Geometric Reasoning and Problem Solving: Keys to Success for English Language Learners

Mark Driscoll
Education Development Center

First Quote

”Broadly speaking I want to suggest that geometry is that part of mathematics in which **visual thought** is dominant whereas algebra is that part in which **sequential thought** is dominant. This dichotomy is perhaps better conveyed by the words 'insight' versus 'rigour' and both play an essential role in real mathematical problems.

"The educational implications of this are clear. We should aim to cultivate and develop both modes of thought. It is a mistake to overemphasise one at the expense of the other and I suspect that geometry has been suffering in recent years.”

Sir Michael Atiyah

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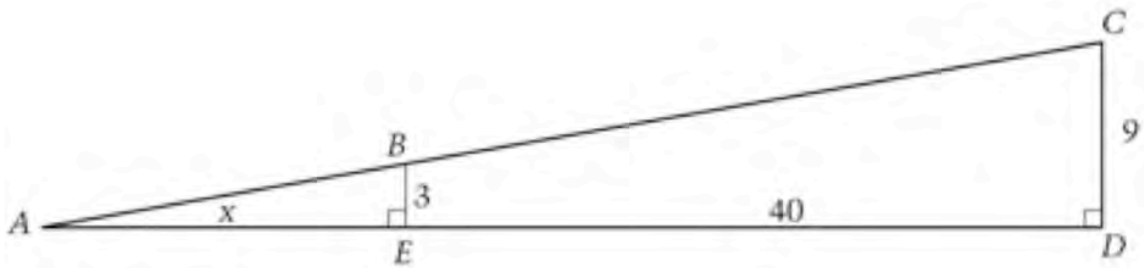
1982

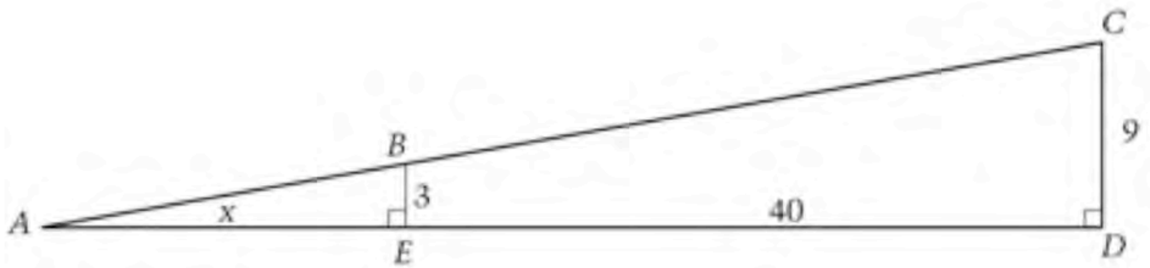
2007 NAEP 8^{th} Grade



The figure above shows two right angles. The length of *AE* is *x* and the length of *DE* is 40.

Show all of the steps that lead to finding the value of *x*. Your last step should give the value of *x*.

2007 NAEP 8th Grade---Only 1% Correct



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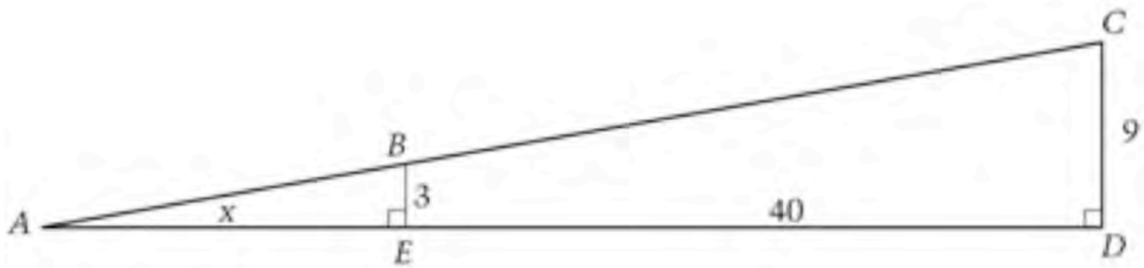

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Second Quote

The back story:

Second Quote

"For ELLs to succeed in learning mathematics, they need to be more *productive* in mathematics classrooms—reasoning more, speaking more, writing more, drawing more."

Maria Santos

Former Director, NYC OELL

Key Question

What knowledge do teachers need in order to support English Language Learners in becoming active participants in mathematical learning communities?

Common Core Standards of Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

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Related question

To what extent do the Fostering Geometric Thinking Toolkit (professional development--Heinemann) materials help teachers learn to support ELLs in becoming active participants in mathematical learning communities?

Research Project

Fostering Mathematics Success of English Language Learners (**FMSELL** project)—A study of the efficacy of the Fostering Geometric Thinking Toolkit materials among teachers of ELLs.

A collaboration of EDC and Horizon Research: National Science Foundation: DRL -0821950

Guiding Principles for Shaping Pedagogy

1. **Challenging Mathematical Tasks Principle.** Engage ELLs at *all* language proficiency levels in mathematical work that challenges them to reason mathematically and solve problems

2. **Multimodal Representation Principle.** Frequent use of pictures, diagrams, presentations, written explanations, and gestures allow ELLs to engage mathematically and express their reasoning and problem solving

3. **Academic Language Principle.** Integrating academic language development with mathematical work enables ELLs to *use* the language in context. Academic language is the driver of mathematical discourse communities. It is much more than vocabulary, incorporating sentence frames that express mathematical reasoning---e.g., "If….then."

Principles have a role in 3 stages

- Lesson planning
- Lesson implementation
- Post-lesson reflection

Challenging Mathematical Tasks Principle

- Silver, E.A. and Stein, M.K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*, 476-522.

- Brenner, M.E. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal 22*(2, 3, & 4), 103-128

Multimodal Representation Principle

- Ng, S.F. & Lee, K. (2009). The model method: Singapore children's tool for representing and solving algebraic word problems. *Journal for Research in Mathematics Education.* 40 (3), 282-313.

- Chval, K.B, & Khisty, L. (2009). Latino students, writing, and mathematics: A case study of successful teaching and learning. In R. Barwell (Ed.) *Multilingualism in mathematics classrooms: Global perspectives*. (pp. 128-144). Clevedon, UK: Multilingual Matters.

Academic Language Principle

- Snow, C., Lawrence, J., & White, C. (2009). Generating knowledge of academic language among urban middle school students. *Journal of Research on Educational Effectiveness, 2*, 325-344.

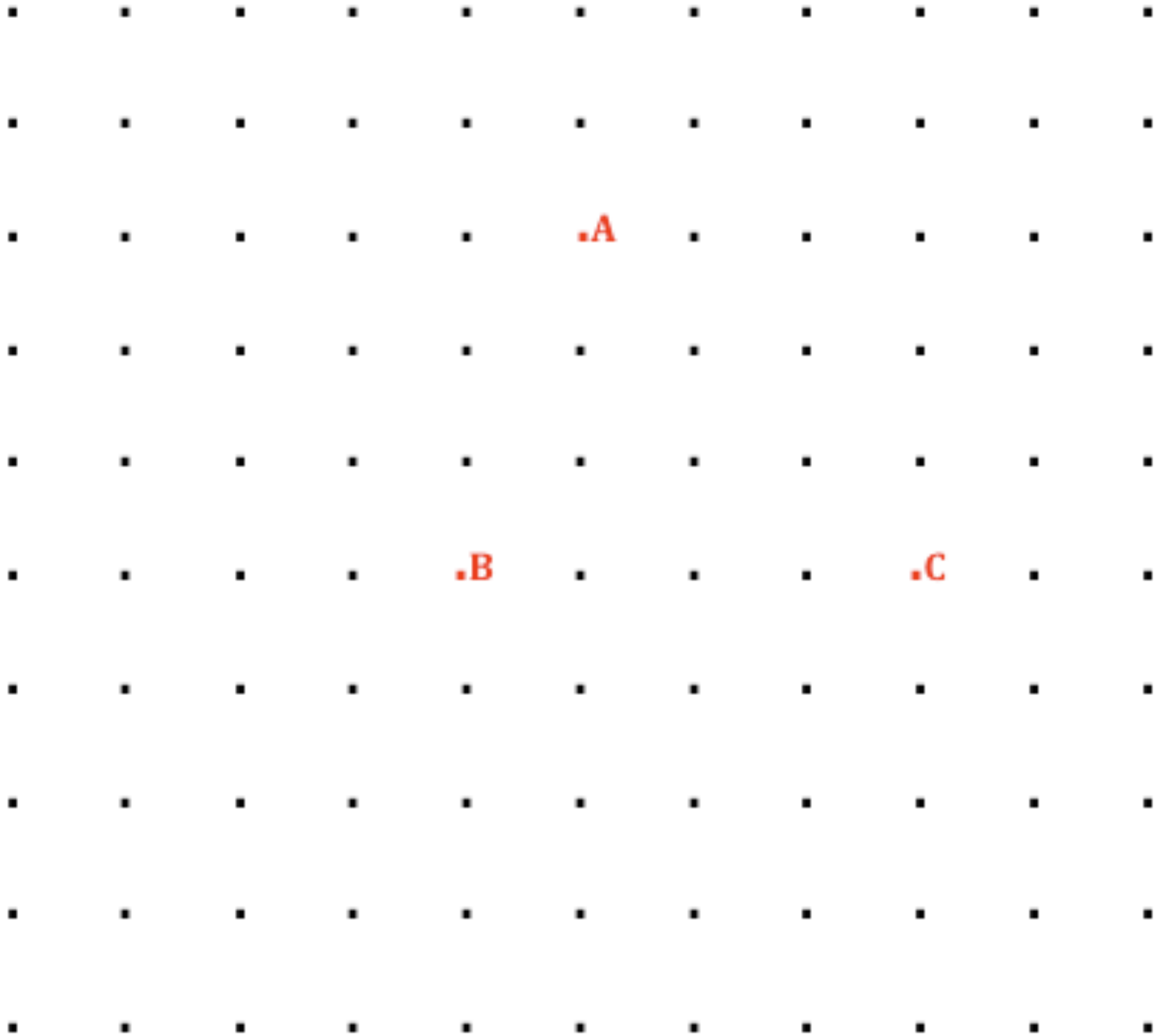
- Schleppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23,* 139-159.

A Core Belief

Geometry provides a context rich in opportunities to enact all three principles.

A problem to think about:

A parallelogram has 3 of its vertices at the points A, B, and C indicated on the following dot grid. Where can the fourth vertex be so that all the four points connect to form a parallelogram? Find as many points as you can that work:


A
B
C

Video Clip

- Piloting student tasks for FMSELL project
- Piloting the tasks *and* the protocols
- 7th grade ELL students

Watch with 2 Questions in Mind

- What opportunities exist here for ELLs to express their mathematical reasoning?

- If this were a teaching situation, what opportunities exist here for ELLs to be active participants in mathematical learning communities?

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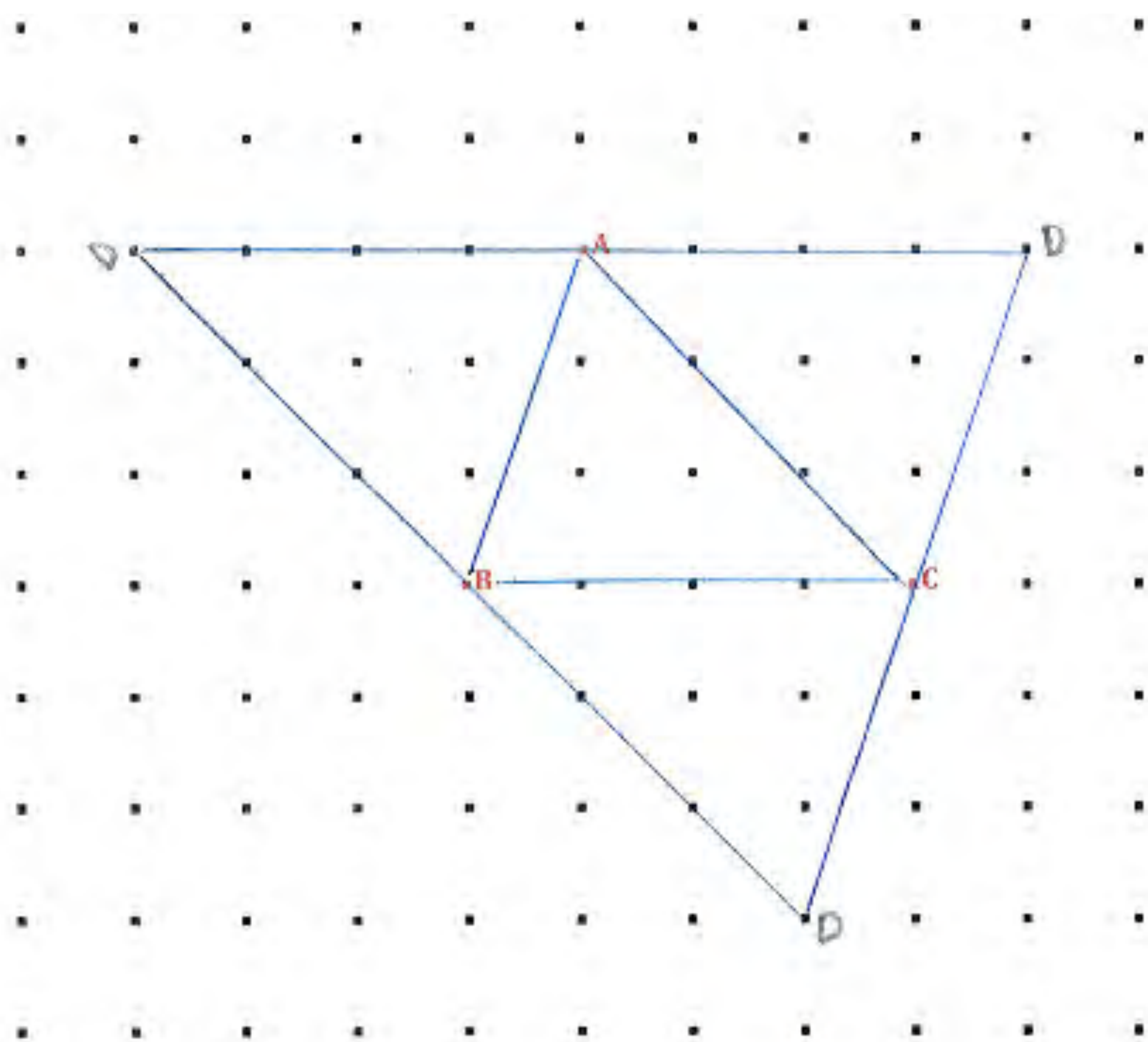
Before the clip

To the questions of the researcher:

- Each of two boys says he has found 2 points as vertices, by “looking opposite the points.”
- Each says he guesses there’s a third point (vertex), but he doesn’t know how to find it.

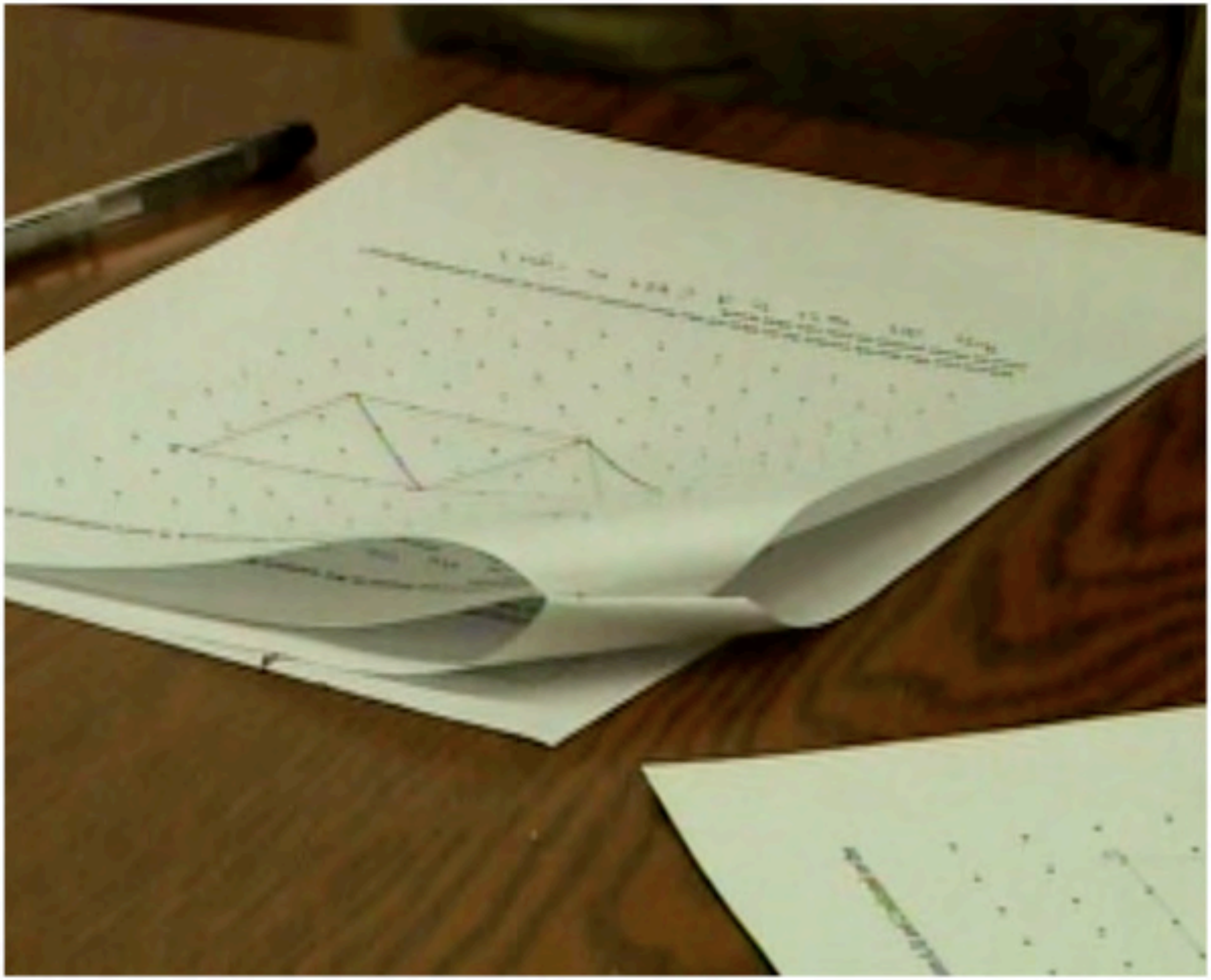
On the clip: “The first one I found because I saw…..”

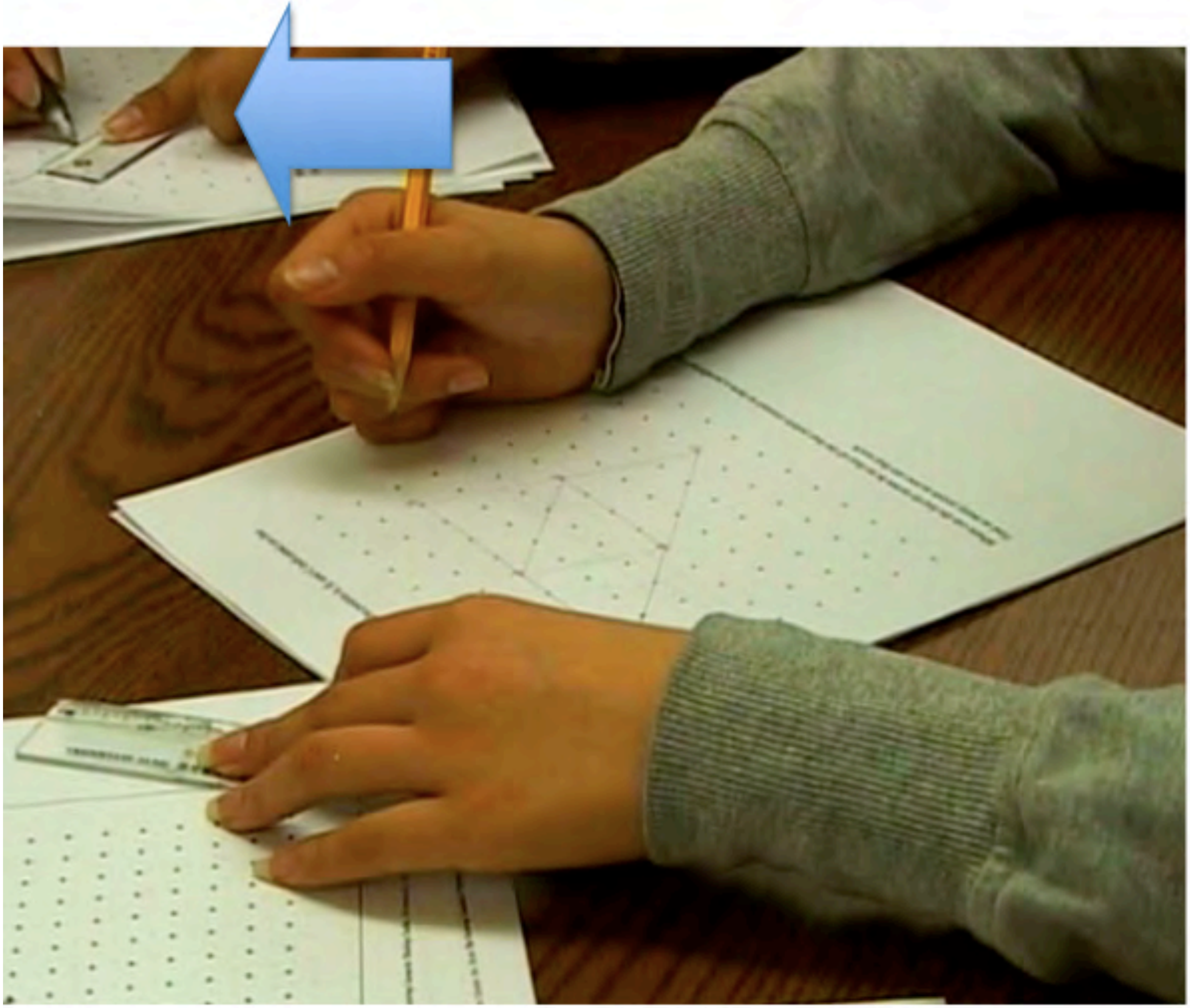
After a minute or so (post-clip) she finishes with: “I saw if I put 2 triangles together, it made a parallelogram.”


A
B
C
D
D
D

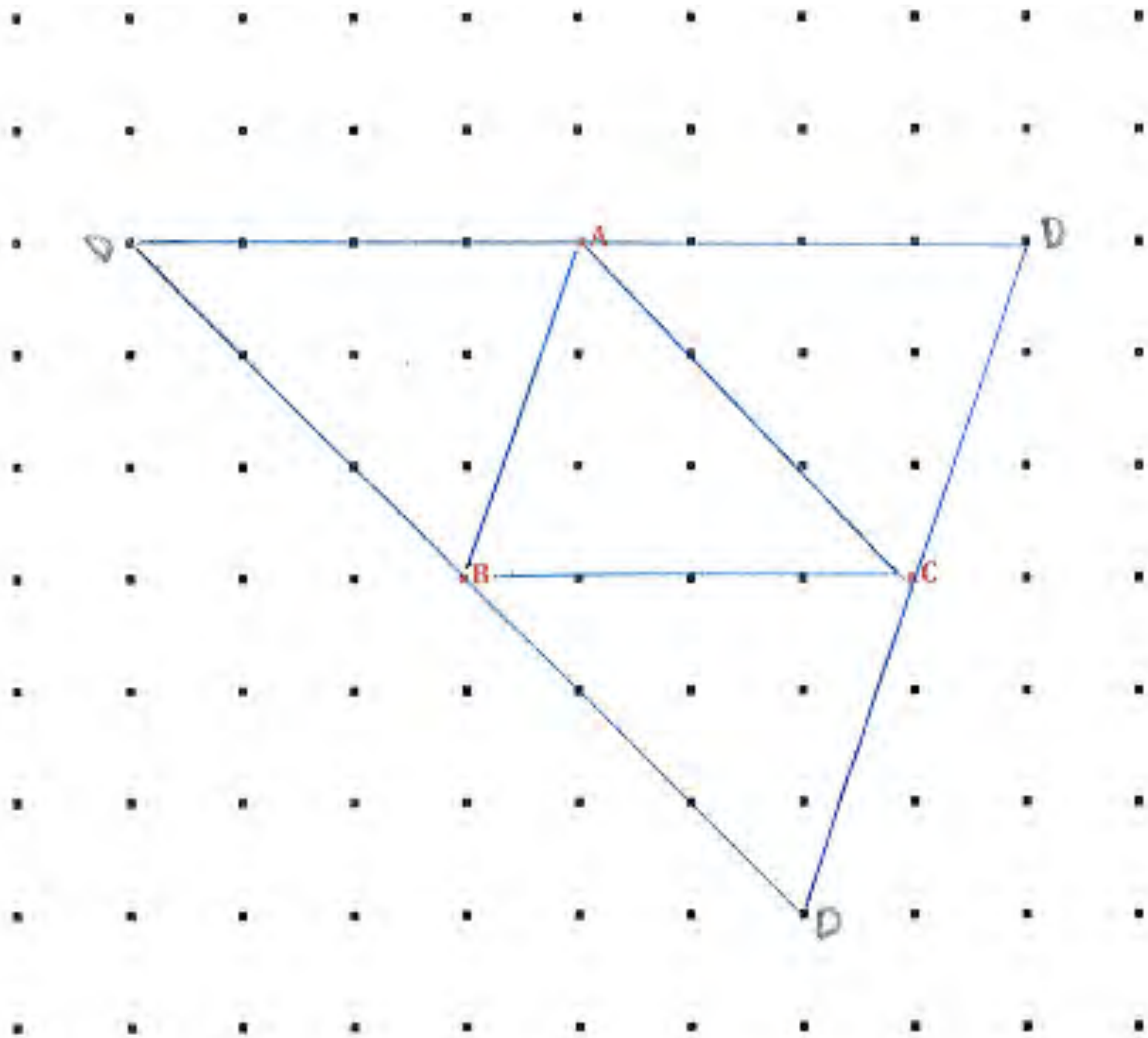
Clip Takeaways

1. There are stages for developing skills in mathematical discourse. Beginnings of mathematical discourse may lack precise language, or even be silent— e.g., observing what others are trying (it's *not* cheating)





2. At next stage, diagrams, writing, gestures, etc., are a bridge to discourse (and geometry provides good bridge material)
3. Rushing academic language should not undercut this development of discourse skills. However,
4. Discourse—the spoken—***is*** an important goal to reach, with precise, mathematical language as the driver of discourse (e.g., the case of 'opposite').

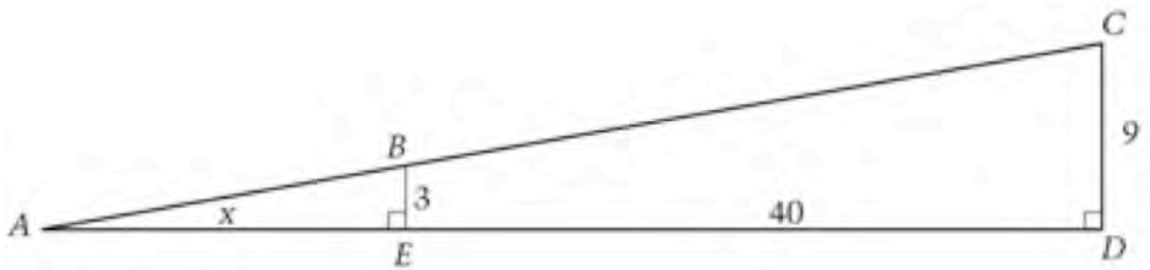

D
A
D
B
C
D

For each point you found in #1, explain how you know the four vertices form a parallelogram.

- they have the parallel
- has the same length with opposite side
- it has 4 angles,
- has 2 angles are the same degree.

Sources of Geometric Reasoning Tasks

Dominance of Rote Procedures in Middle-Grades Geometry



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Shaping Geometric Reasoning Tasks

- Change rote procedure tasks to make reasoning necessary—e.g., “undoing problems”

Change Rote Procedure Tasks

Two vertices of a triangle are located at (4,0) and (8,0) on a coordinate grid. The perimeter of the triangle is 12 units. What are all possible positions for the third vertex? How do you know you have them all?

Shaping Geometric Reasoning Tasks

- Change rote procedure tasks to make reasoning necessary—e.g., “undoing problems”
- Use Paper-folding tasks

Use Paper-folding Tasks

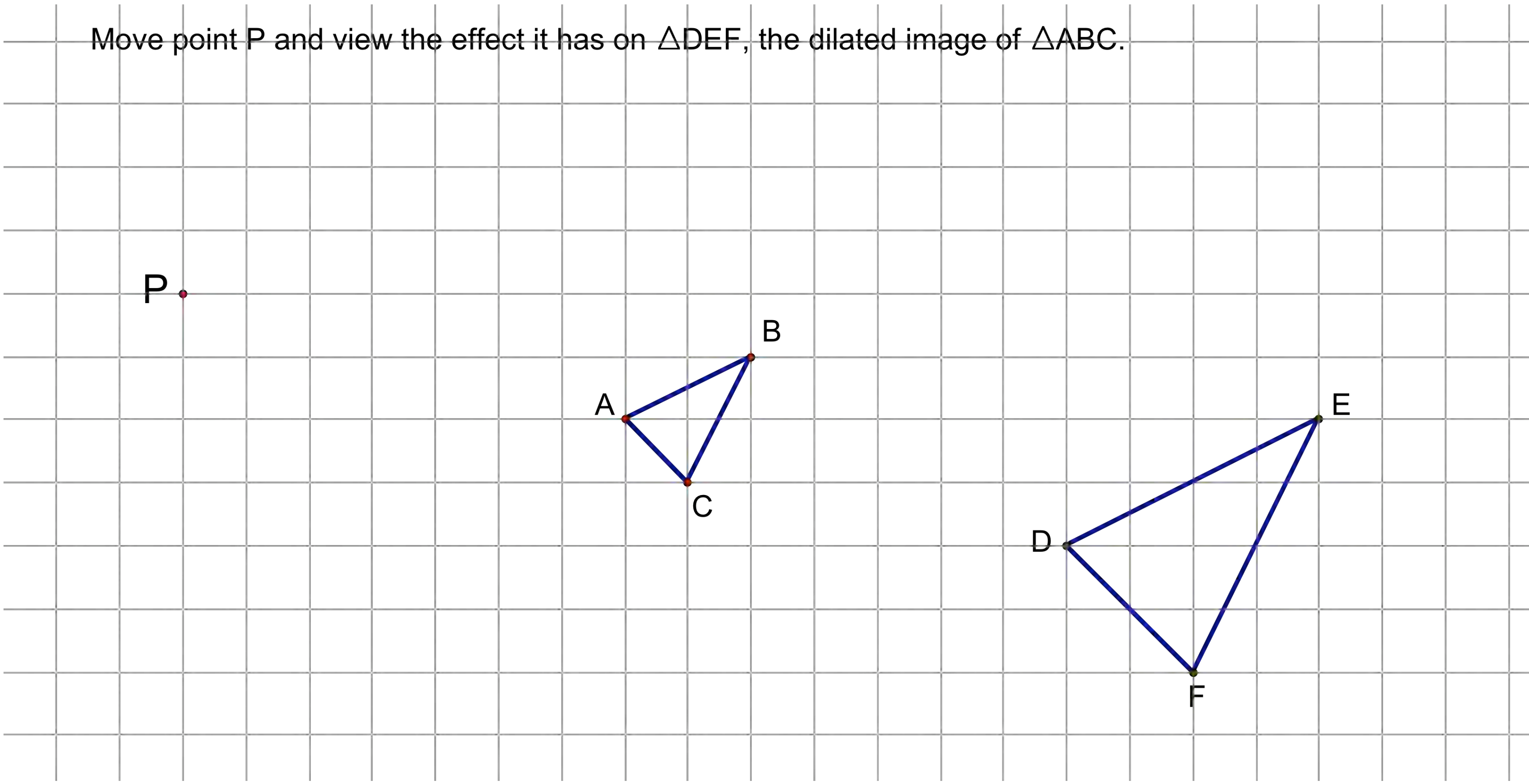
Take a square sheet of paper. Construct a square on the sheet with exactly one-half the area of the original square. How do you know that the new square has one-half the area of the original square?

Take another square sheet of paper. Now construct a non-square rhombus, also with exactly one-half the area of the original square. How do you know the rhombus has one-half the area of the original square?

Shaping Geometric Reasoning Tasks

- Change rote procedure tasks to make reasoning necessary—e.g., "undoing problems"
- Use Paper-folding tasks
- Enhance applet demonstrations

Enhance Applet Demonstrations



Before the Applet:

Draw a new point four grid spaces below the original point P and label it P_2. Predict what a dilated version of ΔABC will look like and where it will land on the grid when you use P_2 instead of P.

- Now draw your predicted triangle on the grid
- How did you decide where to put the new dilated triangle and what it should look like?
- Check your prediction. First on paper, then with the applet

Summary: *Why More Geometric Reasoning and Problem Solving?*

1. It makes possible for *all* students a greater balance of 'insight and rigour' in their mathematical thinking, a la Sir Michael Atiyah's recommendation.
2. Geometry is rich in potential for ELLs to become active and successful participants in mathematical learning communities.

3. It is one route to broader ownership, within school communities, of the challenges involved in helping ELLs succeed academically.

Horwitz, A.R., Uro, G., Price-Baugh, R., Simon, C., Uzzell, R., Lewis, S., and Casserly, M. (2009) *Succeeding with English language learners: Lessons from the Great City Schools*. Washington, D.C.: The Council of Great City Schools. Retrieved on April 1, 2010 from http:// www.cgcs.org/publications/ ELL_Report09.pdf.

- "…collaboration was a carefully orchestrated strategy on the part of the district. Schools, teachers, and subject area departments were encouraged to work together, sharing common planning periods and attending joint trainings. At the classroom level, the instruction of ELs became the joint responsibility of EL teachers, subject area teachers, coaches, and the principal. This was a marked change from the time when ELs and EL teachers were isolated from others." (p.45)