our view of the difference with this example: a teacher who makes a board game using Iron Man characters because a child has demonstrated an interest in Iron Man is drawing on a child's interest. However, this act is not child initiated. Instead, the child's interest is being used as a resource rather than a way of knowing that the teacher adopts and attempts to transfer to other areas (Rodriguez, 2013). In contrast, a teacher who observes a child who is pretending to be Iron Man in play, and then joins the play and learns from the child how Iron Man might solve a story problem, is sharing learning with the child. The difference here is in the agency of the child. While the knowledge brought by the child into the classroom may be more related to popular culture than cultural practices, Rodriguez notes that whether or not it should be considered funds of knowledge has more to do with agency than the type of knowledge (popular or cultural). Funds of knowledge refers to the "extent to which adults (parents or educators) allow for students/children/youth to integrate the knowledge(s) for themselves and contribute that knowledge directly in ways that make sense to them as learners within the classroom setting and beyond (p. 108)."

The ways in which teachers access (and have the opportunity to access) children's multiple mathematical resources varies by context. With young children, observations during play provide a window into their understandings of mathematics, their interests, and how popular and media culture is manifested. Yet this insight does not offer a more thorough understanding of how these ideas are taken up at home. Research in mathematics education that endeavors to support teachers in identifying and incorporating children's home and community practices include practices such as home visits (González et al., 2001), community visits (Bartell, et al., 2010), and focal child projects (Foote, 2010b). In the BRIDGE project (Civil, 2002) teacher-researchers went into children's homes three times and conducted interviews with families. During a presentation and discussion with the teachers in our professional development, Norma González (personal communication, February 16, 2011) explained that the first interview focused on household history; during the second the teachers asked about daily activities, labor history, skills needed for work in and out of the household, and how mathematics is used; and for the third interview they discussed household ideology, the role of parents, challenges/success/joy as a parent, and differences between generations. The teachers in our professional development engaged in a similar series of interviews during the first year of professional development and found they provided a depth of understanding of the child's resources. This experience also supported the teachers in thinking about how to structure a single home visit in the second year.

A focus on one child in a classroom raises the question: What information might a visit to one home reveal about other children? Foote (2010b) found that when teachers develop in-depth knowledge of one child in their classroom they begin to (a) recognize the funds of knowledge and mathematical competencies that extended beyond the classroom, (b) reflect on previous deficit views of families, and (c) extend these notions to other children in the classroom that had not been studied closely. Sudzina and Gay (1993) also found that "by combining in-depth knowledge of one child with scientific inquiry skills and readings from the professional literature, each participant can create a synthesis of principles of human development that is useful in understanding not only the child being studied, but all children as well" (p. 173).

It is from this research on multiple conceptions of funds of knowledge, accessing funds of knowledge, and extending the knowledge about one child to classroom practices that we organized and planned for the action research projects.

Action Research and the Projects

As Cochran-Smith and Lytle (1993, 2009) have argued for over two decades, educators can make important contributions to educational practice and reforms through research in their own classrooms. They suggest turning historically privileged research practices on their ear with teacher research projects that study phenomena embedded in practice. This in turn positions teachers to critically reflect upon and take action in their practice based on their research findings (Lytle & Cochran-Smith, 1992). Action research—also referred to as teacher research, practitioner research, and teacher inquiry—aims to provide an insider perspective on the problem space of teaching through a cycle of inquiry. This cycle or action research process includes: (a) identifying a problem, (b) developing questions, (c) gathering data, (d) analyzing data, (e) interpreting data, and (f) responding to the data in a way that leads to praxis (McNaughton & Hughes, 2008).

A key component to action research is the view that real change comes from those doing the work. We drew on Freire's (2007) definition of praxis as "reflection and action upon the world in order to change it" (p. 51) as we envisioned how teachers would rethink their practice to incorporate children's multiple mathematical resources to provide for a more equitable mathematics classroom. We imagined this change in practice would be based, in part, on their readings and discussions of funds of knowledge, thus linking theory to practice (Bullough & Gitlin, 2001). To support this change we designed the action research course in line with the local district's successful classroom action research program. We met two times per month for approximately 2 hours over the course of the academic year. We followed the action research process described above, providing the teachers with readings and support as they needed. An important aspect of our project was its collaborative nature (Caro-Bruce & Klehr, 2007). Not only did we provide the teacher-researchers with feedback and guidance but they helped each other as well. The only parameters on the action research projects were that the teacher- researchers were to identify a problem connected to early mathematics and funds of knowledge. As we endeavored to remain faithful to the "constructivist spirit of the action research process," (Caro-Bruce & Klehr, 2007, p. 9) the teacher-researchers developed their own working definition of funds of knowledge and, therefore, children's multiple mathematical resources. It was the evolution of these definitions and how they shaped practice that we found particularly interesting to study.

METHODS

In an effort to understand the multiple resources that the teachers drew on to develop their understanding of children's mathematical funds of knowledge, we raised the following research questions: (a) How did Marie and Enid interpret the theories of funds of knowledge and apply them in their own action research projects? (b) In what ways did Marie and Enid draw on the multiple mathematical resources of their focal child to support his learning to count? (c) How did Marie and Enid extend what they learned from one child to rethink their practices more broadly across their classrooms?

Participants

Five teachers participated in the action research course. Although the feedback from the group as a whole is integrated into our data, we are focusing on two teachers in particular, Marie and Enid (a more detailed description of each is provided in their individual stories). We chose to focus on Marie and Enid because they taught in ethnically and economically diverse schools whereas the other teachers worked in pre-schools serving mostly upper middle class White children. We are not suggesting the other stories are any less valid or important. Given the focus of this monograph, however, we thought the outcomes from Marie and Enid's studies were more relevant.

The other participants in this study are the two focal children selected by Marie and Enid. In the first year of the professional development the teachers were asked to select a focal child that was different from them in at least two ways (linguistically, economically, or ethnically). Although we did not make this a requirement for the focal child in the action research study we did want teachers to identify a child who they perceived as struggling in mathematics. Marie's focal child, Donald, was a boy from a White, working class home and received free and reduced price meals. At the time Donald joined Marie's classroom in late October, he would only count four objects. Through the winter he continued to be very hesitant to count things and required significant support from Marie to do so. Enid's focal child, Philip, was a boy from a White middle class home. Philip had a severe physical disability that limited his ability to use his arms, hands and fingers. As a result of this disability, Philip did not actively engage in many of the classroom counting activities.

We also consider ourselves participants in this study. As facilitators of the action research, we played a role in how the teachers engaged with their projects. Our interactions with the teachers on the projects and the ways in which we have analyzed their interactions to craft their stories were shaped by our own experiences and beliefs. As former teachers in high poverty and diverse schools, we are committed to studying and practicing culturally and developmentally responsive pedagogy. Our social justice stance has influenced our choice of focus in this article. For example, we attended to what we perceive as incomplete understandings of home resources by the teachers participating in this group.

Data Collection and Analysis

The data for our study include the action research projects, transcribed audio files of home visits, interviews with the teachers, and teachers' conversations as they developed the projects. We read through the data to identify instances of the teachers discussing funds of knowledge, children's multiple mathematical resources, and the project. We identified comments about the definition, meaning, ways of accessing, and ways of using funds of knowledge. For multiple mathematical resources, we identified references about children's in and out of school practices, skills, and interests. Comments about the action research projects included references to difficulties and Marie's and Enid's use of the project findings in their practice. In addition, we sought out background information about the teachers that might provide insight into their perspectives, including personal, work, and educational experiences. We used these data to write a narrative for the teachers (Clandinin & Connelly, 2000). These narratives provided background on the resources the teachers drew on and insight into how their perspectives changed over the course of the action research project. In using the teacher's words (both written and spoken) we have interpreted their experiences in these projects. As such, the narratives are our stories of the teachers' stories, yet, we worked to foreground those experiences the teachers highlighted (Wortham, 2001).

MARIE'S STORY

Marie was one of the most senior teachers in our professional development. She is a White, middle class woman born and raised in the local community and after considering careers in medicine and business she decided to go into teaching. At the time of this study, Marie had 28 years of experience teaching in preK-5 classrooms: 7 in private preschools, 18 in kindergarten, and 3 years as an elementary school librarian. Over the course of her teaching career, Marie had regularly participated in professional development and graduate coursework (previously in library studies) and this was her second action research project in the district. When the professional development began, Marie was working as a school librarian but was able to secure a position as a 4K teacher in the district during the first year 4K was implemented (the second year of professional development). Marie's class was in an elementary school that predominately served a White, middle and working class community. Approximately 36% of the students in the school were provided with subsidized meals; ethnically, 12% of the students were African American, 11% were Latino/a, 65% were White, 9% identified as two or more ethnicities, and 2% were Asian American. Marie's classroom reflected this population both ethnically and economically.

Marie chose to participate in the professional development because she was very enthusiastic about the plans for 4K in the district. In her initial interview, she stated that she wanted to teach pre-kindergarten for four-year-olds because she disagreed with the increasing emphasis on standards in the district and believed that children learned best in play. Despite this claim, Marie relied heavily on the vast collection of lesson plans and materials she had used in kindergarten for whole class instruction in 4K. In addition, she often expressed that she felt many of the directives from the district in regards to how 4K should differ from kindergarten did not apply to her. Instead, she used her own professional judgment and beliefs to determine what to include in her classroom in terms of both materials and curriculum. This sometimes aligned with district mandates for 4K, but sometimes did not.

Over the two-year program Marie often referred to her experiences teaching and in other courses during discussions. She saw herself as a resource for other teachers in the program. This was most apparent during conversations of cognitively guided instruction (CGI) (Carpenter, Fennema, Franke, Empson, & Levi, 1999) for which she had participated in a number of professional development sessions. Marie felt confident in her understanding of cognitively guided instruction and had used it in her classroom for many years. The construct of funds of knowledge, however, was new for Marie. She seemed to resist the idea of finding it useful in terms of knowing children better. She was also unsure as to how she could connect it to her practice, as evidenced in her reflection following a class discussion with Norma González (personal communication, February 16, 2011):

> After the discussion I still have more questions, the biggest one is how to bring this information into the classroom. As in our reading and in my own practice, I know that we can connect student knowledge to math through CGI math problems. This is easiest in the elementary school classroom. I'm not sure how much of a focus it would be in the 4K classroom.

Not only was Marie unsure how to bring funds of knowledge into the 4K classroom, but her idea was to include home experiences by incorporating them in the contexts of word problems rather than consider the embedded mathematical practices in children's homes (Wager, 2012).

Marie drew heavily on her previous experiences teaching kindergarten as she thought ahead to 4K. During her initial interview, Marie discussed specific ideas to support children's learning to count in 4K such as using the calendar, hundreds chart, and various wall boards (all typical kindergarten materials that may or may not be appropriate with four-year-olds). She planned ahead for different themes she would use in classroom, stating, "We'll have the play area set up as the haunted restaurant at Halloween time" and almost 14 months later that is what she did. This plan was based on an assumption (albeit often true) that all children were interested in Halloween, but the idea came from Marie rather than from the children. Attending to her own experiences was a thread that ran through much of Marie's conversation, which likely made it difficult to conceptualize children's funds of knowledge. Marie's battle with funds of knowledge extended into the second year of coursework and her initial plans for her action research project. Rather than attend to the experiences of her focal child, she drew on her own interest in and experience with music to plan her initial research project. In explaining the reason behind her first plan she stated,

> My son is a percussionist/drummer. His fiancé is also a musician with some background in music therapy. I know a teacher who used drums in his classroom, which really built community. It seemed like a good connection – music and math.

Her plans to bring in music and drumming was also connected to her desire to own a gathering drum as part of the 'fur trade re-enacting' she did with her husband and their interest in Native American culture. Marie is also a talented seamstress and interested in crafts and making things for her classroom. She was so enthusiastic about her music/mathematics project that she made a class set of drums using food cans, and mallets out of dowels and rubber balls. She started by working with a few students who struggled in counting by having them count drum beats as they sang counting songs. She gathered data on their progress, and planned further lessons. All this serves as background to explain how heavily invested Marie was to center her project on drumming, music and math. Then, Marie did her home visit with her focal child Donald and, as she said, “something happened on the way to my project.”

Donald was one of 15 children in Marie’s public four-year-old kindergarten morning classroom. During her home visit in mid-February, Marie learned about the family’s interests in science—they had many different kinds of pets, regularly went to museums and even named their three children after scientists. She also learned that Donald was interested in animals, dinosaurs, and Skylander figures from a video game. Shortly after the home visit, the following event occurred:

> I was sitting next to Donald during breakfast. His stuffed shark was lying on the next table about 5 feet away. I casually said, “I wonder how many teeth your shark has?” He immediately started counting them from the distance. He counted to 11.

This was quite a change from the counting skills she had seen Donald demonstrate in the past. As a result of what she learned from Donald’s family and the shark teeth incident, Marie tried a new tactic. By engaging Donald in counting things he found interesting, he demonstrated an increase in skills and enthusiasm for counting and number.

Multiple Perspectives on Funds of Knowledge

Marie’s initial perspective on funds of knowledge was that it was not a concept with which she planned to engage. During the first year of the professional development she did not seem to recognize the potential in home visits or thinking about home practices. In reflecting on González and colleagues’ (2001) article and the talk by Norma González (personal communication, February, 16 2011), Marie acknowledged the importance of deeper relationships with families but did not seem to make a connection between learning about families’ mathematical practices and how that may or may not align with school practices. Rather, she focused on “looking to find good questions that I can ask to get to know the families better.” This perspective was reflected in Marie’s response to a reading about mathematics activities that bring home into school; here she focused on generic ideas she took from the article such as “kids love to learn and we can do a lot with everyday mathematics,” rather than reflecting on how specific home activities might be brought into the classroom.

Although Marie seemed to resist funds of knowledge in the first year, at the beginning of the second year she focused on her own funds of knowledge as she designed her action research project around her own interests and experiences. It took the combination of a home visit during which she learned some surprising things about her focal child’s family and an observation of her focal child in the classroom to see the opportunities present in ‘non-school’ resources. During her home visit, Marie learned of the family’s interest in science and Donald’s particular enthusiasm for dinosaurs.

> I'm saying ‘okay giving in to the funds of knowledge. We're going to do this.’ So he loves dinosaurs. I went and got dinosaur counters. He was counting them and having a great time … And then he loves Skylanders.

By her own admission, Marie shifts her perspective and ‘gives in to’ funds of knowledge. During the same time, she had come across Hedges’ (2011) article, which reified her new view that popular cultural and children’s interests were important aspect of funds of knowledge, particularly when working with younger children. From this point, Marie developed a newfound enthusiasm for her research project and supporting Donald's counting.

Marie's conclusion in her action research project sums up how her perspective changed. Although she comes to recognize funds of knowledge as an important resource, she adopted a definition that centered on children's interests rather than how those interests were taken up in play. Further, she saw attention to children's interest as only necessary in 4K:

> I did not necessarily put all that much consideration into the funds of knowledge when we were discussing them in [professional development] class. I saw the merits of this idea, but did not realize how important it was in 4k. In my years in kindergarten, I could inspire 5 year olds to become interested in whatever lesson I chose or a topic that was required for kindergarten. When it comes to 4 year olds, they are not as easily persuaded. They are much more likely to get involved in something that comes from their background, something they already know something about and certainly something they are more interested in.

Connecting Multiple Mathematics Resources to Classroom Practice

Marie considered three of Donald's interests as potential resources to support his mathematics learning in the classroom: dinosaurs, Skylander, and the color blue. After discovering his fascination with dinosaurs, Marie purchased dinosaur counters to see if that might engage Donald more than the bug and bear counters provided by the district.

> The counters were an immediate hit. Donald played with them the entire play-time on the first day. He separated them into colors—he preferred to play with only the blue. If I asked how many there were, he didn't hesitate to count them.

Marie found that Donald continued to play with the dinosaur counters and never hesitated to count them when asked. He also began to sort them by color and make up games to play with the other children using the dinosaur counters.

Donald's interest in Skylander was a major focus of Marie's action research project. She purchased a poster that had all the Skylander characters, laminated it, and then cut it into 'cards.' Marie used the cards in multiple ways to engage Donald in counting. Sometimes she would have him count as many cards as he could, other times she had him organize sets of card characters based on the parts of the world they inhabited and count the sets. The data she collected on Donald's counting with the Skylander cards revealed an increase in counting, one-to-one correspondence, and cardinality. On one of her last observations for the action research project she noted,

> I told Donald I was wondering how many of the characters he [& his friend] had at home. So I showed him each card & he told me yes or no if he had those. Then I asked how many is that? He started to count the cards, flipping through the cards, got to 15 [skipped 16], 17, stopped at 19 and then got a bit upset as he could not count on his own.
>
> I said, should I help? So I started flipping through them & we counted together. He skipped 16 again [but I was counting so we did say it], then he quit at 19 as he could not count farther. We got to 23. Then we did the same with his friend—counting together. He had 26. But Donald said, "I have 23." So he remembered what we had counted.

After this event, Marie made a home-school connection by suggesting that Donald go home and count his Skylander figures and come back and report to her how many there were. Though not directly related to mathematics, Marie also connected with the speech therapist that worked with Donald so that she could use the Skylander cards in supporting his pronunciation.

Marie also considered Donald's interest in his favorite color, blue, as a resource. She found that he was much more willing to count anything that was blue, including the dinosaurs and other counting manipulatives in the classroom. In addition to viewing these three interests as a resource, Marie saw play as an important site for Donald's learning. From the beginning of the professional development, she talked about the virtues of play and felt that it provided the best place for young children to learn and an opportunity for teachers to support learning. By considering play as a potential mathematical resource, Marie was able to mathematize the play activities she observed. Further, she saw play as a way to learn more about children's interests.

Extending Knowledge of One Student to Pedagogical Practices

In her work with Donald, Marie learned that other children were interested in dinosaurs and Skylander and she began to use the Skylander cards and dinosaur manipulatives with those children. More significantly, Marie began to attend to the other children's interests and bring those ideas into mathematics in the classroom. She observed that one of the other children in her class who struggled with counting had an interest in fairies. As a result, Marie bought fairy stickers and made cards to replicate the activities she had done with Donald and the Skylander cards. Marie acknowledged that making purchases to engage the interests of every child in the classroom was not possible, but felt that this practice was something she would, and others could, do to support those students who struggled most. Her recommendation after her action research project was,

> Whatever interests the child, try using that in the classroom. The four year old child will be much more interested if it is something they know, understand, and especially like. Teaching will be easier if I know the child, a bit about their family and/or home life, and know what motivates them. Whether it is dolls, dinosaurs, Skylanders or trucks, getting to know the child will help in teaching.

ENID'S STORY

Within our professional development class, Enid was the most recent to have entered the field of teaching. Enid is a late-twenties, White, middle class woman who grew up in a farming community on the outskirts of medium-sized city in the Midwest. After receiving her degree in elementary/early childhood education about ten years ago, she worked in local day care sites for several years. She then spent three years in Japan teaching in an English speaking preschool for Japanese children. After moving back to the area she became an assistant teacher at a well-respected local preschool, The Aetelier Preschool that serves children ages 3 to 5. At the start of our professional development, Enid was still in this position, though hoping to move into a public school 4K classroom with the implementation of district-wide 4K. While this was Enid's goal upon entering the program, during our second year (in which the action research course started), Enid accepted a head teacher position at The Walter Community Preschool, a preschool and childcare located within a larger neighborhood community center. The transition from one setting to another had a large impact on Enid's understandings of funds of knowledge and early mathematics. To untangle this, we first need to consider the contrasting contexts of these two sites.

As early childhood settings go, The Aetelier Preschool and The Walter Community Preschool (Walter) are quite different, both in terms of the families that they serve and the philosophies that they employ in curriculum design. The Aetelier Preschool is one of the most expensive preschools in the area. It serves predominantly White and affluent children and families, with many coming from faculty families at the local university. The preschool uses the Reggio approach to early childhood education (Hewett, 2001), which emphasizes the use of arts and aesthetics to support each child's development. The curriculum, as a result, follows strict philosophical guidelines, and places little emphasis on school readiness tasks. For example, in our observations, we never saw time dedicated to reciting the alphabet, rote counting for counting's sake, or calendar time. Indeed, these types of activities ran contrary to the Reggio philosophy, and this was strongly expressed by the teachers in our project as we went through our coursework together.

Walter, on the other hand, is located across town in a more diverse part of the city. Situated in a community center—hosting a variety of activities for children and families—the preschool predominantly serves low-income children and families, the majority of whom are African-American and Latino/a. While the curriculum is play-based and follows developmental guidelines (such as Developmentally Appropriate Practices (Copple & Bredekamp, 2009)), in our observations, traditional academic tasks were seen on a daily basis. While play was a central tenet of the curriculum, so was getting these children who were considered "at risk" ready for school.

The transition between these two settings was not an easy one for Enid. According to Enid, between the different approaches to curriculum and a revolving door of co-teachers, her first six months at Walter were quite rough. Given this big transition, Enid, early on, expressed concerns about her ability to conduct an action research project. This mostly manifested itself in Enid's difficulty in finding a question around which to frame her research. While many of the teachers in our cohort were concerned about undertaking research, after a few weeks of guidance through the course, they got excited and dove into their work. By early October, all of our teachers except Enid had settled on a question and had begun thinking about data collection. Enid, on the other hand, kept coming back to class each week with a new idea, and had trouble settling on one.

An event in Enid's classroom in mid-October, however, prompted her to consider funds of knowledge in a new way and select her focal child, Philip. As Enid described in a check-in session, "[We were singing] love grows, this…song, it goes '1 by 1 and 2 by 2', and usually in the song we do it with our fingers, but he [Philip] couldn't do it obviously." Given our professional development's focus on early math, Enid had been faithfully trying to integrate more counting experiences for her students. In doing so, she realized that many of the songs, finger plays, and manipulatives that she introduced in the classroom were not accessible to Philip because of his disability. This presented a dilemma that Enid wanted to solve, and we encouraged her to make this the focus of her action research project.

At first, Enid wasn't sure that focusing on a child with a disability would meet the theoretical underpinnings of funds of knowledge that we had been learning about and applying in our work together. We assured her that it would, and that we would help her to consider funds of knowledge from many different perspectives in order to gather enough data to know how to better support Philip. As professional researchers, we saw Enid's project as one with many sources for data. Enid, however, felt that the question was perhaps too big and was unsure of where to start. In fact, uncertainty became the main demon in Enid's work on her action research project. We worked with her to shore up her confidence in the choices that she was making, but each week she would come to us with a fair level of uncertainty about how to proceed. Initially, we pushed her to consider the many different ways in which she could gather rich funds of knowledge about Philip, and worked with her to make a list of access points, such as talking with his occupational therapist who visited weekly at Walter and having a home visit to help draw attention to not only how

Philip's parents accommodated his needs at home but also to what cultural practices around mathematics she might find. With this list under her belt, Enid seemed more confident and proceeded to the data collection phase of her project.

Multiple Perspectives on Funds of Knowledge

To garner as much information about Philip's multiple funds of knowledge as possible, we encouraged Enid to begin to look for ways to better understand both his condition and the ways in which his engagement with mathematical thinking might be hindered by her current approaches to mathematics experiences in the classroom. We encouraged Enid to take a broad perspective on what funds of knowledge she could access to support Philip: knowledge of his condition, familial and cultural practices around mathematics through a home visit, information about how to best support Philip in the classroom, and greater knowledge of Philip's popular culture interests. In addition, we asked Enid to take notes and record her observations about the mathematics curriculum experiences that she planned and whether or not Philip was able to participate in the ways that she had intended for the other children.

Enid, however, wasn't sure how she was going to manage this given the newness of her position at Walter:

> It was hard sometimes because while I was in class, I was so focused on doing my day- to-day class things that a lot of times, it'd be hard to remember or try to narrow in and focus on just that one particular child.

We continued to encourage her in this initial data collection, however, and as the fall semester progressed, she began to see and record patterns in how she would set up mathematical experiences to support Philip's level of engagement. These notes became the data that she used as the basis for her analysis of the mathematics experiences she was making available in the classroom, and to formulate better early mathematics experiences for Philip.

In terms of accessing funds of knowledge from Philip's family, Enid began by asking his mother to explain Philip's condition to her. From discussions with his mother, Enid was able to understand the ways in which the family accommodated Philip's condition in day-to-day life. For example, what Enid first took as reluctance to discuss the condition in fact turned out to be Philip's mother's concern about his safety given his condition. His mother worried that all of the things that they did to keep him safe at home would not be able to happen at school. Whereas at home, large pathways were cleared so that Philip could run around safely, if he fell at school he might bump into any number of things. Over the course of the semester, Enid began to build a rapport with Philip's mother that allowed her to ask more and more questions about his condition. Slowly working to build this relationship and showing respect of the great knowledge of Philip's family about how best to care for and support him, Enid found Philip's mother to be a wealth of knowledge.

With the background information from Philip's family, suggestions from his occupational therapist, and reflections on her work in the classroom, Enid generated some ideas of where to start in accommodating Philip's learning needs. As Enid started to reflect on what she had learned from these sources, she began to formulate new approaches to engaging Philip in mathematical thinking, although she did not realize it at the time and expressed some anxiety about data analysis.

In the early part of the spring semester, we asked the teachers to make sure they included a home visit as part of their data. As we mentioned earlier, home visits are a central part of the funds of knowledge approach; they allow researchers (and in this case teacher-researchers) to better access the cultural, linguistic, and mathematical practices and experiences of their students (N. González, personal communication, February 16, 2011). As in the previous semesters, when we had asked the teachers to do home visits with other students, Enid expressed both excitement and ambivalence at the prospect of visiting Philip's home. After completing her home visit with Philip's family, however, Enid felt like she had a lot more information about his interests and was reenergized to create materials that would grab his interest in Ninjas.

One interesting point to note here is that the focus of Enid's gathering of funds of knowledge was two-fold: first, Philip's popular culture interests and second, his medical condition and the ways in which his parents met his needs given his limited arm, wrist and hand movement. By focusing on these things, Enid shifted the focus from cultural funds of knowledge to accessing information about Philip's popular culture interests and his families' funds of knowledge in terms of raising a child with Philip's particular medical condition. While this is not the traditional approach to funds of knowledge, which focuses on accessing culture practices, for Enid, taking this multifaceted approach to identifying interests and practices as resources allowed her to push on her own instructional practices with Philip in the classroom.

Connecting Multiple Mathematics Resources to Classroom Practice

Given all that she had learned about Philip—his interests (ninjas in particular), his family, and his condition—Enid began to integrate these multiple resources. At first, Philip did not seem to like the special attention and materials that were being directed at him. Using the idea from the occupational therapist and physical therapist that Philip could use his feet to point to objects as he counted, Enid created a large hand, laminated it and put Velcro strips on each finger. She then created several sets of small laminated figures that went with the counting songs they sang in class (e.g. frogs for "Three Little Speckled Frogs" and ducks for "Five Little Ducks"). The idea, she told us, was that Philip could tap each one with his foot and then as they sang each round either he (with his toes) or a teacher could remove or add the laminated figure to each finger. This would help to reinforce the idea of one to one correspondence, an important early mathematics skill.

Over the next several weeks, Enid created several versions of this Counting Hand. At first, Philip expressed little interest. Enid, however, remained confident:

> Now, I think it's more using the tools that I've made and keeping a record of how it goes and just consistently trying to encourage him to do it. And I think the other kids will enjoy doing it too, which will make him want to do it.

Each week, Enid would report to us about changes she had made to the Counting Hand – making it larger, making the bit of Velcro smaller so that the objects were easier to remove, or using magnets instead. Philip was warming up to the idea, she said, but was often distracted by other things. For example, Philip's occupational therapist had begun using the Counting Hand in some of their work together, taking off her own shoe and showing him how he could count on the hand with his foot, as Enid was hoping. Because the occupational therapist came during free play-time, however, she had trouble keeping him on task long enough to use the Counting Hand. Then, a few weeks after her home visit with Philip's family, a light bulb went on for Enid:

> In the beginning, when I was doing it, it was like the typical finger play songs, but then after learning that he likes, well ninjas was one I did before the home visit, but I changed the Velcro pieces to make them ninjas. I think definitely the funds of knowledge, walking away, trying to get more ideas about what they're interested in or what they already know about, evolved in my classroom.

With this success, Enid continued to refine the Counting Hand. In her action research final paper, she described how, with each iteration and change, she would notice another way to meet Philip's needs. Philip, too, began to be excited about the new Counting Hands that Enid was developing, and the new counters for the fingers. With the ninjas capturing his initial attention, Enid found that he was hooked, and she began to expand both the types of counters and the nature of the Counting Hand. One example that she highlighted in her action research paper was the song that went with the children's book *Five Little Monkeys Sitting in a Tree* (Christelow, 1993). Instead of a Counting Hand, Enid created a large tree with five magnetic monkeys. When they sang the 5 Little Monkeys song in class, Philip would use the tree to sing and count along.

As Philip became more confident in his ability to use these tools, so did Enid in her ability to support Philip in his mathematical learning. She brought back out the ducks and frogs that Philip had initially dismissed, and he eagerly began counting those while singing. As Enid and Philip continued to work together, Enid began to notice that Philip was not only comfortable with rote counting from 1 to 11, but was beginning to better understand both one to one correspondence and to recognize numbers. As Enid wrote in her final action research paper:

> Philip's interest in the ducks spread to number recognition. When we first pulled the ducks out of the plastic bag, we realized that there was no Velcro on the backs. I asked Philip if he still wanted to use the Counting Hand or if he wanted to count them alone on the tray. He wanted to continue the use of the counting hands. He placed the ducks onto the fingertips very carefully while noticing the written number below. He asked, "What is this number?" We would then count to see what number it was. When we got to the end of the line he counted "…8, 9, zero." I pointed to the numbers as I said "it's a 'blank' (pointing to the 1) and a 'blank'_(pointing to the 0)" and waited for him to fill in the blanks and then he said "ten!" loudly with a smile. During this exercise Philip was able to successfully count from one to seven with one to one correspondence.

Having gained knowledge of Philip's interests and needs, Enid was then able to take these data and transform them into pedagogical practices that supported his mathematics learning. While Enid did not initially see herself as a successful teacher-researcher, over the course of the year, her uncertainty diminished and she continued to have many successes in supporting Philip. Not only was Enid successful with Philip, but also she began to see how she could use similar pedagogical strategies and tools to support other children in her classroom.

Extending Knowledge of One Student to Pedagogical Practices

As Enid continued her work with Philip, she noticed that other children in her classroom became interested in the tools she had created. Enid began to realize that each of her students had a different learning need and style. In Philip's case, a basic need was fairly obvious, and so Enid was able to seize upon this. The physical accommodations were not Philip's only needs, of course, but rather a starting point for Enid to learn more about how to best support his development of mathematical understanding. In the findings section of her final action research paper, Enid wrote:

> When Philip couldn't do the hand or finger movements during a finger play I made manipulatives, picture cards, and song posters for him to point to with his feet. After noticing that the other children in the classroom wanted a turn I made materials so that everyone could join in.

In helping Philip, Enid discovered new ways to reinforce and rejuvenate the early mathematics learning taking place in her classroom. She found that what she was trying with Philip, from hands-on manipulatives to passing and counting beanbags with his feet, was also a wonderful way to re-enforce early mathematics skills with all of her students.

Even though Philip was moving to a new classroom the following year, Enid felt that she had a new sense of ways to naturally and richly engage young children in counting. While Enid developed skills about engaging children's multiple mathematical resources in her work with Philip, we imagine that she will continue to employ these skills in her daily work with young children. She has reaped the rewards, and there is nothing like helping a child to learn and develop to make you want to do it again:

> During his first interview, when I asked Philip if he enjoyed counting he said 'yes' and then sang his ABC's. Philip's response to the same question at the end of the year, after being exposed to the counting adaptations made was, "Yeah, I really, really love counting." This response was followed by a smile and counting. Philip's counting progress and attitude toward counting leads me to think that this study is an accomplishment.

CONCLUSION

What did these two teachers teach us about (a) the varied perspectives on funds of knowledge, (b) how teachers drew on the multiple mathematical resources of their focal child to support his learning to count, and (c) how the teachers extended what they learned from one child to rethink their practices more broadly? Marie and Enid broadened their notion of funds of knowledge in similar ways. They interpreted Hedges (2011) work to mean children's interest in popular (or media) culture was an example of funds of knowledge. Both used children's interests in popular culture (Skylanders and Ninjas) to develop manipulatives to support counting, though they did not explore how these popular figures were manifest in children's play or activities. Thus, their definition of funds of knowledge differs from our perspective. Yet, these manipulatives and the way the teachers used them were effective in supporting both children's learning; this experience made an explicit connection between their interpretation of the theory and the teacher's practice. In both cases, there were two things that triggered a change in perspective: the home visits and an interaction with the child. For Marie, the day Donald counted the teeth on his toy shark opened her eyes to not only Donald's skills but how he was willing to demonstrate them. For Enid, the day she observed Philip's lack of participation during counting songs in which other children were using their fingers to follow along led to her interest in understanding how to make those experiences accessible.

We consider children's multiple mathematical resources to include children's mathematical thinking, funds of knowledge, and engagement in play. Marie in particular had experience with CGI and recognizing children's thinking. Identifying where Donald was mathematically and how he approached counting was somewhat second nature to her; albeit in a way initiated by the teacher rather than following the child's ideas. Enid, too, recognized Philip's skills but was more focused on the physical strategies he used. Direct modeling is prevalent during counting songs with young children, yet Philip was not able to direct model in traditional finger counting ways. Both teachers came to recognize the children's interests in popular culture as a resource they could connect to the classroom. Enid also reframed Philip's physical disability by considering his strengths (adept use of his feet) rather than focusing on what he could not do. And, both situated the learning in direct interactions with the child and within play.

All of the teachers in the professional development had struggled with how to connect knowledge of one child to the rest of the classroom. The action research projects provided Marie and Enid with the opportunity to see how that might happen. Once they saw this in practice they acknowledged its power. Part of the process of using funds of knowledge is encouraging teachers to trust that as they build knowledge of one child, they will use these same research skills to implicitly respond to other children within the classroom in a meaningful way. A major success of the action research approach was how it married the processes of funds of knowledge with those of teacher research. Since both are research based, and require the teachers to engage deeply with children, families and classroom practice, the direct rewards within the classroom are quite high.

We hope that these examples of meaningful teacher-research will help encourage other teachers and teacher educators to marry funds of knowledge and action research approaches. In doing so, we strongly believe teachers can find ways to better support the learning and development of their students through the curriculum. We recognize that although the teachers found success with a focus on children's interests in popular culture, this was a first step towards understanding and incorporating funds of knowledge and continued critical reflection is needed to push the boundary between theory and practice. We have moved far away from a time when teachers trusted their implicit professional knowledge as the main guide to the curriculum (Crawford, 2004). Instead, the focus on accountability has robbed many teachers of this faith in their abilities and their training (Goldstein, 1997). Our hope is that a return to teacher-led research with a focus on funds of knowledge will help to rebuild this faith in the power of teachers to know their students and create responsive curriculum.

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Making meaningful connections with mathematics and the community: Lessons from prospective teachers[4]

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INTRODUCTION

Research shows that children benefit from instruction that draws upon their cultural, linguistic, and community-based knowledge (Gay, 2009; Ladson-Billings, 1994). Specific to mathematics, teachers need to understand how children's *cultural funds of knowledge*—the knowledge, skills, experiences, and practices found in students' homes and communities—are resources that can support students' mathematical learning (Civil, 2007; Foote, 2009; González, Moll, & Amanti, 2005; Lipka et al., 2005). Furthermore, teachers need opportunities to develop instructional practices for noticing, eliciting, and incorporating children's funds of knowledge in their mathematics instruction. As Grossman, McDonald, Hammerness, and Ronfeldt (2008) note, teachers need to learn to use knowledge of children's cultural and linguistic resources in ways that help children succeed academically. While the field has made strides in understanding how to prepare elementary teachers for other important aspects of effective mathematics instruction, such as how to notice, elicit, interpret, and respond to children's mathematical thinking (Jacobs, Lamb, & Phillip, 2010; Kazemi, Franke, & Lampert, 2009; Vacc & Bright, 1999), less is known about teachers' competencies and practices for connecting to children's cultural funds of knowledge in their mathematics teaching.

The persistent cultural gap between the largely White, female, monolingual and middle class teaching force and the ethnically, linguistically, and socioeconomically diverse student population (Hollins & Guzman, 2005) suggests that making these connections may be challenging for teachers. Specifically, many prospective teachers bring minimal experience with students from diverse cultural, racial, and linguistic backgrounds (Silverman, 2010), and teacher education programs typically provide limited opportunities to interact with families and communities, or to investigate mathematical practices outside the school setting (Burant & Kirby, 2002). Researchers have found that when prospective teachers attempt to connect to children's home and community-based experiences as they plan and adapt mathematics lessons, the connections they make are often superficial. For example, they change names and objects in word problems to reflect students' interests (Nicol & Crespo, 2006; Vomvoridi-Ivanovic, 2012), or design tasks based on assumptions about students' cultural experiences, or generalized notions about what is relevant to children (Aguirre et al., 2013). This tendency towards surface level connections reflects

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the challenging nature of this practice. That said, even limited connections to children's experiences or interests reflect prospective teachers' efforts to build relationships and rapport with their students, and in this way, may serve as a first step towards learning about and connecting to knowledge derived from children's families and communities.

Given the key role of connecting to children's funds of knowledge in effective mathematics teaching, coupled with our limited understanding of how future teachers take up this practice, it is important to study instances when prospective teachers evidence these connections. In this paper, we expand on a prior analysis (Aguirre et al., 2013) to carefully examine promising cases where prospective elementary teachers made substantive connections to children's cultural funds of knowledge in mathematics lesson plans. More specifically, we investigated how prospective teachers positioned and connected to mathematical and other cultural practices in children's homes and communities across different parts of a mathematics lesson.

Connecting to Children's Cultural Funds of Knowledge in Mathematics Teaching

Consistent with the work of Civil (2002, 2007) and Lipka and colleagues (2005), we are particularly interested in teachers' connections to knowledge and experiences related to mathematics, as these cultural funds of knowledge have the potential to support students' school mathematics learning (González, Andrade, Civil, & Moll, 2001; Ladson-Billings, 1994). To contrast with more surface-level connections to students' interests, we use the term *substantive connections* to children's cultural funds of knowledge to signal instances when teachers elicit and build upon children's home and community-based activities and experiences in their mathematics lessons (e.g., drawing on children's experiences estimating the cost of grocery purchases in a lesson on addition and subtraction).

Developing Competencies for Making Substantive Connections

Research suggests that an important task for teacher education programs is helping prospective teachers to recognize students' home and community-based experiences as resources that have the potential to support mathematics learning. This may involve reorienting prospective teachers to the social, linguistic, and cultural practices of diverse communities, so that they begin to position these practices as strengths versus barriers to children's learning (Bartell et al., 2013; Bleicher, 2011; Valencia, 1997). Also important is helping prospective teachers to recognize that families bring knowledge and skills that could enhance teachers' work in the classroom (Graue & Brown, 2003). Prospective teachers might benefit from experiences such as structured interviews that focus on students' knowledge and out-of-school experiences (Downey & Cobbs, 2007), and shadowing activities aimed at identifying children's and families' competencies across multiple spaces (Bartell et al., 2013; Foote, 2009).

While field experiences that include interactions with students from diverse backgrounds are important, prospective teachers also need scaffolded experiences that go beyond the classroom and into the community (Burant & Kirby, 2002). Civil (2002, 2007) described how teams of researchers and teachers conducted home visits to dialogue with students' family members about home and work activities, with the goal of designing mathematics units that drew upon families' funds of knowledge (e.g., planting a garden; game-playing). Other research documents after-school programs as productive spaces for prospective teachers to learn about children's cultural funds of knowledge and to consider how they might draw upon children's knowledge and experience in their mathematics teaching (Vomvoridi-Ivanovic, 2012). Methods courses can also help prospective teachers to learn about community mathematical resources through experiences such as visiting community locations and talking with families and community members about how they use mathematics (Aguirre et al., 2013; Turner et al., 2012). While these studies reflect an increased emphasis on interactions with children, families and communities, our understanding of how prospective teachers use what they learn about children's funds of knowledge in their mathematics teaching is still limited.

Substantive Connections to Children's Cultural Funds of Knowledge in Mathematics Lessons

One instructional practice that may evidence teachers' understandings related to building on children's cultural funds of knowledge is planning and teaching mathematics lessons. Research tends to highlight the work of individual teachers, and to outline in general terms how they honor and connect to children's cultural funds of knowledge. For example, Bonner and Adams (2012) described Ms. Finley, a veteran African American teacher who worked in a predominantly African American community. Ms. Finley drew upon her knowledge of students' home and community experiences in various ways, such as using students' experiences with music and rhythm to support understanding of mathematical ideas, and connecting new mathematics concepts to family experiences (e.g., relating the concept of fact families to relationships among students' family members). While portraits of how veteran teachers connect to children's cultural funds of knowledge in their mathematics lessons are useful, still needed are broader studies of how teachers take up this critical teaching move.

Research by Taylor (2012) and Wager (2012) explored how elementary teachers use what they learn about children's cultural funds of knowledge (in particular their out-of-school mathematical practices) to design mathematics lessons. This emphasis on *mathematical practices* is important, because as González, Moll, and Amanti (2005) note, a focus on what people in specific contexts actually *do* and *say* can help teachers to avoid assumptions based on cultural stereotypes. Both Taylor (2012) and Wager (2012) found that while teachers often used familiar out-of-school activities as the context for teaching particular mathematical concepts (e.g., teaching students about area by posing problems about measuring the area of a soccer field), they rarely connected to ways that students or families used mathematics in home or community settings. However, Taylor found that by using targeted probes to focus teachers' attention on children's use of mathematics outside-of-school, and by presenting examples of how other teachers have connected to children's mathematical practices in their lessons, teachers' capacity to connect to children's cultural funds of knowledge improved over time.

In summary, studies with practicing teachers suggest that while connecting mathematics lessons to children's cultural funds of knowledge, and more specifically to mathematical practices in children's homes and communities, is challenging, it is something that teachers can accomplish with guidance and support. Still needed is an understanding of how *prospective* teachers make these more substantive connections to children's cultural funds of knowledge in their mathematics teaching.

Connections to Children's Funds of Knowledge as a High-Leverage Practice

We consider making substantive connections to children's cultural funds of knowledge to be a "high leverage" or "generative" teaching practice (Franke & Chan, 2006; Franke & Kazemi, 2001; Grossman, Hammerness, & McDonald, 2009). Specific to mathematics teaching, Franke and Chan (2006) have defined high-leverage practices to be "those aspects of mathematics teaching practice that are central to supporting the development of mathematical understanding, generative in nature, and productive starting places for novice teachers" (para. 3).

Grossman and colleagues (2009) present a similar set of criteria, adding that high-leverage practices are accessible and learnable research-based practices that are adaptable across various contexts, curricula, and instructional approaches, and that have the potential to support student learning. We argue that making substantive connections to children's cultural funds of knowledge in mathematics teaching meets these criteria in that (a) it is a research-based practice shown to support student learning; (b) it is adaptable across contexts where teachers at various grade levels and using various curricula can learn about and connect to students' home and community-based knowledge and experiences in their lessons; (c) it is generative in that as teachers begin to elicit and connect to children's cultural funds of knowledge in their mathematics teaching, this move not only supports students' learning, but also enhances teachers' understanding of children's funds of knowledge and their capacity to connect to this knowledge in the future; and (d) it is learnable and accessible to novices.

Mathematics education researchers tend to focus on high-leverage practices that foreground children's mathematical thinking, such as eliciting students' solution strategies (Kazemi, Franke, and Lampert, 2009), identifying patterns in students' thinking and common misconceptions (Ball & Forzani, 2010), and using prepared instructional routines (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). We contend this focus on children's mathematical thinking needs to be coupled with an equally important focus on how knowledge, experiences, and mathematical practices from students' homes and communities can support students' mathematics learning. Expanding the discussion of high leverage practices to include teaching moves such as eliciting and building upon children's home, cultural, and linguistic funds of knowledge positions these practices as core activities of mathematics teaching, versus as a set of peripheral moves, only relevant in certain contexts or with particular groups of students.

Consistent with this goal, this analysis focused on how prospective teachers made substantive connections to children's cultural funds of knowledge in mathematics lessons that also included careful attention to children's mathematical thinking. Examining lessons that coupled connections to children's cultural funds of knowledge with attention to children's mathematical thinking is important, because it demonstrates how a focus on cultural knowledge can be integrated with (rather than detract from) other widely valued high-leverage teaching practices. While we examined lessons evidencing this integrated focus, our analysis centered on connections to children's cultural funds of knowledge. The following research questions guided the study.

1) How do prospective teachers make substantive connections to children's cultural funds of knowledge in mathematics lesson plans that also attend to children's mathematical thinking? More specifically,

a) How do prospective teachers identify and connect to mathematical and other cultural practices in children's homes and communities?

b) How do prospective teachers elicit and connect to children's cultural funds of knowledge across the different parts of a mathematics lesson?

c) How do prospective teachers position children's families and communities in their lessons? (e.g., positioning cultural knowledge as a resource, or a deficit; positioning families as actively contributing to their child's learning).

Understanding different ways that future teachers make substantive connections to children's funds of knowledge in their mathematics lesson plans is important because it provides insights about different entry points into this critical, high leverage practice, which mathematics teacher educators can use to support prospective teachers' learning.

METHODS

Project Background

The data for this analysis come from a multi-site National Science Foundation funded research project called Teachers Empowered to Advance CHange in Mathematics (TEACH MATH, http://mathconnect.hs.iastate.edu). A primary project goal is to support K-8 mathematics teachers to develop understandings and practices that connect to children's mathematical thinking and children's community and cultural funds of knowledge (Aguirre et al., 2012, 2013; Bartell et al., 2013; Foote et al., 2013; Turner et al., 2012). One way we attempt to accomplish this goal is by developing instructional modules for elementary mathematics methods courses that explicitly foster these connections. The focus of this analysis is on the lessons that prospective teachers prepared as part of the Community Mathematics Exploration Module (described below).

Prospective Teacher Participants

For this analysis, we focused on the work of 29 prospective teachers from across three research sites. These research sites reflect a diversity of both geographic contexts (e.g., suburban, borderland, and a mix of urban and suburban) and program contexts. Overall, participants reflected the prospective teacher population nationwide—predominantly White, middle-class females in their early 20s (Hollins & Guzman, 2005). More specifically, 27 of the 29 participants were female, and 24 identified as White (the remaining five participants identified as Latina, Indian American, or Asian American).

Community Mathematics Exploration Module

The Community Mathematics Exploration module was designed to help prospective teachers learn about mathematical practices in community settings, and to utilize what they learned to design a problem solving-based mathematics lesson. Projects were typically conducted in pairs or small groups. After talking with students in their field experience classrooms about places they frequented in the community, prospective teachers visited one or more sites, closely observing mathematical practices, and talking with students, parents, and community members about their activity. Next, prospective teachers designed, and in some instances taught, a problem-solving based mathematics lesson that connected to their community mathematics experience. They also reflected on their experiences, describing what they learned about their students' communities, and the benefits and challenges of mathematics teaching that connects to community contexts (see Appendix A for the Community Mathematics Exploration assignment).

Data Sources and Analysis

Data sources for this study included the written artifacts that prospective teachers produced during the Community Mathematics Exploration module: group and individual written reports, accompanying lesson plans, and individual reflections. In a previous analysis (Aguirre et al., 2013), we examined 70 Community Mathematics Exploration projects from across three research sites. We identified three categories of projects that reflected a progression of connections to children's cultural funds of knowledge and children's mathematical thinking: emergent connections (n=37, or 53% of projects), transitional connections (n=21, or 30%), and meaningful connections (n=12, or 17% of projects). In this analysis, we focused on the 12 projects (representing the work of 29 PSTs) that were identified as making *meaningful*, or more substantive, connections to children's cultural funds of knowledge and children's mathematical thinking (see Table 1).

Table 1. *Overview of 12 Community Mathematics projects with meaningful connections*

Grade	Lesson Title and Context	Mathematics Content	Lesson Task	Connections to Practices
K	*Musical Hula Hoops* at the Community Center	Addition and subtraction; combining and separating sets	Model addition and subtraction by joining or separating sets of objects (students, hoola hoops) that have 10 or fewer total objects.	Children's game playing practices
1st	*Shortest Route* between neighborhood landmarks (school, park)	Adding whole numbers; comparing distances; longer vs. shorter	Find the shortest distance to walk from a specific community location to the school. How many blocks is the shortest route?	Family practices walking to and from school and other places in community.
2nd	"Carnival Rides" at the annual *Church Carnival*	Finding different number combinations that equal to exactly 25	Find at least two different ways to spend all 25 tickets for carnival rides.	Individual and family decision-making practices attending a local carnival
2nd	Planning Party at the local *Mexican Bakery*	Problem-solving; multiple operations	Solve problems related to planning a birthday party on a budget.	Planning and purchasing practices for birthday parties.
3rd	*Bus Pass* Math on Wheels	Single-and multi-step word problems with whole numbers	Determine whether more cost effective, to by a weekly bus pass or pay individual fares.	Decision-making practices related to purchase of bus pass.
3rd	*Pool Pass* at Community Swim Center	Problem solving; addition and multiplication	Determine if a “better deal” to buy a family pass or individual swimming passes.	Decision-making practices related to swim pass purchase.
3rd	Abuela’s shopping List at *Las Socias* Tienda	Problem solving with whole numbers	Determine if they have enough money to purchase items on Abuela's list. Use multiple representations (e.g., Pictures, equations)	Purchasing practices at the community store.
3rd	Planning a *Pizza Party* at Round Table Pizza	Whole number problem solving; estimation; graphing; fractional representations.	Consulting families to determine a reasonable amount of pizza per person. Deciding how much pizza to order for class.	Decision and purchasing practices at the pizzeria.
3rd	*La Lavandaría,* a local laundromat	Multi-step problems using various operations including addition and multiplication.	Determine how much it will cost to wash (not dry) the clothes, calculate maximum and minimum possible price.	Student and family decision-making in doing laundry.
4th	*Laundromat Specials* at a local Laundromat	Multi-step word problems; multi-digit multiplication.	Determine which laundromat deals are the best deal for an imaginary family and own family.	Decision-making based on price comparisons and quantitative reasoning.
4th	*Library Late Fees* at the local public library	Problem solving involving multiplication and division; reasoning about remainders.	Calculate library late fees for various scenarios. Write a position letter to about whether the library late fee policy is fair.	Patron practice of checking out books and the policies and costs associated with this practice.
4th	Skate Ramps at the local *Skate Park*	2-dimensional area (area formulas); Surface area of 3-dimensional shapes.	Find the surface area of a skate board ramp to determine how much paint is needed to repaint the ramp.	Possible decision-making practices to repair skateboard ramp at park.

The lessons included multiple opportunities to elicit and build on children's home and/or community experiences and their mathematical thinking, and the problem solving tasks were both cognitively demanding and connected to authentic practices that occurred in the community setting. In this analysis, we further examined these 12 projects to better understand different ways prospective teachers make substantive connections to children's cultural funds of knowledge in mathematics lessons. Drawing on key ideas outlined in our framework, and following the principles of analytic induction (Bogdan & Biklen, 1992), we coded each of the 12 projects according to multiple dimensions. These included:

a) ways of eliciting and building on children's cultural funds of knowledge across the different parts of the lesson (e.g., launching lessons by asking students to share about out-of-school experiences; posing small group tasks that require students to use knowledge from home or family practices);

b) the home and community-based practices, particularly mathematical practices, that prospective teachers identified and linked to their lessons (e.g., mathematical practices of families, students, consumers, workers, or community members; imagined or assumed mathematical practices versus those that were observed or reported by children; cultural and linguistic practices); and

c) how prospective teachers positioned students, families, and communities, including their cultural and linguistic practices, in their lessons (e.g., positioning cultural knowledge as a resource or a deficit).

In the initial round of coding, the first three authors independently coded the same subset of projects (n=3 of 12) and then met to discuss and compare codes. Disagreements were discussed and code definitions were refined until agreement was achieved. In the second round of coding, the remaining nine projects were each coded by two members of the coding team. During this second round, coding differences were minimal, and those that did occur were discussed until agreement was reached. We then created an analytical memo for each of the 12 projects that summarized and provided evidence for each of the codes. Analysis across analytic memos resulted in themes related to our three research questions. We report on these themes next.

FINDINGS

Identifying and Connecting to Mathematical Practices in Community Contexts

A common feature of the projects with substantive connections to children's cultural funds of knowledge was that the problem-solving lessons connected to authentic (or at least potentially authentic) mathematical practices from the community setting. By authentic, we mean mathematical practices prospective teachers witnessed or engaged in themselves, or practices reported by families, children, or other community members during interviews. These connections were typically inspired by teachers' interactions with community members, including customers, families, children, and workers. In a few instances, prospective teachers drew on their own experiences in the community to imagine how children and families might use mathematics as they participated in a given site. Next, we describe different ways that prospective teachers learned about and connected to community mathematical practices.

Connecting to specific practices of children and families. Parents and students often served as vital resources that guided site selection and lesson design. For example, in the *Lavandería* project, two prospective teachers were interested in learning more about the growing Latino/a community surrounding their field placement school. They asked a Latina parent volunteer if she would be willing to show them around the community. They met on a Saturday morning and walked around the neighborhood, visiting the mobile home park where many of the Latino/a families lived, *una tienda* where she shopped for Mexican food products, and the *lavandaría* where she met other mothers to wash clothes every Saturday. According to their report:

> The laundromat, where women's voices competed with the sounds of the *telenovela* blaring from a grainy television set, was particularly interesting to us. The student's mother introduced us to several of her friends who were filling washers with tiny children's clothing while other women looked on with amusement as we took pictures and recorded data.

During their guided tour, the prospective teachers learned how many loads of laundry families washed per week, and the average amount of money spent. They also gathered information about the cost of standard and double-load washing machines, and talked with families about their laundry decisions (e.g., estimating the number of loads, deciding which washers to use). The information they gathered about families' practices inspired the following third-grade lesson task:

> Lesson Task: You live with your three siblings, your two parents, and your grandmother. Every week, your family has lots of dirty laundry that needs to be washed – today, in fact, you have 10 loads of laundry! You go to the laundromat to help your mother. She wants to know how much it will cost to wash (not dry) all the clothes. Can you help her? How many solutions are there? What is the maximum and minimum that you might pay? (You already have the detergent). After the washing…now you and your mother have 10 loads of wet laundry. How much will it cost to dry all the loads? (On average, each load of laundry will need 45 minutes to dry).

The prospective teachers described the lesson as a way to celebrate the knowledge and resources in students' communities, including how families reason mathematically in out-of-school settings. They explained, "if we are allowed to present this lesson, we will attempt to bring in the student's mother (or other parents) to encourage collaboration with the community."

In this and other projects, it is not surprising that families often described their activity in ways that did not make explicit mathematical ideas and processes (Civil, 2002). We found that to make substantive connections to children's cultural funds of knowledge in their mathematics lessons, prospective teachers analyzed children's and families' practices to identify possible mathematical connections. For example, in the *Shortest Route* project, two prospective teachers went on a community walk with their cooperating teacher and her first grade son. During the walk, prospective teachers learned that many students walked to school, and that some, including their first grade tour guide, preferred particular routes because they included "shortcuts." One prospective teacher noted,

> The student we were walking with mentioned that he knew of shortcuts to get home faster. When we asked him why he chose to take these shortcuts he said, "So I don't have to walk as long." This statement made us aware that this first grader understood shorter distances, and switched the light bulb in our heads!

While the student did not describe in detail how he figured which routes were the "fastest," the prospective teachers recognized that comparing different routes to school (and to other neighborhood locations such as parks) was an authentic activity for children, and one that involved mathematical ideas. They used this information about children's activity as the inspiration for a problem-solving task: Find the shortest distance to walk from a specific community location to the school. How many blocks is the shortest route? Prospective teachers explained that this task would help students consider how mathematical reasoning was connected to an out-of-school activity such as deciding how to walk to and from school.

Connecting to broader community practices. In other cases, projects were inspired by broader activities observed during the community walks. This sometimes included explicit mathematical activity, and in other instances, cultural and linguistic practices in a community setting. For instance, in the *Bus Pass* project, prospective teachers were struck by the level of activity at numerous bus stops in the neighborhood. They knew many families rode the bus to and from school, and observed the key role public transportation played in the community. They noted:

> The community-walk exploration revealed a strong local usage of public transportation. Bus riders included local business owners, local residents, and schoolchildren. Therefore, participating in this community means having knowledge of the bus system as a transportation resource.

The prospective teachers then designed a third-grade problem-solving mathematics lesson that connected "to the mathematics involved in bus riding," particularly calculating fares and comparing the price of different bus pass options. They explained:

> This math lesson focuses on personalized calculations for bus routes. [In preparation for the lesson] students, along with their parents, are asked to note their bus riding destinations and frequency of rides in a week. Students note the cost of the legs of their trip, using single-fare and weeklong-pass rates. The information is charted and compared, students drawing conclusions about cost-effectiveness, based on their own data.

While prospective teachers did not have specific information about how families reasoned about bus pass options, by inviting families to discuss their transportation needs and to generate data for the lesson, the teachers opened a space for families to talk about how they might use mathematics to inform their decisions. Also, families' knowledge about transportation was framed as a resource to support students' reasoning in the lesson.

Another way that prospective teachers connected to community practices in their projects was by attending to cultural and linguistic activity. For example, in the *Mexican Bakery* project, teachers visited a neighborhood bakery and noted that workers and patrons interacted almost exclusively in Spanish. The prospective teachers felt that some of their Latino/a students, who had Mexican heritage and spoke Spanish at home, might more readily engage in conversations at the bakery than at school where English is the language of instruction. In their group report they noted:

> The Mexican Bakery stood out for a couple of reasons. First, the staff spoke Spanish and … we reflected that many of our students could converse better here than in our classroom. Incorporating Spanish helps Spanish-speaking students to become excited about our lesson and proud to use their knowledge of Spanish vocabulary in the lesson.

The prospective teachers then drew upon their knowledge of how children in the community celebrated birthdays to design a lesson that asked students to plan a birthday party. They explained, "We noticed that the store had almost everything necessary for a birthday party. The store had piñatas, candy for the piñata as well as their well-known (within the community) *pasteles de tres leches*, three-milk cakes." The prospective teachers decided to use the Mexican bakery as a context for their lesson both because it honored the use of Spanish in the community and because it connected to cultural practices surrounding birthday celebrations.

Leveraging own experiences to imagine mathematical activity. In a few instances, prospective teachers drew on their own experiences in community settings to imagine ways that children and families might use mathematics. For example, the two prospective teachers that designed the third grade *Pizza Party* lesson knew that many students frequented a local pizzeria. The teachers visited the pizzeria with their own families and reflected on how they used mathematics as they placed their order.

> We immediately faced several questions that many customers face there. How many and what size pizzas did we need to feed our party? How could we equally divide a pizza between people based on how many slices were in each size? What combinations of pizza topping choices could you use so everyone gets the topping they want (i.e. half pepperoni, half sausage & black olive)?

Then, they crafted the lesson task asking students to figure out how much pizza they needed to order for their upcoming class party. Solving the task involved gathering information about how much pizza each student would eat, and then using that information to calculate the total number of pizzas needed, a process that reflected how the prospective teachers used mathematics when visiting the pizzeria with their families.

Similarly, in the *Laundry Dilemma* project, a prospective teacher visited a small laundromat in a commercial center adjacent to her school. She knew students participated in laundry practices at home, and that some students had experience with this site. However, the specific lesson tasks were inspired not by knowledge of specific family practices (as occurred in the *Lavandería* project) but by the teacher's own visit and her reflections on how she would calculate the cost of washing and drying clothes, or determine which of the laundromat specials "might be the better deal." She noted:

> It presented the problem almost effortlessly as I thought about how I would figure out how to do my own laundry there. I think that putting myself into the mindset of a person who would visit one of these local shops helped me to think about the kinds of mathematic[s] that are required in order to use those services or facilities.

In examples like the *Pizza Party* and the *Laundry Dilemma* project, prospective teachers focused on their own mathematical practices in the community setting. Whether prospective teachers' own practices mirrored those of children and families is unclear. Teachers would certainly benefit from conversations with families about their activity, as basing lessons on assumptions about families' practices may result in lessons that misrepresent or fail to connect to children's cultural funds of knowledge (Hedges, Cullen, & Jordan, 2011). That said, we see the fact that prospective teachers drew on their own experiences in community settings as important, because it represents another possible entry point to making connections to mathematical practices in the community.

Drawing on Children's Funds of Knowledge in Lesson Design

Another important feature of projects with substantive connections to cultural funds of knowledge was the lessons included multiple and varied opportunities for children to draw on home and community-based knowledge and experiences to support their participation and sense making. In these lessons, prospective teachers not only considered students' experiences as they designed the task; they opened up additional spaces for drawing on students' cultural funds of knowledge across different phases of the lesson, including the introduction, task exploration, and lesson summary.

Connections in the lesson introduction. In several cases, prospective teachers elicited students' community-based knowledge and experiences in the lesson introduction. In the *Skate Park* project, prospective teachers learned during an informal home visit with a student and her foster mom that a neighborhood skate park was a "social hub" for local youth. Based on this conversation and observations at the park, the teachers designed a fourth-grade lesson that involved calculating the surface area of a new ramp at the skate park to determine the amount of paint needed to paint the ramp. The lesson launch elicited students' experiences with skate parks, via questions such as "Who has skateboarded before?" and "What do you find in a skate park?" The teachers planned to show images of skate park activity to further promote discussion. The lesson introduction also probed students' ideas about the concept of area through questions such as, "What do you know about finding area? How do you think the builders of the skate park might use area?" and "How do you think you would find the [surface] area of a skate park that has ramps and different levels?" Their goal for the lesson was for students to leverage knowledge of area of two-dimensional shapes such as rectangles and triangles to investigate the surface area of three-dimensional objects such as a skateboard ramp.

The *Las Socias* project provided another example of how prospective teachers opened a space for students' cultural funds of knowledge in the lesson introduction. The teachers began the lesson by having two bilingual (Spanish/English) students role play an interaction between a grandmother and a grandchild who were preparing to go shopping at Las Socias. The unscripted role play asked students to discuss what to buy, how much different items cost and the total amount of money that could be spent. The purpose of this introduction was to elicit and connect to students' experiences shopping with family members (including possible experiences comparing costs or considering a budget), and also to honor the language practices in many students' homes (i.e., many bilingual students interacted with family members in Spanish). The introduction positioned both the linguistic and cultural knowledge children brought to the lesson as important resources for supporting their participation and sense making.

Connections as students work on lesson tasks. Prospective teachers also planned spaces for drawing on students' cultural funds of knowledge during the main lesson task. In the *Bus Pass* project, students were asked to talk with family members about how often they used the bus, and for what purposes, and to bring the information to class. During the main lesson task, students made a table to organize their data. The table included information about "where they take the bus to and how many times they use the bus in a week." Students then analyzed the data to determine which bus pass was cost effective for their family. By framing the task in this way, the prospective teachers opened a space that invited students' knowledge about their family's public transportation usage.

The *Shortest Route* project, in which first grade students figured out the shortest distance between two neighborhood locations, offers another example of how students were invited to draw on out-of-school-based knowledge and experiences as they solved lesson tasks. For example, the prospective teachers noted that students "had experience walking from these places in the community in their own lives. They could easily visualize this [walking from one location to another] because it was something very real to them." Elaborating on this idea, the lesson plan noted students may consider "cutting through yards" as a way to decrease the distance between two locations. Similarly, prospective teachers recognized students may generate multiple possible routes or "use their personal experiences to route how *they* go from place to place (in a car or walking)," which prospective teachers described as strategies "to be applauded" and that could help students determine which route was the shortest. In short, prospective teachers not only anticipated ways students might leverage out-of-school knowledge as they worked on lesson tasks, but the lesson included strategies for eliciting and celebrating this knowledge as a resource for mathematical understanding.

Connections in the lesson summary and lesson extension. Other Community Mathematics projects opened spaces for students' cultural funds of knowledge in the lesson closure. For example, in the *Laundry Dilemma* project, the lesson concluded with a homework task that invited students to draw on what they learned during the lesson to consider the laundry needs and practices of their own families.

> Homework Task: Students will take the handouts home and find the most cost effective way to wash their own family's laundry at the laundromat each week.

Solving this task required students talk with family members about the amount of laundry to be washed each week, and to consider ways to meet their family's needs (i.e., Should they purchase detergent at the laundromat? Do they want to dry everything in the dryer?).

Another example of how lesson closures opened spaces for students' cultural funds of knowledge was the *Library Late Fee* project. Prospective teachers visited a local library that was a strong partner with their elementary school. Through conversations with the librarian, they identified different ways children and families might use mathematics in the library. Their fourth-grade lesson focused specifically on the library's late fee system, including a "Book Bucks" program in which late fees for children's books were reduced by $1.00 for every 30 minutes a student read. During the lesson, students calculated and compared late fees for adults' versus children's books, and reasoned about different payment options. The lesson closure asked students to "speak their opinions by writing a letter to the library," explaining whether the library late fee system was fair. The lesson plan stated:

> Is the late fee system fair? Is it fair to charge adults more than kids? Write a letter to the Library, and defend your stance on the late fee system and back it up with mathematical evidence. If you have a better system include it in your letter.

Prospective teachers encouraged students to draw not only on the mathematical analyses they completed during the lesson, but also on their own experiences and opinions related to the late fee system. In this way, the lesson summary created an opportunity for students to consider how their experiences outside of school connected to the mathematical concepts and skills they explored in the classroom.

These examples highlight various ways prospective teachers connected to children's home and community-based knowledge across different components of their lessons. We see opening multiple spaces where students can use out-of-school experiences to support their sense making as a critical component of connecting to children's cultural funds of knowledge in mathematics instruction.

Positioning Students, Families, and Communities

Our third set of findings focuses on how prospective teachers positioned students, families, and communities in their lesson plans. A key rationale for the Community Mathematics Exploration module was for prospective teachers to get to know the communities and neighborhoods of their students, with the aim of counteracting deficit views prevalent in school and societal discourse (Valencia, 1997). We found that most prospective teachers acknowledged home and community resources, and made efforts to build upon families' knowledge and practices in their lessons. However, a few projects reflected more inconsistent or mixed perspectives, at times emphasizing the strengths of children and families, and in other instances framing families as lacking important understandings. In the next section, we describe these patterns related to positioning.

Positioning children and families as active decision makers. Prospective teachers' lessons often positioned children and families as knowledgeable and active decision makers. This positioning was reflected in tasks that revolved around decision-making practices observed at the community site, such as party planning, doing laundry, buying a bus or swim pass, walking to school, or paying fees for late library books. For example, in the *Mexican Bakery* Project, prospective teachers designed a task that positioned students as active decision-makers that could weigh multiple options, generate questions, and justify decisions as they planned a party. As the group report noted:

> This was a great opportunity to get the students to explore those different options and to present more questions as extension problems…Some students may also come up with their own questions such as "What can I do if I don't want to invite that many people?" or "I think I could shop around and find supplies that are less expensive. Then can I (have another piñata, invite more friends, get more gifts, etc.)?"

Other projects positioned *families* as active decision makers that utilized mathematics to solve problems. For example, in the *Bus Pass* lesson, the reasoning and decision-making practices of students and families were framed as resources for solving problems. As students collected and analyzed public transportation data to determine the best deal for their family, conversations with family members about their transportation-related decision-making were essential.

(Re)Positioning children and families to resist deficit perspectives. In some instances, prospective teachers noted that an explicit aim of their lesson was to (re)position families and communities to combat feelings of isolation, negative stereotypes, and deficit views that prospective teachers encountered in schools. In the *Pizza Party* lesson, one prospective teacher noted that connecting family practices and school mathematics would "open doors with the families of my students" that often feel isolated from the dominant school culture, specifically English language learners.

> This [project] has made me even more committed to opening doors with the families of my students, especially the English Language Learners whose families seem to hang back and not jump into the school culture.

Her comment hinted that schools and teachers are responsible for building relationships with families. She suggested that some families, such as the families of English learners, may not feel openly welcomed by the school and she hoped to change that situation.

Similarly, the *La Lavandaría* lesson aimed to challenge pervasive negative views of the Latino community held by cooperating teachers at the school site. The prospective teachers explained,

> We have heard many of the teachers (nearly all of whom are White, English-only speakers) make degrading comments about the students and families from the community, which is largely of Mexican origin. When one of the resource room teachers heard about our Community Math lesson project, she exclaimed that it was a great opportunity to "show them how to fix some of their problems. Maybe you can somehow make a lesson that will make parents care about their kids." We felt passionately that this bias against the community was unfair – clearly parents in the school community care deeply about their children. As such, we wanted our lesson to be a tiny step in the opposite direction; we wanted our project to recognize (and even celebrate) students' families and values rather than criticize them.

While the prospective teachers considered this a "small step" to counter these deficit views, they explained that showcasing community knowledge was a way to combat the "negative impact" of such remarks on students' views of themselves as mathematical learners. Their lesson included spaces "to bring community members into the classroom" both to open communication between home and school and to "allow students to see" how out-of-school knowledge connected to school mathematics.

Mixed positioning of children and families. There were a few projects that reflected a mixed positioning of children, families, and communities. On one hand, these prospective teachers made notable and explicit efforts to position children and families in positive ways, while on the other hand they expressed views that seemed to emphasize perceived deficits. For example, the *Library Late Fees* lesson included opportunities for students to analyze and critique library late fee policies, and to share their reasoning and recommendations with library staff. However, the prospective teachers also viewed their lesson as a way to address a perceived need for increased responsibility among youth in the community. They noted most families in this community "probably struggled with money," and that an added benefit of their project was it would teach children personal and community responsibility. As one prospective teacher noted,

> This is also a great lesson to teach students responsibility, and that if they do not want to pay any late fees, and then they must return their books or items before they are considered late. Another benefit is that this lesson and problems are helping students to become active members of their community, and by knowing rules and consequences can help form students into responsible citizens.

This perspective, which suggests students lack responsibility for themselves and their communities, is inconsistent with other ways prospective teachers positioned students and families in the lesson—as knowledgeable decision makers able to critically analyze policies impacting their communities.

The *Skate Park* lesson included a similar mixed-positioning of students and their communities. While students were framed as active contributors to their neighborhood (e.g., capable of designing and painting a new skate ramp for a community park), prospective teachers also talked about the lesson as a way for students to develop a presumed lack "of agency, purpose, and control over their lives and their world." Similarly, prospective teachers at times emphasized the sense of pride and ownership that the community felt towards the skate park, and the vibrant activity the park facilitated, and in other instances described the community in deficit-based terms, as "disjointed" and having "issues with crime" and poverty. Awareness of these mixed perspectives is important, because it suggests that even as prospective teachers begin to take up the high leverage practice of eliciting and making substantive connections to children's cultural funds of knowledge, they may hold perspectives about families and communities that are fragmented or even contradictory (Mason, 2008).

Discussion and Implications

We found that prospective teachers were able to identify mathematical, cultural, and linguistic practices in students' homes and communities, and to design mathematics lessons that included multiple spaces for eliciting and building upon these funds of knowledge. In this way, our study contributes much-needed examples of what this high leverage practice looks like in mathematics teaching, at least in prospective teachers' planned instruction. Our analysis also highlights how some prospective teachers resisted deficit-based discourses by repositioning families and communities in terms of their strengths and contributions, while others evidenced inconsistent perspectives. In this section, we situate our findings relative to prior research, and discuss the implications both for mathematics teacher educators and for future research.

Opportunities to Learn about Community Practices

The 12 Community Mathematics Exploration lessons highlighted in this analysis connected to a broad range of home and community activities including transportation (walking, bus), consumer purchasing (ordering food, shopping), routine family practices (doing laundry), family gatherings (party planning), library visits, and game-like activities (hula hoops, skateboarding). We suspect that these varied connections may reflect both specific prompts and scaffolds in the Community Mathematics module that encouraged prospective teachers to explore a range of community settings, as well as the orientation of these particular prospective teachers to focus on interactions and practices (rather than more superficial aspects) of the community settings. These prospective teachers interacted with community members and carefully observed their practices, they walked the same streets as their students, and they visited social hubs frequented by their students' families, sometimes guided by parents and children. Other prospective teachers visited and participated in the setting with their own families, as a way of imagining how their students might participate in the setting.

The varied ways prospective teachers learned about and identified practices in children's communities are important to note, because they reflect multiple entry points to the practice of connecting to children's cultural funds of knowledge in mathematics teaching. These multiple entry points in turn suggest different ways teacher educators might scaffold prospective teachers' participation. For example, prompting prospective teachers to notice and inquire about mathematical practices during their community visits seems to be an important way to move beyond more superficial connections to children's interests. Similarly, opportunities for prospective teachers to learn about communities via the perspectives of children and their families seem promising. At least for some prospective teachers, such interactions may result in more robust knowledge about families, including knowledge of ways that children and families reason mathematically outside of school, which can in turn support the development of mathematics lessons where families, students, and communities are viewed as mathematical resources (Aguirre, Zavala, & Katanyoutanant, 2012). Furthermore, while research has long demonstrated that teacher-family relationships are important for students' success in school (Henderson & Mapp, 2002; Sheldon & Epstein, 2005), teacher education programs continue to lack opportunities to learn about and interact with families and communities (Broussard, 2000; Chavkin, 2005). Our analysis highlights the critical role of such interactions in helping prospective teachers develop this high leverage practice.

Expanding Understandings about Mathematical Practices

While our findings demonstrate that some prospective teachers can connect to cultural funds of knowledge in their planned mathematics instruction, we acknowledge that this is challenging practice, because it requires, among other things, (re)orienting oneself to mathematics, and what might be considered mathematical practices outside the confines of the school mathematics curriculum (Civil, 2002, 2007). The mathematical practices that seemed most salient to prospective teachers were those related to financial transactions, budgets, and purchasing decisions. These findings suggest that prospective teachers may need support in recognizing mathematical practices that extend beyond consumer decision-making. Mathematics teacher educators could begin with a practice identified during the community walks, such as the first grade student's reasoning about the shortest route to school, and then guide prospective teachers to build on this practice in ways that address key mathematical ideas. For example, a Shortest Route lesson could address expectations for geometric thinking in the early grades, such as "describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance" and "find and name locations with simple relationships such as 'near to' and in coordinate systems such as maps" (NCTM, 2000, p. 96), or expectations about measuring length indirectly or by iterating units (National Governor's Association, *Common Core State Standards*, 2010,

p. 16). In short, while many prospective teachers are committed to connecting mathematics to children's lives, they need scaffolded opportunities to notice the varied ways that families and community members use mathematics as part of their daily activity.

Connections to Cultural Funds of Knowledge as an Ongoing Practice

While prior studies have focused on how teachers connect to knowledge about families as they design mathematics tasks (Taylor, 2012; Wager, 2012), our analysis extended beyond tasks to also consider how prospective teachers opened spaces for children's home and community-based knowledge and experiences across multiple components of a mathematics lesson (Drake et al., in press). We see connecting to children's cultural funds of knowledge as something that should permeate routine instructional practices, such as designing tasks and introducing lessons (Jackson, Shahan, Gibbons, & Cobb, 2012). In fact, it is the potential routineness of this teaching move, along with its role in supporting student learning, that makes it a high leverage practice. While not all students share the same funds of knowledge (i.e., not every student rides public transportation), by talking to students, families, and community members, prospective teachers gain important insights that can be used to create opportunities for students to learn mathematics in ways that leverage their own and peers' experiences. Over time, this practice can validate multiple experiences as mathematical resources.

Moving beyond isolated connections to children's cultural funds of knowledge, however, is not an easy task. Even when teachers have positive relationships with families and learn about children's out-of-school experiences, they tend to connect to this knowledge in a rather piece-meal fashion (Hedges et al., 2011), or connections to children's cultural funds of knowledge are not always coupled with an equally important focus on children's mathematical thinking (Aguirre et al., 2013). An important goal for mathematics teacher educators is to help prospective teachers integrate connections to children's funds of knowledge with focused attention on children's mathematical reasoning. The examples offered in this study may support mathematics teacher educators as they begin to work towards this goal. Additionally, while our analysis focused on planned instruction, we stopped short of investigating lesson implementation. Future research might focus on how prospective teachers' planned connections to children's cultural funds of knowledge play out in their teaching practice, including the challenges that arise, as this will advance our understanding of the support that prospective teachers might need as they move from planning to enacting instruction.

Supporting Strengths-based Orientations towards Children and Families

Positioning children and families as mathematical thinkers and decision makers, with valuable contributions that could enhance students' mathematics learning was also a powerful feature of the Community Mathematics projects included in our analysis. However, a few lessons reflected mixed perspectives on children, families, or communities. This inconsistent, or fragmented awareness (Mason, 2008), suggests the fragile nature of prospective teachers' (re)orientations towards students and families as mathematical resources and serves as a guidepost for teacher educators to provide additional supports to help strengthen prospective teachers' views.

In addition, an important implication of this analysis is to better understand why only a small percentage (17%) of Community Mathematics lessons were able to make consistent and substantive connections to cultural funds of knowledge. Similar to Graue's (2005) use of cultural and narrative frameworks to study how biography shapes prospective teachers dispositions towards families, future research might examine how the culturally-based knowledge, stories, and experiences that prospective teachers bring to mathematics methods courses (their own cultural funds of knowledge) shape the ways that they orient to children's families and communities, such as from a strength-based or deficit-based perspective. Enhancing our understanding of prospective teachers' orientations may support the broader goal of resisting and dismantling deficit discourses in education.

Conclusion

Prospective teachers must be given opportunities to learn more about the families and communities of the students they serve. This knowledge is crucial beyond building rapport with students. By identifying mathematical practices in home and community settings, prospective teachers can build lessons that leverage children's funds of knowledge to learn mathematics. The 12 community mathematics exploration lessons provide various examples of how these meaningful connections were made within the community and across the lesson plans. Furthermore they provide insights about prospective teachers' orientations toward strength-based and deficit-based perspectives of students, families and communities. Helping prospective teachers develop this high leverage practice will contribute to producing a new generation of teachers with the knowledge and skill set to meet the mathematics learning needs of culturally and linguistic diverse students.

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Opening Classroom Spaces for the Mathematical Success of African American and Latino/a Students

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INTRODUCTION

A number of studies have found that mathematics instruction for African American and Latino/a students with low socioeconomic status often emphasizes disconnected concepts, mathematics vocabulary out of context, following steps, and answers rather than explanations (Anyon, 1981; Ladson-Billings, 1997; Lubienski, 2002; Means & Knapp, 1991). For example, using data from the National Assessment of Education Progress (NAEP), Lubienski (2002) found that even when controlling for socioeconomic status, African American students were more likely to experience instruction that framed mathematics as being based on fact memorization, allowed for only one correct strategy, and assessed student's knowledge using multiple choice. These practices restrict the relationships that students can build not only with mathematics, but with their teachers as well. Some scholars assert that the often-impoverished instructional practices found in urban schools (often schools with high percentages of African American and Latino/a students with low socioeconomic status) are tied to teachers' negative attitudes towards African American or Latino/a students (Baron, Tom, & Cooper, 1985; Ferguson, 1998; Stiff & Harvey, 1988). A meta-analysis of 16 studies performed by Baron and colleagues (1985) found that there was a statistically significant relationship with teachers having more negative views of African American students compared to white[5] students. In noting research on instruction and teacher attitudes, Stiff & Harvey (1988) suggest that the teaching of low quality mathematics is connected to negative teacher attitudes about the ability of African American students.

When negative teacher attitudes manifest themselves in interactions with African American and Latino/a students, they often result in students' disengagement, misbehavior, or dropping out (Feagin, Vera, & Imani, 2001; Solórzano, Allen, & Carroll, 2002). For example, Solórzano and colleagues share findings from three different studies using focus groups, a survey, and a historical analysis to show the negative racial interactions students experience with their teachers and the depressed achievement, disengagement, and drop outs that result. This was not the only response to this treatment from African American and Latino/a students, but these kinds of responses were more frequent for African American and Latino/a students than their white counterparts. Mainly reported are the negative school experiences of African American and Latino/a students, which include harsher discipline, higher referrals for special education, and

[5] "White" holds a particular social meaning in society and has to do with privilege and treatment others attribute to certain people for perceiving their skin color as "white," whether or not the actual color is white. It is a perception based on light skin color and facial features that extends to Europeans, European Americans, and light skinned persons from Africa, the Middle East and Latin America.

behavior perceived as lower achieving and threatening (Downey & Pribesh, 2004; Lewis, 2003; Neal, McCray, Webb-Johnson, & Bridgest, 2003; Skiba, 2001). Neal and colleagues (2003) had teachers evaluate videotapes for the same behaviors of African American and whites and found that teachers rated the African American students as lower in achievement, more aggressive, and more in need of special education services. While this is very disturbing, the experiences of African American and Latino/a students are diverse in both shortcomings and successes.

Some scholars suggest that successful teachers understand the cultural richness African American and Latino/a students embody by acknowledging the importance of attending to students' ways of being and by opening spaces for varied styles of teaching and learning (Gay, 2002, 2010; Ladson-Billings, 1992, 1995, 2009; Howard, 2001a, 2003; Lee, 2003; Lee, Spencer, & Harpalani, 2003). This type of approach has been called culturally responsive or relevant teaching and describes a pedagogy in which teachers are intentional in acknowledging (or valuing) students' cultural backgrounds as an asset. When teachers value the prior experiences of students, students feel connected to their teacher and feel cared for within the classroom (Howard, 2001b). In that same vein, enriched learning spaces are created in classrooms when teacher practices allow for multiple forms of participation.

Within mathematics, researchers have all too often found that the ways of being offered to African American students are limited and constrained to procedural forms of mathematics (Anyon 1981; Ladson-Billings, 1997; Lubienski, 2002; Means & Knapp, 1991). When classrooms focus on limited mathematics, they constrain students' ways of being in classrooms and their stances towards learning and doing mathematics (Franke, Kazemi, & Battey, 2007). In contrast, when teachers open multiple ways of being in mathematics classrooms, they legitimize standard and non-standard discourse practices, connect to forms of participation outside the classroom, and position students as capable mathematically (Hand, 2012). The importance of creating spaces for multiple forms of participation in the mathematics classroom is summarized by Franke, Kazemi, and Battey (2007) when they state that, "Students' ways of being and interacting in classrooms impact not only their mathematical thinking but also their own sense of their ability to do and persist with mathematics, the way they are viewed as competent in mathematics, and their ability to perform successfully in school" (p. 226). In opening such spaces, teachers make room for students to bring outside school experiences, culture, and non-standard discourse practices across the classroom boundary.

To create spaces that open multiple ways of being within mathematics classrooms, Bartell (2011) suggests that teachers must demonstrate care by developing relationships with students and knowing them well. This means a dual relationship must be developed where teachers share of their own personal lives as well as participate in and value their students' lives. When teachers both share of themselves and value students experiences, it shows that teachers have a willingness to develop classroom environments that students view as supportive, nurturing, and caring (Bartell, 2011; Howard, 2001b). In fact, students' perception of teachers as caring is seen as a way for teachers to develop effective relationships with students, including African American and Latino/a students (Good & Brophy, 2000; Howard, 2001a; Noddings, 1992).

Developing caring relationships can be embedded in the teaching of mathematics through teachers being intentional in reducing issues of power and status, positively acknowledging student contributions, framing students as mathematically capable, and attending to language and culture (Battey, 2013; Hand, 2012). In these ways, a more culturally relevant mathematics instruction can be provided for all learners, especially for students from linguistically and culturally marginalized backgrounds. Without a reciprocal caring perspective between teachers and students, teachers could unknowingly create contentious learning environments for students (Hackenberg, 2005, 2010;

Noddings, 2001, 2002).

Positive Relational Interactions that Promote Successful Teaching in Mathematics

In general, classroom mechanisms are not well understood other than that poor quality instruction can affect outcomes for African American, Latino/a and low income students (Lubienski, 2002). One mechanism that needs to be better understood is the association between traditional dimensions of mathematics instruction and relational interactions among teachers and African American and Latino/a students. As it stands, moment-to-moment episodes of interactions between African American and Latino/a students and teachers within mathematics classrooms have not been well researched (Battey, 2013). We define *relational interactions* as moment-to-moment communicative actions between teachers and students, occurring through verbal and nonverbal behavior, which convey meaning and mediate student learning beyond the mathematics or instructional techniques (Battey, 2013). In particular, understanding positive relational interactions that promote the mathematical success of African American and Latino/a students in mathematics is needed. Presently, within mathematics classrooms, little is known about the communicative interactions between teachers and students that convey meaning about engaging in mathematics.

Research on teacher-student relationships has found that many teachers misinterpret their relationships with low income African American and Latino/a students and lack an understanding of what students' consider to be meaningful relationships with teachers (Murray, Waas, & Murray, 2008). For instance, Saft and Pianta (2001) noted that teachers generally rated their relationship with African American students higher in conflict. Murray and colleagues (2008) found that, unlike teachers' ratings of white students, teacher ratings of their closeness or conflict with African American students did not relate to the students' ratings of liking school. The authors raised the possibility that teachers interpreted compliance and behaving as closeness and liking school for African American students, but that these behaviors did not mean the same thing for African American students themselves. In prior work, the second author documented a teacher who had more negative than positive relational interactions with her mathematics students (Battey, 2013). The teacher's negative interactions devalued students' home language practices, ignored student contributions, and framed student behavior as problematic. This occurred despite quality mathematics instruction in terms of eliciting student explanations, having students share strategies, and focusing on critical mathematical concepts. This is consistent with Stiff and Harvey's (1988) claim that teachers of African American students often hold negative views of their students leading to inadequate academic instruction. These impoverished views and relationships with African American students negate the fact that they enter into classrooms with a rich set of personal resources (Howard, 2001b, 2003).

In contrast, culturally relevant mathematics instruction positively acknowledges student contributions, reframes deficit stereotypes about mathematics abilities, and attends to language and culture, opening multiple ways of being for students to engage both the teacher and the mathematics (Brenner, 1998; Ladson-Billings, 2009; Lipka & Adams, 2002; Nelson-Barber & Estrin, 1995). It is important to say that teachers cannot practice these dimensions in isolation from one another and that caring relationships involve reciprocity between teachers and students (Bartell, 2011). In order for students to perceive a teacher as caring, the teacher's behavior and relational interactions with students must demonstrate that they value students' mathematical contributions and cultural backgrounds. Thus, mathematical caring relationships between teachers and students must involve a communication of caring that result in students feeling cared for (Hackenberg, 2005, 2010; Noddings, 2001, 2002). This paper showcases the positive ways in which two teachers acknowledged student contributions, framed students' abilities, and accessed culture and language in one urban school.

Specifically, both teachers engaged in high quality math instruction, but also developed caring relationships with students. Through our work, instead of teachers viewing cultural differences that can exist between school and a student's background as a deficit, we highlight examples of teachers engaging in positive relational interactions during quality mathematics instruction with African American and Latino/a students in ways consistent with culturally relevant pedagogy by opening spaces for multiple ways of being in the mathematics classroom.

METHOD

A Mathematics Classroom as Context for Learning that Goes Beyond the Curriculum

When conceptualizing quality mathematics instruction, instructional practices and teacher knowledge are elements commonly cited (Wilson, Cooney, & Stinson, 2005). However, to focus on these two elements exclusively ignores relational, cultural and racial aspects of classrooms (Battey, 2013). For those reasons, we used Battey's (2013) Relational Interactional Framework as a lens to better understand teacher-student interactions as an aspect of mathematics instructional quality.

This framework was also chosen because it specifically takes into consideration teacher-student interactions and provides a viewing frame that allows the researcher to deconstruct episodes of relational interactions into micro communicative acts for the purposes of observing exchanges with students, and in this case, African American and Latino/a students during mathematics instruction.

In this study, we examined the verbal and nonverbal communicative action between teachers and students for the purposes of determining its influence on quality mathematics instruction. In previous research during the development of this framework (see Battey, 2013), relational interactions were defined as teacher-student interaction that went beyond mathematics and five different dimensions of relational interactions between students and teachers influential to quality mathematics instruction were identified: (a) addressing behavior, (b) framing mathematics ability, (c) acknowledging student contributions, (d) attending to culture and language, and (e) setting the emotional tone of the classroom. The aforementioned dimensions were also used in this study because these dimensions are particularly helpful in allowing teachers to develop an understanding of students that goes beyond the curriculum, by considering relational interactions when conceptualizing quality mathematics instruction.

The forms of emphasis, such as word choice and facial expression, were determined by multiple coders from different cultural backgrounds. This occurred because it was important to have coders with multiple interpretations involved in the analysis. This coding process also allowed the study to adjust to teachers because some talk louder than others or use more hand gestures and so on. Therefore emphases could be coded with respect to the norms of individual teachers. Instances of relational interactions were identified as teacher-student communicative interactions (verbal or nonverbal) that conveyed meaning and went beyond mathematics or instructional techniques. Forms of emphasis were identified as word choice, physical gesture, facial expression, stance or posture, vocal stress in syllabication, repetition, and extension.

The next layer of coding involved intensity and quality of interaction; and was based on identifying emphases as low, medium or high, and positive or negative. Positive interactions were actions characterized by affirmation, favorable, non-aversive, and without negation. Negative interactions were actions marked by hostility, sarcasm, denial, or expressed negation and considered adverse or unfavorable. Inter-rater reliability of coders from various cultural backgrounds was 92% across the codes. After discussing any disagreements, 99% reliability was achieved. Any interactions that coders could not agree on were not included in the analysis (see also Battey, 2013, for a more detailed account of analysis).

This study was conducted in one elementary school located in a large city in the southwestern United States. This particular school was struggling to serve their students mathematically; only 16% of African American and 41% of Latino/a students achieved proficient or higher on the state mathematics test in fourth grade the year prior to the research. The teachers in this study participated in ongoing professional development with the second author one year before and after the teachers were videotaped for this research. The goal of the professional development was to support the teachers in designing instruction to build off of their students' mathematical thinking.

Two 5th grade teachers working in this school participated in this research. Mr. Thompson[6], a white male, and Mr. Gray, a black male, were both in their first three years of teaching. Each classroom consisted of approximately 30 students. All students involved in this study were African American or Latino/a and from lower socioeconomic backgrounds. To keep the content as constant as possible, both teachers taught the handshake problem for one lesson (see Kaput & Blanton, 2001):

> Twenty people are at a party. If each person is to shake everybody else's hand once, how many handshakes will take place at the party? How many handshakes will take place for 21 people? How does the number of handshakes grow every time someone new arrives at the party?

While the teachers worked on this problem with students, they made adjustments and pedagogical decisions based on their students. The two videotaped lessons ranged from 30 to 50 minutes.

Through our analysis, we found that teachers implemented a variety of quality instructional strategies that not only built upon students' skillsets, but also elicited high levels of positive student engagement from students who might have otherwise been withdrawn from learning, or left feeling disconnected from the classroom. For example, teachers used word problems, had students explain and justify their thinking, asked clarifying questions of students, made student explanations explicit through revoicing, and pressed students to detail their mathematical thinking (Carpenter, Fennema, & Franke, 1996; Carpenter, Franke ,& Levi, 2003; Franke, Kazemi, & Battey, 2007; Franke et al., 2009; Kazemi & Franke, 2004). In addition to what is typically considered mathematics instruction, we also documented the relational interactions in the classroom.

We want to make clear that we are not promoting a cookie-cutter approach for teachers to engage with students. We fully recognize that teachers' individual ways of engaging in quality mathematic instructions looks very different among teachers and varies across grade levels and mathematical topics. Our portrayal of teachers in this study is not holistic, but instead consists of examples of what acknowledging student contributions and accessing language and culture looks like in practice. We believe that we can learn something about teachers' construction of high quality relational interactions with African American students, thus informing teachers and other practitioners about how their own everyday teaching practices can cultivate classroom learning communities that are supportive of all learners.

Our choice to use a mathematics classroom as context for learning that can go beyond the curriculum was to document the experiences of African American and Latino/a students in the hopes of interrupting the deficit notions that often surround their schooling in mathematics. Our goal in writing this paper is to provide models of student-teacher interactions that can promote ways of thinking for teachers themselves to engage in quality mathematics instruction that make connections between math and everyday language and build

[6] All names are pseudonyms.

upon the opulent resources students bring with them into classrooms. It is the interconnectedness of the practices, however, that conveys a teacher's valuing of students' culture and mathematical thinking. In the next section, we discuss how the two mathematics teachers went beyond the curriculum and incorporated the resources of students and their families within their everyday classroom practices.

RESULTS

Helping Teachers to Build Upon the Resources of Students and Their Families

In this section, we elucidate how teachers can build upon the resources of students and their families through the incorporation of culturally relevant teaching practices and strategies (Franklin, 1992; Gay, 2010; Howard, 2001a, 2003; Ladson-Billings, 1995, 2009). Specifically, we highlight episodes of positive relational interactions involving two teachers, Mr. Gray and Mr. Thompson. Both recognized the importance of positively acknowledging student contributions in ways that are personally meaningful to students. Mr. Gray, however, also used his own cultural forms of movement and language, positively framed students' ability in mathematics, and informally engaged students in contrast to the traditional formality of many mathematics classrooms. In engaging these practices, Mr. Gray is caring for the mathematical ideas that students' share as well as opening spaces for multiple forms of language and culture to enter the mathematics classroom. In this way he simultaneously opened spaces for students' learning within the mathematics classroom while showing an acute awareness of students' ways of being.

Illustrations of Acknowledging Student Contributions

We conceptualize *acknowledging student contributions* as a form of relational interaction that can occur in a variety of ways in classrooms (Franke et al., 2007). Most often it includes an act of recognition that is a relational aspect outside the category of content instruction. Typically this form of relational interaction involves a teacher valuing, devaluing, withering, or praising student thinking shared through a student's work or talk.

Within the lessons captured for this study, both Mr. Gray and Mr. Thompson engaged positively around students' mathematical contributions and in doing so opened up more forms of participating in mathematics than are available in traditional classrooms. Combined, these teachers acknowledged student contributions twenty times, of which only one was negative. That means, across all of the interactions coded as acknowledging student contribution, 95 percent of teacher-student interactions in this domain were positive experiences in the two classrooms. Mr. Gray and Mr. Thompson provide us with models of how teachers may use their own classroom practices as a way to recognize and validate students' mathematical thinking. Instead of viewing student misconceptions as signifying a lack of mathematical ability (Battey & Stark, 2009), both created learning opportunities for students that promoted high levels of positive student engagement and valuing of their mathematical contributions. They each displayed communicative interactions with students that conveyed messages marked with competence, knowledge, and skill in mathematics.

Mr. Thompson, a fifth grade teacher, was observed acknowledging student contributions 12 times, of which one was negative, during a 50-minute lesson. These interactions included praising a student's reasoning ability, prompting students to share their mathematical strategy with classmates, affirming students' efforts to solve problems, and encouraging different ways of thinking and problem solving. He routinely provided encouragement to the students and communicated to students that it is acceptable to struggle with mathematical concepts as a way to be successful. For instance, after introducing the handshake problem to the class, he assured students they could solve the problem. Later in the lesson, as Mr. Thompson walked around the classroom stopping at a student's desk, he pointed to their work and said, "You guys are on the right track." As he followed up with another group of students working together, he said, "You guys are really close." A few minutes later, Mr. Thompson again commented to yet a different group of students, "Interesting, interesting, that's a good thought." Across these episodes, we see how Mr. Thompson consistently recognized students' thinking and encouraged students to keep working to solve the problem. The message being conveyed is that mathematics is something to struggle through and he wants students to keep going.

Within the same lesson, Mr. Thompson displayed another series of interactions where he acknowledged student contributions as he encouraged and reinforced students' thinking while they figured out the solution to the problem. For example, while walking around the room looking at students' work, a student raised his hand and Mr. Thompson walked over to the student's desk and said: "Thank you very much, Othello." Bending over to get a closer look at the student's work, after about 30 seconds, Mr. Thompson said, "Very close on this." Without pause, Mr. Thompson walked to another group and said directly to a student while pointing to his paper, "James, I really like what you're doing with these numbers." He then stretched as he stood up, all the time keeping his eyes on the student's paper, and then leaned back down to the student's desk to reassure him he was on the right track in solving the problem. In this instance, Mr. Thompson used proximity and verbal repetition as a way to reinforce a student's individual contributions as legitimate. Throughout the lesson, students were positioned as resourceful and competent mathematical learners, necessary components of culturally relevant instruction.

Later, he invited a group of students to display their work to their peers, asking them to write out their strategy on the board in front of the class. After probing this group of students about their approach and checking for understanding with the rest of the students in the classroom, Mr. Thompson said in front of the entire class, "I knew you guys could do it!" These interactions are examples of ways teachers can positively acknowledge student contribution as they struggle to find the correct strategies. On a difficult problem for fifth graders, when students were reaching a frustration point, Mr. Thompson simply encouraged them to continue without putting down any student's thinking and reasoning. He later noted their success in solving the problem. These interactions went beyond how he addressed the mathematics of the problem to demonstrate the ways in which he was communicating for students to continue to struggle and that he valued the thinking they were sharing with the class. The consistency of his practices around supporting, challenging, and questioning students' thinking in mathematics positioned students as being skilled mathematicians. Notably, this is an oppositional stance to how society typically frames the mathematical abilities of African American and Latino/a students. When classrooms are consistently structured in this type of manner, students receive messages they are competent. If the practices were not consistent, then some students might have received messages of incompetence or not trusted Mr. Thompson's belief that they achieve mathematically.

In our observations of Mr. Gray, we saw him acknowledging student contributions in a different manner. We share this because it is important to represent the diversity of teaching practices that accomplish positive mathematical environments rather than to narrow quality teaching to one set of practices. He used physical gestures and facial expressions to show excitement about student thinking and to encourage mathematical strategies. All eight of Mr. Gray's interactions around acknowledging student contributions were positive within the 35-minute lesson. For example, early in the lesson as he watched a student solve a problem at her desk, he looked over her shoulder and stated, "That's great!" While she worked, and stopped briefly to gaze at him, he patted her on the shoulder and smiled broadly, validating her mathematical thinking. Immediately after, he moved to another student and said, “Alright!” as he gave a fist pump in the air and proceeded to pat the student’s shoulder. These comments and gestures speak to his style of positively acknowledging students’ contributions. Mr. Gray showed a high level of enthusiasm and encouragement for students’ mathematical strategies and successes.

While students shared during whole class time, Mr. Gray exhibited the same behaviors. While a student was in front of the class solving a problem on the board, Mr. Gray asked the student to explain his work. After listening to the student’s explanation, Mr. Gray stated, smiling, “Alright doc, that’s really good.” As the student continued to solve the problem in its entirety and explained his thinking process, Mr. Gray (still smiling) walked over to the student and said, “Interesting, alright doctor.” Following this he began clapping and the entire class joined in to recognize the student’s strategy. The use of doctor is notable and we will speak to this in the following section of the paper. The encouragement of students to clap in recognition of student sharing, and calling the strategy ‘really good’ and ‘interesting,’ showed respect for the students’ thinking. This same recognition was given to each student who came to the board to share. Later in the lesson, a different student’s sharing is followed with, “That is really nice… I like that so far [as class claps]. Yeah!” Mr. Gray regularly exhibited excitement in his interactions with student’s thinking individually and in front of the whole class. Again, it is important to note that Mr. Gray was consistent throughout the lesson both in his encouragement of different ways of thinking and in enjoying student success.

While the teachers both positively acknowledged students’ mathematical contributions, and positioned students as competent by recognizing the valid mathematics thinking of students, they did so in different manners. Mr. Thompson encouraged students to struggle and persevere through difficult mathematics. He also noted when students triumphed in solving the problems and he validated students’ strategies. On the other hand, Mr. Gray was more enthusiastic using vocal stress in positive ways, calling students doctor, and initiating the whole class applauding to recognize students’ contributions. Additionally, Mr. Gray used physical gestures to acknowledge student progress, patting students on the shoulder and fist pumping, showing his excitement in students achieving mathematical success. Across these classrooms, the teachers supported multiple ways of engaging mathematically beyond the procedures, disconnected vocabulary, and rote facts often found in urban school classrooms.

Illustrations of Accessing Language and Culture and Reframing Student Ability

While both teachers were adept at acknowledging student contributions, one in particular opened up multiple ways to engage in cultural ways of being within the classroom. Specifically, Mr. Gray used informal language practices and reframed students’ ability in relation to broad deficit narratives to support students in building positive mathematical identities. Important to note is that these practices cannot be separated from the ways in which teachers acknowledge students’ mathematical contributions. If the message students receive is that they can use informal language and are capable mathematically, but the teachers’ practices contradict this by not allowing spaces for students to share their thinking, then students might disengage or not feel cared for mathematically. Therefore, it’s the interconnectedness of the two categories and relational practices that support a caring mathematical environment rather than isolated practices.

We conceptualize *attending to culture and language* as teachers incorporating cultural and multi-linguistic forms, which open opportunities for students to engage in mathematics instruction using their personal knowledge and thinking. In some cases, this may include a teacher’s acceptance of multiple vernaculars within classroom spaces while students are in the process of understanding mathematical content or demonstrating mastery of mathematical content. By doing so, teachers reframe African American and Latino/a students, from a view of student's culture as deficient or an impediment, to one of cultural resources as valuable in learning mathematics. Important to understand is that attending to culture and language is complex, in part because of the inseparability of language and culture. When teachers make meaningful and personal connections with students and engage in interactions that build upon students' strengths, a student's personal cultural knowledge and linguistic practices are considered assets.

Dressed in professional attire, donning a long sleeved, white-buttoned, collared shirt, paired with a dark pair of dress slacks and black tie, Mr. Gray began class by telling the students, "Class, today we would like to do in math, a problem that will probably enhance your thinking." After introducing and explaining the handshake problem, Mr. Gray asked, "Does anybody have some ideas?" as he solicited the class for a volunteer to come up and solve the problem on the board. Mr. Gray's style of interaction with the students involved him walking around the classroom, visiting students as he stopped to look at their work. He seemed to stroll through the classroom, comfortably shifting his hands in and out of his pockets as if he was on a leisurely walk. He constantly smiled at students and patted particular students softly on their backs to encourage them as they worked individually at their desks. These constant and consistent types of interactions with students were a way for Mr. Gray to display his level of excitement and interest in student's thinking, as noted earlier.

Almost 30 years ago, Boykin (1986) identified nine dimensions of culture that can be of importance with regard to African Americans. His more recent work (Cole & Boykin, 2008) suggested that the physical movement styles embedded within a traditionally structured classroom can be restrictive for some students. For students from non-dominant groups, expressive movements can be "part of their everyday learning and communicative behavior" (Cole & Boykin 2008, p. 333). Cole and Boykin (2008) contend that an Afrocultural meaning system exists that connects music, verve, communalism, affect, rhythm, kinesthetic movement, and gestures. In their study, they compared classrooms that specifically allowed for more varied movement expressions with those that did not and found that African American students learned significantly more in classroom environments that encouraged more expressive, high energy, and affective behavior. In contrast, traditional mathematics instruction usually limits communicative and movement styles in the classroom (Battey, 2013; Brenner, 1994).

Mr. Gray opened up multiple ways of being in the classroom—through movement, speech, and informality—that are typically not observed in mathematics classrooms. He seemed to strut around the classroom, swinging his arms from side to side; he was continuously upbeat, displaying high levels of enthusiasm. In this sense, Mr. Gray's personal movement style opened up non-traditional ways of his being in the classroom. A great illustration of this is at the end of the class, after the students succeeded in solving the mathematics problem. Mr. Gray engaged in a type of "victory dance," clapped his hands, and said rhythmically, "Give yourselves a hand, yeah! Alright! You guys are the best, not like the rest!" The class applauded, he pointed his fingers and extended his arms outward, walked in a celebratory manner, waived both his hands high in the air, smiled, looked at the class, and said, "Alright, alright, alright!" In addition to the excitement in seeing students generate new strategies, his physical gestures displayed cultural movements that represented more varied ways of being than typically found in mathematics classrooms. We are not suggesting that Mr. Gray's expressions were about students' cultural expressions or ways of being, but simply noting his own style of teaching displayed movement patterns not typical in mathematics classrooms.

Continuing to make personally meaningful connections with students, Mr. Gray called a student "doctor" after the student shared his thinking with the class. Simultaneous to calling the student doctor, Mr. Gray led the entire class in clapping as a way to praise the student's efforts to solve the problem. A few seconds later, Mr. Gray asked for another student volunteer to show their work to the class. As the second student approached the board, Mr. Gray said, "Sir, Professor Suarez. Here is my other mathematician. Let's see what you come up with," and handed the marker to the student so he could begin showing the class how he solved the handshake problem. Mr. Gray's word choices to name students doctor, professor, and mathematician framed students as having ability to achieve mathematical success. These representations are in stark contrast to the ways African Americans and Latino/a students are, unfortunately, generally portrayed mathematically. Mr. Gray's regular use of these terms allowed students to view themselves as capable mathematically and as members in these professions. Again, this practice in isolation would have little impact on students' mathematical selves. Coupled with supporting students' mathematical thinking and opening cultural ways of being within the classroom, however, Mr. Gray's use of these terms reinforces his belief in students' mathematical ability and cultural value.

In the prior section, we noted moments when Mr. Gray acknowledged students' contributions in a variety of positive ways during class. Some examples included smiling and nodding to affirm a student's thinking and patting them on the back. These interactions were further supported by the words he chose to encourage a student's thinking, reasoning, and problem solving abilities. It is in these episodes of interactions that students' contributions are not only positively affirmed, but relayed in a way to students that demonstrated the belief they are capable math students, thus interrupting the negative notions of a deficit perspective. Also important to note is Mr. Gray's intentional effort to create a learning environment where students feel safe to take risks and display their work publically to other students, consistent with teaching with caring (Bartell, 2011).

The value in teachers accessing more varied linguistic and cultural forms is paramount. This interactional domain not only entails what a teacher says, but also includes ways of thinking and knowing as it relates to students. Through the use of culture and language, teachers can reposition students as having cultural resources rather than deficits (Artiles, 1998; Gutiérrez & Rogoff, 2003; Howard, 2003). Much the same as reframing deficit narratives about ability, this form of interaction positions students' ways of being as valid (Battey, 2013; Civil, 2007; Civil & Bernier, 2006; González, Andrade, Civil, & Moll, 2001). Mr. Gray was skillful in his shifting of language and his ways of interacting with students that disrupted many notions of deficit thinking regarding African American students. For example, while standing next to a student, looking at her work, he noticed she had drawn a picture as a way to help solve the problem. Mr. Gray said directly to her, "Oh this is deep! The emperor and his new clothes, didn't know that." While still next to the student, he stated to the rest of the class, "You guys can't see this yet, only me and Sarah can see this. This is deep. This is a teacher-student thing, only she and I can see it." With the student smiling and laughing at her desk, Mr. Gray holds up her work so others can see it and says, "She got 380," then displays a "thumbs up" physical gesture indicating his approval of her strategy. Students can be heard giggling and laughing in the background during this time. While this is going on in the classroom, students briefly stopped working to pay close attention to Mr. Gray's more informal exchange with Sarah.

During the previous episode, Mr. Gray used informal language and slang in his classroom. In moving to more informal language, he opened up more linguistic ways to engage mathematics rather than strictly enforcing a formal mathematics register. This is evident in the student responses to his language shift. Throughout the lesson, students can be heard laughing, seen smiling, and seen working diligently for long periods of time to solve the complex mathematical problems.

Another example of where Mr. Gray intentionally opened up more ways of being within the mathematics classroom is when he shared with the class his personal struggles with mathematics. After a number of students had shared strategies for the problem, he noted his own mathematical difficulties, but assured students that they can still understand math.

> Now I told you this before, my mind does not think algebraically, and um if I looked at this, I would be thinking, what is a way that I could do this and get the same answer, do it quickly, and I would do this, so, for my mathematician over there, you would probably like this, (pointing to how the problem was solved on the board using an algebraic formula) you guys are sharp, really smart.

Erasing the board to create space to demonstrate another way to solve the problem, Mr. Gray said, "If you are like me, and think math is not my cup of tea, but I want to be able to break this down and at the same time understand it, you might want to try this way. I thought this was nice." Then he began to explain and show on the board how the handshake problem could be solved using an alternative method. In this interaction, unlike prior ones, he personalized struggles with mathematics, and brought vulnerability and authenticity to the mathematics classroom. Further, Mr. Gray shared with students that he worked at understanding mathematics and students could too, but they had to be willing to work hard.

Later in the lesson, Mr. Gray again shared with students his personal mathematical goals. He furthered this narrative by adding,

> You couldn't be ashamed for not understanding math in the class. Is there anybody that doesn't understand this problem still? Don't be ashamed. You know you can't be ashamed in this class because I tell you all the time, there's a lot of stuff I don't understand in math and math, I'm going to make it one of my... It's one of my goals. If you're working on that too, it's okay.

Though not mentioned during the class period, Mr. Gray previously shared with the researchers that he was taking an Algebra course at the community college so he could better think algebraically. Mr. Gray felt this would give him a stronger grounding in the subject matter and therefore give his students more access to the mathematics. Next in his classroom interaction with students he asked, "Still don't understand? Let's model it." Immediately after sharing his personal goals, a student volunteered that she didn't understand and he responded by modeling the shaking for the entire class. This speaks to the students' comfort with him and the safety they feel in how he will respond to their not understanding something mathematically.

Through his personal narrative, Mr. Gray embraces the idea of his own struggle to understand mathematical concepts and reassures students that everyone in the classroom is capable of engaging in quality mathematical reasoning. This narrative challenges the metanarrative that mathematics is for the elite to understand and that you either get it or you don't. In sharing his personal struggles, he reframes himself as an authority of the mathematics, validates multiple strategies in the classroom, and creates spaces where it's okay to have to work hard to understand math. In his teaching practice, Mr. Gray was vulnerable, authentic, complimentary toward students, personally engaging, and legitimating of students' effort to understand the mathematics. This personal and informal interaction again opened up ways for students to participate mathematically and one student in particular took the risk of admitting that she did not understand.

Mr. Gray made spaces for different linguistic vernaculars and cultural practices within his everyday teaching practice. In creating a more informal space, he supported more cultural ways of being and in turn, developed more caring relationships. By calling students doctors, lawyers and mathematicians, Mr. Gray sanctioned students' ways of knowing and positioned them as successful learners. He later commented that he called students doctors and lawyers in an effort to build confidence and push back against African American and Latino/a students being stereotyped in math.

Mr. Gray supported students by bringing his own way of being to the teaching of mathematics through movement, speech, and emotion, but these are only part of the story. Although the consistency across these practices support an ethic of caring, practices such as strutting around the classroom, being excited, or using informal language do not capture the complexity of relational support in this classroom. Likewise, calling students "professor" and "mathematician" in isolation does not support students in persevering in the mathematics, unless coupled with believing in students' abilities, supporting their effort, and noting their successes through establishing and maintaining meaningful teacher-student interactions.

Conclusion

Adopting Battey's (2013) Relational Interactional Framework as a lens, we demonstrated the application of the framework in two mathematics classrooms and showcased exemplary episodes of teachers engaging in everyday practices. While the typical mathematics practices were also of high quality, it was the building on students' thinking, linguistic knowledge, and personal resources that exhibited relational aspects that built caring classrooms. Although African American and Latino/a/a students were the focus of this research, the framework can easily be applied more broadly in multiple contexts and with other student groups.

Mr. Thompson went beyond positively acknowledging students' contributions by validating their thinking. Mr. Gray consistently maintained a learning environment filled with communicative messages that relayed care for students. He also consistently reframed students as resourceful and knowledgeable beings. Further, he frequently complimented students on their thinking and mathematical contributions through such statements as, "I thought this was nice" or "That is really nice, I like that so far" and regularly encouraged them to "Come on, work with me. Talk to me." Moreover, Mr. Gray's manner of interacting with students communicated care, concern, and genuine kindness in addition to supporting students to learn complex mathematics. Through his own movement, informal speech, and word choice he brought culture across the boundary of the mathematics classroom, affirming more ways of interacting in classrooms than are typically allowed. In bringing his personal self into the classroom, Mr. Gray opened different ways of participating in mathematics classrooms while maintaining connections with students.

These types of relational interactions demonstrate the intentionality of the teacher to position students as mathematicians working to solve a problem. Noddings (1988) suggests that students "will work harder and do things…even odd things like adding fractions…for people they love and trust" (p. 10). As demonstrated by Mr. Gray, we contend that by teachers being deliberate in how they interact with students and purposeful in engaging in relational interactions with students that promote willful participation, teachers will positively influence students to engage mathematics.

A teacher's intentional use of language can be a powerful way of interacting with students that not only builds trust among students but also fosters a learning environment that is rich in opening opportunities for students' to bring culture into the mathematics classroom. This can be seen in a teacher moving away from misconceptions and deficit thinking to reframing students' abilities as mentally able and skillful.

As researchers, we strive for teachers to develop strategies of how they can incorporate practices within their everyday pedagogies that take into account culture, ethnicity, and potential differences in ways that quality instruction may be a common reality for all students, but especially for students from linguistically and culturally diverse backgrounds. We are suggesting that teachers not transcend cultural aspects of students' background, but acknowledge students' cultural holdings and realize some students are framed as incapable mathematically through certain societal vantage points. It is our aim that teachers and practitioners challenge deficit perspectives of African American and Latino/a students and find meaningful ways to connect with students through incorporating language and cultural practices within their student's everyday classroom experiences as a way to open more ways of being mathematically and of being a whole person.

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Reconocer: Recognizing Resources with English Learners in Mathematics

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Introduction

The fast-paced routines that characterize the culture of numerous U.S. mathematics classrooms along with politics-based decisions that discourage or even prohibit the use of some students' strongest resource—their native language and bilingualism—promote an uncritical use of material resources for teaching mathematics to English learners—e.g., vocabulary lists, translated worksheets, and technologies—that are often ineffective as very little learning is achieved through them (Clarkson, 1992). Often, when material resources exist in schools, they tend to be underused or misused as something that is good for all English learners (Stein & Bovalino, 2001), a cure-for-all that comes from sources external to the cognitive relationship that ought to be established between the students and the teacher. Clarkson (1992) has presented relevant evidence that bilingual students in schools with serious lack of ready-made teaching resources, but with teachers who specially created and built their own ad hoc resources, outperformed on two measures (a general mathematics test and a problem solving test) a comparison group of monolingual students in schools with abundant, ready-made teaching resources that included various aides, games, and even computers.

In times when material and human resources are becoming scarce or are inequitably distributed across schools (Flores, 2007), particularly schools with English learners, rethinking resources not as something that schools have or that students bring to school but as something located inside teacher-student interactions is critical. Re-envisioning resources in this way is not intended to disparage material resources. It is a fact that when these resources exist in classrooms, however, often they are either not used at all or they are used improperly (Ball, 1992; Stein & Bovalino, 2001). A problem that, in my view, is more interesting to solve than that of material resource availability is how to support teachers of English learners to view students not as resource deprived simply because ready-made material resources are scarce or unavailable in their contexts. Boaler (2008) has characterized this resource deprivation in U.S. schools as learning without thought, learning without talking, and learning without reality. In contrast, shifting our focus to instructional interactions as occasions for recognizing resources contributes to rethinking the whole problem of resource availability, accessibility, and deprivation. Material and human resources are important, but failing to recognize resources in teacher-student interactions is a less visible but more serious resource deprivation. Thus, this paper suggests a new conversation about resources in relation to English learners. The conversation begins with a conceptual component in the form of a critical review of literature that highlights resources for English learners in mathematics. The conversation continues with an empirical component by presenting a case selected from a longitudinal project in a Latino bilingual classroom. This case serves to illustrate the main arguments outlined in the theoretical component.

Current Understandings of Resources for English Learners in Mathematics

Current conversations about resources for English learners suggest looking into the students' home language, culture, or community as sources of such resources. Of course, there exists research that views language, culture, or community as deficits that interfere with school learning. I will not discuss such deficit views as they have been discredited in educational research (see Moschkovich 2002 for a critical review of research perspectives that have created deficit views based on narrow views of language and mathematics). Instead, I will critically and respectfully examine perspectives that recognize resources in language, culture, and community.

As a resource for learning mathematics, students' languages—Spanish and English—have figured in various ways in research, including students' propensity for problem solving collaboration and risk taking in Spanish (Dominguez, 2011); students' appropriation of mathematical language as modeled by a teacher in English (Khisty & Chval, 2002); teachers' discourse as facilitating a participatory or non-participatory environment for students (Khisty, 1995); students' multimodal ways of communicating and accessing mathematical ideas (Morales, Khisty, & Chval, 2003); students' degree of bilingualism as a possible predictor of strategy selection in mathematical problem solving (Secada, 1991); and language considered within other situational resources such as gestures and objects (Moschkovich, 2002). Cutting across this research is the recognition that language matters for learning mathematics, an important recognition for working with English learners. In this study, I take this recognition to mean that language matters not as something that participants "bring" to interactions, something that pre-dates the interaction, but rather as something that is created and recreated in interactions. Instructional interactions are the site in which teacher and students make language into a common resource, allowing them to arrive at a common ground (Staples, 2007) where meaning transcends correctness, proficiency, or any other issues that threaten the participants' goal for common understanding. Moschkovich (2002) provides examples of language as a common resource that is characterized by English learners' common focus on meaning, an interest by participants in understanding each other's language, and even the invention of words that emerge during interactions. This emphasis on instructional interactions highlights the responsibility accrued to the teacher to notice, explore, and eventually inventory the resources that are recognized in interactions with students. The interest in language as a resource is consonant with emphases in teaching and learning mathematics such as communicating ideas, creating multiple representations of these ideas, and creating inclusive and equitable learning environments for all students (National Council of Teachers of Mathematics, 2000; Common Core State Standards, 2010). These processes emphasize creating resources with students, and I add that these resources can be created in daily interactions.

Another strand of research locates resources in culture, specifically in students' cultural backgrounds, as potential sources for meaning-making in school mathematics. According to this research, children use cultural resources differently when they participate in everyday activities that include mathematics than when they participate in school mathematics. These everyday activities are part of students' cultures and include street vending activities (Carraher, Carraher, & Schliemann, 1985); candy selling (Saxe, 1988); carpet laying (Masingila, 1994); grocery shopping (Lave, Murtaugh, & de la Rocha, 1984; Lave, 1988; Taylor, 2004), and tailoring and pattern making (González, Andrade, Civil, & Moll, 2001). Other research has proposed culturally relevant pedagogy (Ladson-Billings, 1995) as a means to use cultural referents in teaching to support students' learning. For example, Gutstein, Lipman, Hernandez, and de los Reyes (1997) used the approach known as culturally relevant pedagogy with Mexican-American students, concluding that resources are to be located in the connections teachers must establish with the students' families in order to create cultures in the classroom that resonate with the students' cultures. Similarly, Torres-Velasquez and Lobo (2005) used students' home culture as a resource for learning mathematics topics such as graphs and data representations. Finally, Gerdes (1988) has provided examples of cultural groups' use of mathematical concepts that are not recognized as resources for learning school mathematics. A general finding from this body of research confirms students' successful and resourceful participation in a variety of out-of-school activities that include mathematics. Such a finding—consistent across this research—has been produced without any assumptions regarding students bringing resources to their non-school mathematical activities. Rather, supporting evidence for the finding suggests that the environment itself supports these students to be successful in their mathematical activity (Carraher & Schliemann, 2002). This finding resonates with my interest in a similar support for developing resources with students in the environment constituted by the moment-to-moment teacher-student interactions.

Finally, students' communities also have been conceptualized as depositories of resources that can be explored in classrooms. This approach includes the construct of funds of knowledge applied to games and geometry concepts (Civil, 2002) or tailoring and pattern making (González, Andrade, Civil, & Moll, 2001). For example, Civil (2007) explored and viewed a gardening project as family knowledge and experiences that could be used as resources for mathematics classroom instruction. Other studies have used students' community and home practices as resources to support the students' conceptual development in mathematics (Lipka, 1994, 1998, 2002, 2005; Brenner, 1998; Simic-Muller, Turner, & Varley, 2009; Barta, Sánchez, & Barta, 2009; Cicero, Fuson, & Allexsaht-Snider, 1999). Similarly, mathematizing the school's cafeteria food was viewed as a problem that belongs to the school and the broader community in which the school is located (Brenner, 2002). Resources, according to these studies, are to be found in the communities from where students come, as these communities are viewed as repositories of knowledge and ideas that are meaningful in the students' lives and that can be used to enhance school mathematics learning.

The idea of recognizing resources in the spontaneity of interactions is, as mentioned earlier, a way of recalibrating the more common idea of resources for students to be found in their cultures, languages, or communities. Neither the teacher nor the student approach mathematical interaction explicitly considering resources that are part of their cultures, languages, or communities. But skillful conversations with the students can lead both participants to consider such resources in the school context, thus making the *recognition of resources,* not the bringing of resources, the reason for coming together. Moreover, communicating to teachers that students bring language, culture, and community resources to classrooms can mislead teachers to focus more on nonmathematical aspects of these resources, and ultimately to contribute to the "othering" of English learners (Banks, et al., 2005; Sowa, 2009). The critical view of resources I propose here does not emphasize that students bring resources from language, culture, or community. Instead, the view emphasizes the mathematical ideas, conceptions, or insights that students describe, talk about, and discuss in instructional interactions. Since the teacher has access to various representations of these ideas, it is in them that the teacher can recognize resources that may have linguistic, cultural, or community roots. Further, this recognition is grounded in the teachers' practice, specifically in the most important moment of teaching: the instructional interaction. This is the view that I develop in the following conceptual framework.

A New Conversation for Understanding Resources for English Learners in Mathematics

Language, culture, and community offer, according to the literature reviewed, rich possibilities for thinking about resources for English learners. But how can teachers begin to use these resources in the fast-paced environment of teaching school mathematics? Published research has not articulated an explicit answer to this question. Some studies, though, offer promising approaches (e.g., the TEACH MATH project: Aguirre et al., 2013; Turner et al., 2012). This article offers another possible answer. To begin articulating this answer, I would like to propose two scenarios. In the first, teachers could be expected to first know (*conocer*) the resources before starting instructional interactions, which seems like a formidable, if not impossible, task. Alternatively, teachers could use instructional interactions as the sites for recognizing (*reconocer*) and building with students such resources. By being based on the recurrent point of contact between students and teachers — the instructional interactions — this alternative offers multiple and repeated opportunities for teachers to re-envision the classroom in general and the instructional interactions in particular as language-culture-community spaces in which resources can be recreated with students. Resources, in other words, should not be thought of as pre-existing to teacher-student interactions, lurking in the students' home language, cultures, or communities. Instead, teachers must learn to talk with students to recognize, together, resources that can enhance teaching and learning simultaneously. Resources in this view are less visible (they may not have a name yet until they are recognized) but once recognized they may be more powerful and meaningful for the immediate users—teachers and students—than the distant visions of pre-existing resources that many teachers may have. They are not things to be found in a student's background but instead constructed with students as part of constructing their foregrounds (Skovsmose, 2005). This view is consonant with the idea of re-sourcing (Adler, 2000), which implies looking at resources as a verb and also as a new way of thinking about resources. Implicit in this view of resources as nested in interactions is the imperative of addressing the quality of teacher-student relationships. As Hamre et al., (2012) argue, teacher-student relationships are impoverished and require attention. This is a problem that tends to be more acute for English learners who often are placed in learning contexts such as low track classes in which the teacher-student relationships are weak and based on low student expectations (Oakes, 1985; Valenzuela, 1999).

The first idea that is relevant for this new understanding of resources is illuminated by Thom and Roth's (2011) calling attention to the etymological roots and educational meaning of the word *recognize*:

> We conceive learning as a process of re-cognizing (from the Latin word *recognoscere*). Our use of the term should not be taken in its first everyday signification of the word, which is to 'identify as already known,' but in the second meaning, to 'know again' (p. 282).

When considered in relation to resources to be used with students, *to know again* implies to know such resources in a different way, not as pre-existing an interaction but as sharing the "life" of an interaction. In this recognition of resources, teachers necessarily start where students are (Mercer, 1995). In turn, such recognition of resources should create cognitive affinity between students and a teacher. Once resources have been recognized, then teachers can begin to associate these newly recognized resources as having something to do with language, culture, or community. And because these resources share the same life as instructional interactions, they can be accessed by teachers as part of well-cultivated and deliberately planned instructional interactions with all students.

Central to recognizing resources in teacher-student interactions is talk. The focus is on talk, not language, because talk is alive, reciprocal, and it is always located in a present that is incessantly becoming the future or the foreground of students' mathematical resources. But talk about mathematics is a special kind of talk that, in order to foreground resources, requires learning how to talk. In particular, many children need to be taught how to use talk as a resource (Rojas-Drummod, Pérez, Vélez, Gómez, & Mendoza, 2003; Rojas-Drummond & Peon Zapata, 2004; Mercer, 1996; Mercer, Wegerif, & Dawes, 1999; Wegerif, 1996; Wegerif & Mercer, 2000).

A very important related idea for recognizing resources with English learners is the understanding that once recognized these resources become a common good for the teacher and the student. My work with teachers, both in university classrooms and in elementary school classrooms, has caused me to think about the need for teachers to think of common resources between them and their students instead of the traditional thinking of resources *for* students that may have nothing to do with the students' mathematical ideas or with the teaching-learning relationship that is formed in every instructional interaction. Resources for English learners that these teachers typically use include vocabulary lists, hands-on activities, concrete representations, translations, use of key words, language specialists, or use of simple word problems. While well intended, there is nothing in these resources that suggests the developing of something common between students and teachers. My thinking of a common resource also helps to resolve what Adler (2000) has conceptualized as the dilemma of transparency of resources. According to this dilemma, to make a mathematical concept visible (e.g., volume), the resources supporting the teaching and learning of such a concept must become invisible, at least momentarily. Adler argues, "…if the resource is to enhance and enable mathematical learning, then at some point it will need to become invisible – no longer the object of attention itself, but the means to mathematics" (p. 216). The process of making resources common between students and teachers is instrumental for dissolving this dilemma. For example, a teacher who talks like a student creates a common language, which makes the language into an invisible resource for both teacher and student, liberating both participants to focus on the mathematical object of their talk (e.g., volume). In contrast, teachers who focus on the visible resources are less likely to develop common resources between them and their students. Real examples of this focus include a teacher over-concerned about English learners not being able to distinguish the pronunciation difference between *half* and *have* (English language is made too visible); or using the actual objects in a word problem to facilitate understanding of a concept (material resources made too visible); or creating social representations of the English learners (Gorgorió & de Abreu, 2009) that emphasize what the students cannot do (removing the responsibility and the opportunity of recognizing resources together with students.)

Finally, my interest in supporting teachers of English learners to recognize common resources in interactions responds to the fact that many teachers do not share a language, culture, or community that is common between them and their students (Hollins & Guzman, 2005; Gay, 1993). When there is little in common between a teacher and their students, it does not matter how abundant other material and human resources are in that classroom (see Clarkson, 1992) because students will not see or use them in the same way that teachers do. In the following section, I will present an interaction with an English learner to illustrate how she and I recognized resources [from RE (again) CO (together, in common) GNOSCERE (to know)] that shaped our interaction.

Recognizing Resources with an English Learner

To illustrate the process of recognizing resources with English learners, I used a purposive sampling (Cohen, Manion, & Morrison, 2000); that is, I selected a case in which an English learner participated in an interaction filled with moments in which resources were recognized. First, I present my conversation with Marifer, a third grade English learner whom I met in a professional development project aimed at supporting first year teachers develop common resources with English learners in mathematics. The teacher, overwhelmed by the fast-paced environment of her first year of teaching, referred Marifer to me as someone who needed help understanding the concept of volume. The teacher's request for help was based on the fact that Marifer had performed poorly on the concept of volume as measured by a benchmark test.

I approached my interaction with Marifer with two questions in mind: (1) What does she know already about volume that I can recognize as possible resources to use in our interaction? (2) How can I join her in this knowledge of volume so that the resources that I recognize can become common resources for advancing her understanding of this concept? The following transcript of our conversation is presented in three parts, with the analysis of the process of recognizing common resources segmented accordingly. The English translation is bracketed in italics.

Interviewer: Muéstrame una pregunta con la que batallaste mucho. [*Show me one question with which you struggled a lot.*]

Marifer: (Opens test booklet and carefully looks for one question. When she finds one, she presses on it with index finger strongly and emphatically.)

Interviewer: Ahí está, vamos a ver, dice, pues léemela tú primero. [*There it is, let's see, it says, well, you read it to me first.*]

Marifer: El siguiente modelo está hecho con cubos de un centímetro. ¿Cuál es el volumen de este modelo? [*The following model is made with one centimeter cubes. What is the volume of this model?*]

Interviewer: OK, ¿cuál es el modelo? [*OK, what's the model?*]

Marifer: (Points to model on paper).

Interviewer: Esta cosa, ¿verdad? Y está hecho con cubos de un centímetro. ¿Me puedes encontrar un cubo de un centímetro? [*This thing, right? And it's made with one centimeter cubes. Can you find me a one centimeter cube?*]

Marifer: (Looks around, then looks at paper in front of her) ¿Aquí? [*Here?*] (Points to paper)

Interviewer: En el modelo, uh-huh (Marifer pone el dedo índice en varios cubos en el modelo). OK. Éste es uno, y aquí hay otro. Es como un bloque. ¿Cuántos cubos de un centímetro crees que hay en este bloque? [*In the model, uh-huh. (Marifer puts index finger on various cubes in the model) OK, that's one, and here's another one. It's like a block. How many one centimeter cubes do you think there are in this block?*]

Marifer: Eighty-two.

Interviewer: ¿Cómo sabes? [*How do you know?*]

Marifer: Because…yo los conté ayer. [*…I counted them yesterday*]

Interviewer: ¿Y cómo los contaste? [*And how did you count them?*]

Marifer: De uno por uno. [*One by one*]

Interviewer: Pero éstos están atrás, ¿cómo sabes contar si no se ven, cómo le hiciste? [*But these are in the back, how do you know how to count them if they cannot be seen, how did you go about that?*]

Marifer: Yo nomás conté todos éstos, y éstos, y éstos [*I only counted all of these, and these, and these*] (Points to each of the 3 visible faces of the model)

Interviewer: Ah, contaste…[*Ah, you counted…*] (Marifer interjects)

Marifer: Como, conté esta parte (circula cara frontal), y esta parte (circula cara superior), y éstos de al lado (cara lateral), 82 en total. [*Like, I counted this part (circles front face), and then this part (circles top face), and the ones from the side (circles side face), 82 in total.*]

To begin the process of recognizing resources with Marifer, I first let her pick one problem that was challenging for her. Her careful selection of the volume problem suggests her desire to share with me what she knew about volume, signaling the first opportunity to begin recognizing resources in her ideas. Then I asked her to read the problem, with the intention of reacquainting her with her knowledge of volume. Then by asking her to find a 1-cm^3 cube in the model, a common object between us was found, given our cultural familiarity with the metric system. This common object I knew was going to be pivotal for finding additional resources with Marifer. First, the 1-cm^3 cube served to reveal the misconception of volume as being only the three visible faces of the block. Noticing that Marifer was not seeing through the model drawn on the paper (material resource was too visible), I suggested to build together a three-dimensional model using connecting cubes. This new model helped me and Marifer locate her misconception of counting only the visible faces of the two-dimensional model on the paper. Using this misconception as the focus of our talk, she and I continued finding more resources in our interaction. In the second part of the transcript, I continue our conversation right after she and I had helped each other to stack two layers of 40 cubes each (4 sticks of ten cubes together). In this part of the conversation I am asking her how to proceed from there.

Interviewer: ¿Le ponemos otra capa o ya así? [*Should we put another layer or is that it?*]

Marifer: No. Otra. [*No. Another.*]

Interviewer: Otra, ¿verdad? ¿Nada más otra, u otra más? De acuerdo al modelo. [*Another one, right? Just one more, or another one? According to the model.*] (Point to model on paper). ¿Cuántas capas más necesitamos? [*How many more layers do we need?*]

Marifer: ¿De arriba, aquí arriba? [*On the top? Here on top?*]

Interviewer: Uh-hum.

Marifer: (Refers back to the two-dimensional model on paper, she counts the four rows on the top (from back to front) and then continues counting on the front side, counting two more rows. Next she refers to the cube model under construction and counts the 4 top groups of ten connected cubes, by tapping each one with a finger. Finally, she transfers this count onto her hands as she counts 1, 2, 3, 4, and then 5, 6): Seis más. [*Six more*] (Her counting indicates an interesting idea: to get to the top of the three-dimensional model, she would physically have to go up two cubes (the height of the model so far) and then walk across the width of the top, which is 4 cubes wide, and that is how she is getting 4 and 2, or "seis más.")

Interviewer: OK (I hand her one stick of ten cubes and she places it on top. At this point a non English learner comes to ask us what we are doing and she tells me that she got a very high score on the same test that Marifer got a low score and that she took it in English and that math is her favorite subject. As I talk with the other student, Marifer finishes forming and placing rows of cubes on the top of the model, not six as she said but only 4)

Interviewer: OK, entonces ¿se ve más o menos como éste? [*OK, so does it look more or less like this one?*]

Marifer: Sí. [*Yes.*]

Interviewer: Sí, ¿verdad? A ver, ¿en qué se parece este modelo Marifer a éste? Dime por qué es igual a éste. Vamos a compararlo. [*It does, right? Let's see, how does this model look like this one, Marifer? Tell me why this one is the same as this one. Let's compare them.*]

Marifer: Porque, está, está, uh, en cuadros, los dos están divididos en cuadritos. [*Because, it's, it's, uh, in squares, both are divided in squares.*]

Interviewer: Uh, huh, en cubos de un centímetro. [*Uh, huh, in one centimeter cubes.*]

Marifer: Sí. [*Yes.*]

Interviewer: ¿Y en qué más se parece este modelo de los cubos al modelo del examen? [*And how else does the cubes model look like the model in the exam?*]

Marifer: Es la misma, uh (mueve ambas manos hacia arriba y abajo, con un espacio en medio) altura. [*It's the same,* (moves both hands up and down, with a space in between) *height.*]

Interviewer: La misma altura, OK. ¿Cuál es la altura? [*The same height, OK. What's the height?*]

Marifer: Esto [*This.*] (points to the top of the model on test).

Interviewer: ¿Esto que está arriba? [*This, what's on the top?*]

Marifer: Sí. [*Yes.*]

Interviewer: Es como el techo. OK, ¿cuál es el largo? [*It's like the roof, OK. What's the length?*]

Marifer: That one. (Points to bottom of the model on test.)

Interviewer: OK, aquí es el largo, y acá en los cubos, ¿cuál sería el largo? [*OK, here's the length, and over here with the cubes, which one is the length?*]

Marifer: Aquí el de abajo. [*Here on the bottom.*] (runs finger along the base of the block of cubes.)

Interviewer: Uh-huh. Y el ancho, ¿cuál sería el ancho? [*Uh-huh. And the width, which would be the width?*]

Marifer: Aquí, como el ancho. [*Here, like the width.*] (points to base of block at one end.)

Interviewer: OK, aquí es el ancho. Entonces tú me dices que aquí contaste 82, o sea ¿nadamás contando los que se ven? [*OK, here's the width. So you told me that here you counted 82, I mean, only counting the ones that are visible?*]

Marifer: Uh-huh.

Interviewer: Qué te parece aquí, en este modelo, ¿cuántos cubos de 1 centímetro hay aquí? [*What about here, on this model, how many one centimeter cubes are there?*]

Marifer: (Takes a careful look at front side for a while, then tilts head for an easier and closer look at one end side of the block, then announces): Cien treinta [*One hundred thirty*] (a common way among Mexican Americans to say numbers larger than one hundred; the standard way is ciento treinta.)

Interviewer: Ciento treinta. ¿Por qué ciento treinta? [*130. Why 130?*]

Marifer: Porque uh, conté uh, primero conté éstos, de esta línea, y había 10 cuadritos, entonces pensé que en cada línea había de estos 10, y nomás conté todos. [*Because uh, I counted uh, I counted these first, in this line, and there were 10 little squares, and so I thought that in every line there were 10 of those, and so I just counted all of them*] (points to several of the sticks in descending order.)

Interviewer: (Echoing Marifer as she speaks): A ver, ¿y por qué son ciento treinta? [*Let's see, and why there are 130?*]

Marifer: Porque [*Because*], uh, I don't know if I'm right…

Interviewer: A ver. [*Let's see*]

Marifer: (Points to each stick of ten as she skip counts by 10): 10, 20, 30, 40, 50, 60, 70, 80, 90 (aspirates 90 as she runs out of breath), 100, cien diez, cien veinte. [*One hundred ten, one hundred twenty*] Oh no, never mind, cien veinte. [*one hundred twenty*]

Interviewer: Estabas cerca, ¿verdad? (Marifer: Uh-huh.) Ciento veinte. Pero cuando tú me dijiste aquí (le muestro el examen), no me dijiste que eran ciento veinte, ¿qué pasó ahi? [*You were close, right?* (Marifer: Uh-huh). *120. But when you told me right here* (show her test), *you didn't tell me it was 120, what happened there?*]

Marifer: I think, um, I counted, I count wrong.

Interviewer: You think you counted wrong (she nods) OK. ¿Qué más piensas? ¿Por qué son dos respuestas diferentes? [*OK, what else do you think? Why are these two different answers?*]

Marifer: Oh! Oh, porque, uh, en éstas (apunta a la pregunta del examen), hay como de cuatro (apunta a los 4 grupos en la parte superior del modelo con cubos) aquí 4 (apunta a los extremos de los 4 grupos en un lado del modelo con cubos) y aquí hay de diez (apunta al largo de un grupo). Como, en cada, en cada línea, como, el techo, como usted dijo, hay, están, tiene 4, y acá en el largo (ahora apunta al modelo con cubos) tiene 10, y allá (apunta al examen) tiene cuatro, como en cada línea está, como en cada lado tiene diferentes, uh, números, como así (coloca el filo de la mano en el ancho del modelo con cubos). [*Oh! Oh, because, uh, in these* (points to test item), *there are like (lines) of four* (points to 4 sticks on top of cubes model) *right here 4* (points to the ends of the 4 sticks on one side of the cubes model) *and here there are (sticks) of ten* (points to the length of one stick). *Like, in each, in each line there is, like on each side it has different, uh, numbers, like this* (puts edge of hand along the width of the cubes model)]

Interviewer: Uh-huh. Entonces ¿cuál crees que está bien, como los contaste aquí o como los contaste acá? [*Uh-huh. So, which one do you think is correct, the way you counted them here or the way you counted them there?*]

Marifer: Como los conté aquí [*The way I counted them here*] (points to cubes model.)

Interviewer: ¿Por qué? [*Why?*]

Marifer: Porque…allí, aquí, uh, tenemos, la foto, y está como, aquí (redirige la atención del examen al modelo con cubos) tenemos las líneas que t-, como en cada uno está el mismo número de líneas. [*Because…there, here, uh, we have, the picture, and it's like, here* (shifts attention from test to cubes model) *we have the lines that, like in each one there's the same number of lines*]

Interviewer: Uh-huh.

Marifer: Y [*And*]…I think.

Marifer and I constructed a new resource together: A three-dimensional model inspired by the drawn model on the test. There is evidence in our talk, however, that this model did not become a common resource immediately. It took us multiple iterations of our thinking and talking together in order to make the new model our common resource. For example, Marifer first seemed to be thinking differently about my question regarding how many more layers we needed to finish the three-dimensional model, as evidenced by her counting of still visible faces (4 on the top, and then 2 on the side). Marifer was beginning to recognize (or to know in a different way), though, in the two-dimensional rendition of the model, the intended three-dimensionality. Evidence of this important recognition is in how originally she had only counted the faces of cubes as separate groups, but now she was counting 4 rows on the top, and then continued with 2 rows on one side. This manner of counting suggests that she was connecting the dimensions of this model, even on the paper rendition. This indicates that she was beginning to see the idea of volume through this model, which contrasts with the earlier isolated counting of visible faces. Also, I noticed that Marifer correctly recognized and named the attribute of height. My initial question was, "What does she know already about the concept of volume?" Marifer certainly knew a lot, much more than what she had been able to demonstrate on the benchmark exam. Her existing knowledge, and the knowledge activated by my questions, helped Marifer to transfer the attributes of height, length, and width to the paper model on the test. For example, she "lifted" the attribute of width by gesturing with her hands, leaving a space to indicate the width. This gesturing was enacted between the two models. She recognized that she had counted wrong in the two-dimensional model and was able to explain why she had counted wrong. In her explanation there is reference to "la foto" as something that prevented her from seeing all the dimensions that she was able to see in the common resource constituted by the cubes model.

Marifer shifted her attention back and forth between the two-dimensional and the three-dimensional model. I followed her as she moved back and forth between the two models, because I wanted her to decide which one was going to be our common resource. We also followed each other linguistically, sometimes talking in English, sometimes in Spanish, and sometimes bilingually, so in this way we made our talk a common resource. An important moment when Marifer "sees" the concept of volume through the resource constituted by the cubes model occurred when she was mentally counting the connected cubes. She never touched the model; instead, she was surveying the model, tilting her head to gain a different perspectival side view. This is what Adler (2000) calls the transparency of resources. Just like the paper version, the cubes model was not showing all the cubes that it was made of, but Marifer was seeing through it this time. In her explanation of how she counted the invisible cubes, she declared: "*…and so I thought that in every line there were 10 of those*." The models became transparent, and her reasoning through them became visible both for me and for her. Finally, there was one instance, at least, in which she and I did not create a common resource: When she referred to 130 by saying cien treinta, whereas I referred to it by saying ciento treinta. This may not have played a significant role in our interest in recognizing common resources, but it illustrates a moment in which we did not share a linguistic form in our talk. In the concluding part of the transcript and as a result of noticing her miscount of 130, I asked Marifer to invent a different way of counting the cubes, one less prone to counting errors.

Interviewer: ¿Podrías contarlos de una manera más rápida, o sea, podrías inventar una manera de contar esto más rápido? [*Could you count them in a faster way, I mean, could you invent a way of counting these faster?*]

Marifer: (Looks at model, purses lips slightly, looks at me while putting her finger on her chest as in disbelief but also with a smile showing an unexpected challenge): Me?!

Interviewer: Uh-huh!

Marifer: (Continues looking at model for a while, then with a big smile exclaims): ¡No!

Interviewer: ¿No? Está bien, está muy bien lo que hiciste, de 10 en 10, pero, a lo mejor ¿verdad? (Asiente) a lo major hay una manera más rápida de contar. ¿Quieres pensar un poquito más, a ver si se puede más rápido? (Asiente) OK, te doy ahí un minuto a ver si lo haces. [*¿No? That's alright, what you did is great, by tens, but maybe, right? (She nods), maybe there's a faster way to count. Do you want to keep thinking to see if you can do this faster? (She nods) OK, I'll give you one minute to see if you do it.*]

Marifer: (Looks at the corner of the block model where she can get a perspective of the three dimensions): Oh! uh, hay, en cada, en cada de éstos hay 4 uh de estas líneas, entonces, [*Oh! Uh, there's, in each, in every one of these there are 4 uh of these lines*] I think, y en cada linea hay 10 cuadritos [*and in each line there are 10 little squares*], so we can do, I think uh, ten times four, diez por cuatro.

Interviewer: Oh! Sí, está muy bien, y cuánto es ten times four? Diez por cuatro, cuánto es diez por cuatro? [*Oh! Yes, that's great, and how much is ten times four?*]

Marifer: Forty.

Interviewer: Forty. OK.

Marifer: But I think it's not it.

Interviewer: Huh?

Marifer: I think it's not the answer.

Interviewer: Well, not the final answer, but, I can see the forty right here. Can you see the 40 right here? (I lift the top layer of 4 sticks of ten, Marifer nods). OK, so you got like part of the answer, and then what? I like what you're doing! Y luego, ¿qué más harías, Marifer? [*So then what else would you do, Marifer?*] So you got 4x10. ¿Qué más? [*What else?*]

Marifer: (Looks at model for a long time. I do not say a word. I am holding up the top layer.): We get another 4, (she lifts the layer of 4 sticks of ten that was in the middle) then these ones (points to the base layer).

Interviewer: Uh-huh, ¿puedes escribirme lo que acabas de descubrir? A ver, vamos a escribirlo. [*Can you write what you just discovered? Let's see, let's write it out.*]

Marifer's miscount of 130 made me curious about what she knew already about counting, therefore I asked her to invent a different way. I also wanted Marifer to move with me to a common understanding of a multiplicative way of thinking about volume, to prepare her to have a resource that she could share in the future with others. Her surprise with my invitation indicated that she was not expecting this challenge, but in the end she successfully created this new common resource between she and I. Her exclamations after reflecting ("Oh!") could indicate many possible things, including a sudden recognition of relevant knowledge for more efficient counting. Perhaps Marifer did not "invent" a different way of counting, as suggested by my request. It is possible that she was able to recognize that a multiplicative way of counting was a relevant idea that she could use to enhance her existing knowledge of volume. She began this multiplicative counting with one layer (4x10), but she knew that was not the final answer. There was a final moment in our interaction when Marifer created yet another common resource: In that moment, Marifer lifted the layer that was in the middle, an action that I had initiated and that she appropriated. By doing this unprompted action, Marifer finished constructing our common resource for recognizing the dimensions of the model and using these dimensions to count the one centimeter cubes multiplicatively.

DISCUSSION

The words in Marifer's talk with me were loaded with her language, bilingualism, culture, and community. However, the words that she chose to talk with me did not pre-date our conversation. They were created between the two of us, as we began to struggle together to recognize—to know in a different way—resources that we wanted to make common for the two of us, in order to understand the important concept of volume. Marifer and I knew that we wanted to talk about mathematics, and that we needed resources in order to do that. In this process, we used material resources (paper model, cubes model) and talk in order to recreate something common between us. At times, we failed to make these things common. I did not begin my talk with Marifer by declaring language or anything else to be a resource, simply because I wanted to work with her and recognize those resources together. Marifer and I are both Mexican, bilingual, and from working class communities. Yet, our language, culture, or community does not warrant that linguistic, cultural, or community resources pre-exist our work together in mathematics. Instead, we must work together to recognize something that is common and therefore useful to advance our understanding of mathematics.

In this paper, I have suggested a specific way in which teachers of English learners can begin to recognize resources as inseparable from their instructional interactions with students. While research suggests the existence of such resources in the language, the culture and the communities of students, the point of access to these resources for teachers has not been specified. In my instructional interaction with Marifer, I was not expecting her to "bring" resources to our talk. Instead, I was interested in recognizing and constructing those resources with her. And so we did. We constructed a cubes model. With that model as our initial common resource, we moved on to recognize key attributes of three-dimensional shapes. As these common resources accumulated in our interaction, we used talk that was at times bilingual and at other times monolingual. Finally, we were able to recognize increasingly sophisticated ways of counting that supported Marifer's reasoning about the concept of volume. I believe that all of the resources that we recognized in our interaction were impregnated with language, culture, and community; only we did not start our interaction with them in mind.

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Corey Drake is an Associate Professor of Mathematics Education and Director of Teacher Preparation at Michigan State University. Recent publications span the areas of mathematics education, curriculum studies, and teacher education. Her long-term research interests include the design of teacher education experiences and contexts to support new teachers in learning to connect to students' resources, as well as the use of curriculum materials as learning tools for teachers.

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