The ideas, most slides, and most handouts come from ProofBlocks: A Visual Approach to Logic and Proof
By Jenni Dirksen & Jinna Hwang

www.proofblocks.com
The Plan

- Why is proof hard for ELLs?
- Using Conditional Statements
- Learning the support
- Trying it yourself
- What are the benefits?
What makes proof tough?

Why is this hard for students?

Given: ΔPRF is equilateral

\[ RO \ bisects \ ∠PRF \]

Prove: ΔPRO ≅ ΔFRO

Use your chat bar to respond.
What makes it hard?

What if you started with the idea that not all students see three triangles?

Before even getting to the postulates, theorems, givens, and format---**HOW** can we support our visual or kinesthetic learner’s and our ELL’s ability to see all of the figures?

**Let’s go on a triangle hunt!**

Can this work with **midpoints and bisectors**?
Theorem: The sum of the interior angles of a triangle is 180°.

Let’s rewrite this as a conditional statement:

At home, rewrite this as a conditional statement.

Use your control bar to raise your hand when you are done.

You probably wrote something like, **If a figure is a triangle, then the sum of the interior angles is 180°**
Conditional Statements

- If a figure is a triangle, then the sum of the interior angles is $180^\circ$.
- What is the input? -- respond by chat bar
- What is the output? -- respond by chat bar
- Can we draw a picture of the input? The output? -- Draw this at home on two opposite sides of a strip of paper.
Conditional statements

Did it look similar to the one below?

Labeling is important, so if you missed labeling your angles and writing the equation, please go ahead and add this to your picture.

If you want to try this with students or colleagues, you may want to use the Basic Theorems and Postulates handout and the conditional block handout.
Conditional Statements

Now, write the theorem in the center of the strip of paper.

**Triangle Sum Theorem**

![Diagram of a triangle with angles A, B, and C, and the equation A+B+C = 180°]

The sum of the interior angles of a triangle is 180°.

- How does this *support* understanding the theorem?
- **Take a minute and look at the proofblocks including HL.**
- **What observations do you have? Share them in the chat bar.**
Proofblocks turn theorems, postulates, and definitions into tools, manipulatives.

The tools have a visual cue, symbolic information, and the text of the theorem, postulate, or definition once students make their set.

This supports students’ use of theorems, postulates, and definitions as well as students’ understanding that proof is a structured connection of inputs and outputs.
Proof Blocks

Given: $\overline{MT} \perp \overline{AH}$  
$H$ is the midpoint of $\overline{MT}$

Prove: $\triangle MAH \cong \triangle TAH$

First we will place the information on the picture as we did in the marking definitions activity.
Given: $\overline{MT} \perp \overline{AH}$
H is the midpoint of $\overline{MT}$

Prove: $\triangle MAH \cong \triangle TAH$

Now we have to find a proofblock that has these inputs.
Given: \( MT \perp AH \)
H is the midpoint of \( MT \)

Prove: \( \triangle MAH \cong \triangle TAH \)
Proof Blocks

It really helps ELLs to make a picture on the block!
Proof Blocks

\[ \text{MH} \cong \text{HT} \]

\[ \angle XXX \cong \angle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

SAS Postulate

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]
Proof Blocks

\[ \overline{MH} \cong \overline{HT} \]

\[ \angle AHM \cong \angle AHT \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]
Proof Blocks

\[ \overline{MH} \cong \overline{HT} \]

\[ \angle AHM \cong \angle AHT \]

\[ \triangle XXX \cong \triangle XXX \]

\[ \overline{AH} \cong \overline{AH} \]

\[ \overline{XX} \cong \overline{XX} \]

\[ \triangle XXX \cong \triangle XXX \]
Proof Blocks

\[ \overline{MH} \cong \overline{HT} \]
\[ \angle \text{AHM} \cong \angle \text{AHT} \]
\[ \overline{AH} \cong \overline{AH} \]

SAS Postulate

\[ \triangle \text{MAH} \cong \triangle \text{TAH} \]
Definitions

H is the midpoint of MT

[Diagram showing the definition of a midpoint with labeled points X and H, and the midpoint property represented as XX ≅ XX]
Definitions

H is the midpoint of MT

\[ X \text{ is the midpt of } XX \]

\[ XX \cong XX \]

Definition of a Midpoint
Connecting Blocks

**Definition of a Midpoint**

\[ X \text{ is the midpoint of } XX \]

\[ \overline{XX} \cong \overline{XX} \]

\[ \overline{MH} \cong \overline{HT} \]

**SAS Postulate**

\[ \triangle XXX \cong \triangle XXX \]

\[ XX \cong XX \]

\[ \triangle XXX \cong \triangle XXX \]
Connecting Blocks

\[ \overline{MH} \cong \overline{HT} \]

Definition of a Midpoint

\[ XX \cong \overline{XX} \]

SAS Postulate

\[ \triangle XXX \cong \triangle XXX \]

\[ \triangle XXX \cong \triangle XXX \]
Proof Blocks

G

H is the midpoint of MT

Defn of a Midpoint

\( \overline{MH} \cong \overline{HT} \)

SAS
Definitions

\[ MT \perp AH \]

Definition of Perpendicular Lines

\[ \angle XXX \cong \angle XXX \]

or

\[ m\angle XXX = 90^\circ \]
Definitions

\[ \overline{MT} \perp \overline{AH} \]

**Definition of Perpendicular Lines**

\[ \angle ABC \cong \angle DBC \]

or

\[ m\angle ABC = 90^\circ \]
Connecting Blocks

Definition of Perpendicular Lines

\[ \angle AHM \cong \angle AHT \]
Connecting Blocks

Definition of Perpendicular Lines

\[ \angle DAB \cong \angle DBC \text{ or } m \angle DAB = 90^\circ \]

\[ \triangle ABD \cong \triangle BCD \]

\[ \angle AHB \cong \angle AHT \]

SAS Postulate

\[ \triangle ABC \cong \triangle DEF \]

\[ \triangle ABC \cong \triangle DEF \]
Proof Blocks

H is the midpoint of MT

Defn of a Midpoint

MT ⊥ AH

Defn of ⊥ Lines

MH ≅ HT

∠AHM ≅ ∠AHT

SAS

ΔMAH = ΔTAH
Proof Blocks

G: H is the midpoint of MT

G: MT perpendicular to AH

Defn of a Midpoint: MH congruent to HT

Defn of Perpendicular Lines: Angle AHM congruent to angle AHT

SAS

Triangle MAH congruent to triangle TAH
Proof Blocks

- H is the midpoint of MT
- MT \perp AH

Definitions:
- Defn of a Midpoint
- Defn of \perp Lines

Statements:
- MH \cong HT
- \angle AHM \cong \angle AHT
- AH \cong AH

Conclusion:
- \triangle MAH = \triangle TAH

Questions:
- What block looks like this statement?
- Where did we learn this?
Proof Blocks

- H is the midpoint of MT
- MT \perp AH
- Defn of a Midpoint: MH \cong HT
- Defn of \perp Lines: \angle AHM \cong \angle AHT
- Reflexive Prop.: AH \cong AH
- SAS
- \triangle MAH = \triangle TAH
Proof Blocks

- G: H is the midpoint of MT
- G: MT ⊥ AH
- Reflexive Prop.: AH ≅ AH
- Defn of a Midpoint: MH ≅ HT
- Defn of ⊥ Lines: ∠AHM ≅ ∠AHT

SAS → ΔMAH = ΔTAH
An Extension

Given: \( \overline{MT} \perp \overline{AH} \)

\( H \) is the midpoint of \( \overline{MT} \)

Prove: \( \overline{AM} \cong \overline{AT} \)
An Extension

Given: \( \overline{MT} \perp \overline{AH} \)

H is the midpoint of \( \overline{MT} \)

Prove: \( \overline{AM} \cong \overline{AT} \)
An Extension

Given: $\overline{MT} \perp \overline{AH}$
$H$ is the midpoint of $\overline{MT}$

Prove: $\overline{AM} \cong \overline{AT}$

**Corresponding Parts of Congruent Triangles are Congruent**
An Extension

I have found color pictures work well with this block.

Corresponding parts of congruent triangles are congruent

$\triangle XXX \cong \triangle XXX$

CPCTC

$XX \cong XX$

$\angle XXX \cong \angle XXX$
An Extension

Given: \( MT \perp AH \)

H is the midpoint of MT

Prove: \( AM \cong AT \)
Given: \(O \text{ is the midpoint of } BK\) 
\(<B \cong <K\)

Prove: \(BL \cong CK\)

Did you find the Definition of a Midpoint block?

Did you find the vertical angles theorem?

Did you use ASA?

Did you use CPCTC?

Did you mark vertical angles as a “P”? 
Proof Blocks in Action
Given: \( \overline{DG} \parallel \overline{EF} \) and \( \overline{DG} \) is extended within \( \overline{EF} \).
Proof: 
\[ \overline{DG} \parallel \overline{EF} \]
\[ DG = DF \]
Given: O is the midpoint of \( \overline{CD} \), \( \overline{CY} \) is parallel to \( \overline{OD} \), \( \overline{CO} \) is congruent to \( \overline{OY} \), \( \overline{OD} \) is congruent to \( \overline{OD} \), \( \triangle CYD \) is congruent to \( \triangle ABO \) (SAS), \( \triangle AYO \) is congruent to \( \triangle AYO \) (CPCTC).
Benefits

- Theorems, postulates, and definitions *become manipulatives*
- Student focused
- Visual, kinesthetic, and ELL friendly
- Easily checked logic
- Flexible: Front to back, back to front, middle to ends
- An unexpected benefit is that students are able to discern information from a picture and information from given statements as different (not apparent in traditional instruction).
Questions?
I will stay and answer your questions, send them via your chat bar.
Thank you all for joining me this evening!

Thank you to Jenni Dirksen & Jinna Hwang for their creation of proofblocks!
Their materials can be found at:
www.proofblocks.com