Mathematics of the Americas

ACTIVITY BOOKLET

TODOS
Mathematics for ALL

Activities to use with the poster set

TODOS/Key Curriculum Press

Activity Booklet

Mathematics of the Americas

TODOS Poster Set Activities

The TODOS/Key Curriculum Press Activity Booklet for the *Mathematics of the Americas* TODOS Poster Set is a joint project of TODOS and Key Curriculum Press. Purchasers of the *Mathematics of the Americas* poster set receive this activity booklet with the posters. It can also be purchased separately. The *Mathematics of the Americas* poster set and the activity booklet can be purchased from Key Curriculum Press.
A portion of all sales goes to support the work of TODOS: Mathematics for ALL.

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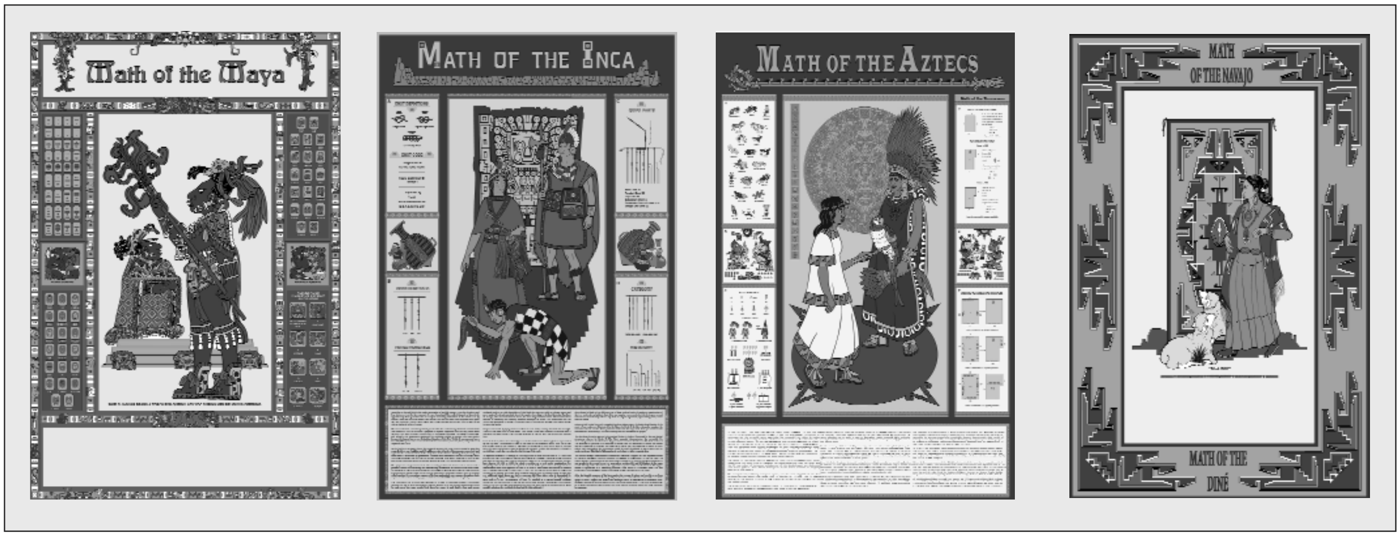

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Credits and Acknowledgements

A special thank you must be extended to Key Curriculum Press and *TODOS: Mathematics for All* for instigating and supporting these efforts to create a booklet to accompany the Key Curriculum poster set—*Math of the Americas*. This four-poster set includes *Mathematics of the Maya, Mathematics of the Inca*, *Mathematics of the Aztec*, and the *Mathematics of the Navajo*.

We greatly appreciate the guidance and direction provided by Susie W. Håkansson, California Mathematics Project (CMP) Executive Director and TODOS Board Member, and the financial support of the CMP.

TODOS gratefully acknowledges the contributions of the following reviewers:

Cynthia Anhalt, Gloria Brooks, John Clay, Carol A. Edwards, Susie W. Håkansson, Sharon Hoffert, Rosemary Klein, Miriam Leiva, Bob McDonald, Luis Ortiz-Franco, Nora Ramirez, Tod Shockey, and Rick Silverman.

Throughout our study of these indigenous cultures, our respect grew for the mathematics by which these people of the Americas are so often defined. We hope that our efforts allow teachers to more effectively use the poster set to enhance the instruction they provide their students and help students see the beauty of mathematics when experienced in its many cultural forms in the Americas and throughout our world.

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Welcome and Foreward

TODOS
Mathematics for ALL

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students—in particular Hispanic/Latino students—by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

The Intriguing World Of Culture, History, and Mathematics

Foreword by Miriam A. Leiva

Over time, through many years in teaching, I discovered that in order to teach successfully I had to "learn" deeply about ***each*** of my students: their personal history, culture, life experiences, their family and friends, and their previous academic background. This information varied greatly among each of my pupils, whether they were born in our area, or in far away lands. What I found was that we, teachers, had to give each one of them an equal opportunity to be different, and utilize these differences to reach them individually and collectively. Finally, I recognized that my mathematics lessons had to connect to other academic subjects, to the student's world, and to the greater world outside of school.

The poster set, *Mathematics of the Americas*, offers teachers an opportunity to make historical, cultural, and mathematical connections in the classroom. These connections transcend barriers and reinforce both the mathematics and the cultures being studied; cultures such as the Maya, Inca, Aztec, Navajo, and others to come.

In 2006, Steve Rasmussen, then President of Key Curriculum Press and a TODOS member, approached the TODOS Board of Directors with a generous offer: Key Curriculum would *bundle* their posters dealing with Native Cultures of the Americas and give TODOS a monetary gift for each set sold. In addition, he invited TODOS to write this document to accompany the set when distributed. We accepted Key Curriculum's offer and this booklet is the result.

On behalf of the Board of Directors of *TODOS: Mathematics for ALL*, I thank Susie W. Håkansson, Project Chair, as well as Jim Barta, Writing Team Chair, and co-authors: Lenie Galima and Silvia Llamas-Flores. It is our intention that you will use the information here to give students a glimpse of the proud past and knowledge of the civilizations featured. Hopefully, these will enhance their knowledge of their own mathematical skills, as well as culture and history. Finally, we hope that this helps open doors for each student to witness the diverse beauty of other cultures and the role mathematics has played in their development.

Miriam A. Leiva

Founding President
TODOS: Mathematics for ALL

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Understanding the Relationship Between Mathematics and Culture(s)

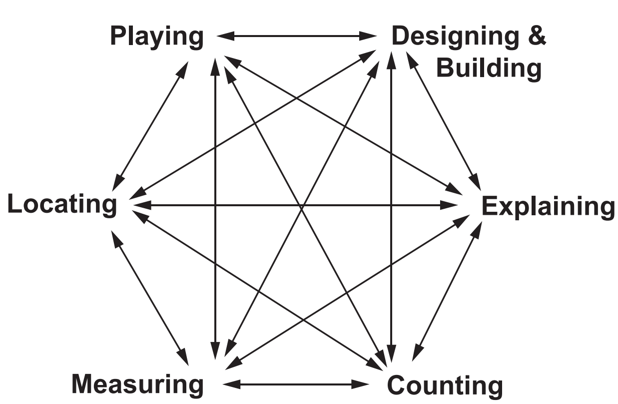
Mathematical applications are especially evident as the people who throughout the world and over time use them to solve problems of daily life. Mathematical applications of people of diverse cultures using mathematical concepts and procedures often become a reflection of that community affected by the vocabulary used to describe it, the tools used to implement it, and the thinking used to create it. In a real sense, our understanding of societies, whether past or present, is enhanced as we learn to witness the relationships between mathematics and cultures.

Teachers who learn to see mathematics as a key component of culture can help their students grow to realize the reciprocal role of mathematics in helping to shape a community and how, at the same time, mathematics is shaped through its use in the community. Emphasizing mathematics in context can often help to confirm and even restore the cultural dignity of cultures previously thought of as backwards, primitive, or less than sophisticated. As our students explore and experience multicultural mathematical activities reflecting the knowledge and behaviors of people from diverse cultures, they may learn not only value the mathematics, but, just as importantly, also to develop a greater respect for those different from themselves.

Finding Applications of Mathematics in Daily Life

Counting, measuring, locating, designing, explaining, and playing are "universal" human activities we find in applications of daily life, whether they are historical or contemporary. These activities are illustrated by various cultures in many different levels of representation and abstraction, formality or informality, concreteness, or mental abstractiveness (Bishop, 1988).

Shockey, Mitchell, and Barta Conceptualization of Bishop's 6



These universals are clearly evident when teachers incorporate a human context in the mathematics they teach. Educators can use the twelve activities presented in this booklet as lenses through which to better identify, understand, and describe mathematical concepts and procedures of diverse communities being studied and help their students not only gain new mathematical insights but new perspectives of other cultures as well.

For instance, we can use counting to examine how a culture keeps discrete records of quantities and names those values; measuring to quantify, in a continuous sense, the length, width, volume, and duration of an object or event; locating to note a position of one object in relation to another; designing to describe the patterning, geometric shape, or transformational movement of those shapes in a building design, decorative textile, or artistic display; explaining to communicate and analyze data and provide proof in a variety of representations such as through the use of graphs, spreadsheets, and statistics, or algorithmic procedures; and, lastly, playing to define scoring procedures, game rules, game shapes, motions, and gaming features.

The diagram of the six universals depicts these universals as intricately dependent and dynamically interwoven, rather than isolated, disconnected aspects of human behaviors.

The mathematics of the cultures depicted in the *Mathematics of the Americas* poster set can be explored using these universals. Teachers may ask, for instance, how did the Maya count, the Inca measure, the Aztec explain, or the Diné design? Answers to these questions provide a bridge that connects mathematics instruction to cultures or contexts where applications abound. Educators can use these universals to seek additional instructional opportunities for examining the mathematics in the cultural communities in which they live and work.

How To Use This Guide

For some of you, this may be your first attempt to incorporate cultural connections into the mathematics you teach. For others, more familiar with this practice, you are adding to the insights you already possess as you experience these activities with your students. Our efforts are intended to serve two purposes in your development as equity-oriented educators:

- At one level these activities are ready-made to use as directed in your current instruction. You will see how mathematics instruction can be presented within a cultural and historical context, which will allow students to gain a deeper understanding of the role of mathematics in shaping cultures and, conversely, the role of cultures in shaping mathematics.

- The second purpose is to provide teachers who wish to create their own culturally responsive curriculum a model to follow. Using several simple techniques and viewing mathematics as a "cultural event," you will be able to gain information about the cultural community, which reflects the students you teach or another you wish to study. Your mathematics instruction can then be shaped to include a "cultural context" illustrating mathematical (cultural) nuances, specific traditions, or activities of the community being studied.

This poster booklet is intended to be more than a series of activities for teachers to implement in the classroom. Educators must know the foundational issues of culturally responsive teaching that help situate this instruction in the curriculum and illustrate its purpose and benefits. This guide explains:

- The importance of understanding the role of culture in teaching and learning and the necessity of knowing your students.
- The benefit of identifying six human behaviors involving mathematics that are found in nearly every cultural community.

The activities depicting the mathematics of the Maya, Inca, Aztec, and Navajo are shared in three grade ranges: K–3, 4–8, and 9–12. The booklet was written to provide links to the posters, which hopefully hang from your classroom walls.

- Each activity begins with brief background information that is more fully explained and illustrated on its accompanying poster.
- Hands-on activities are included to encourage both teachers and students alike to engage critically and thoughtfully in expanding their knowledge and skills.
- The mathematics instruction incorporates aspects of research-based best practice and leads students from the concrete, interactive stage, to the drawing of their thoughts and ideas, and finally to the description of mathematics in its most abstract form—that of numbers and symbols.

Activities noting the mathematical progression across grade levels follow.

The Mathematics of the **Maya** activities include:

- Number and Number Sense for Grades K–3, where students explore our base-10 counting system and learn about the Mayan base-20 number system.
- Place Value and Number Operations for Grades 4–8, where students explore different Mayan calendars and focus on the least common multiple of two numbers.
- Place Value and Number Operations for Grades 9–12, where students calculate perimeter, area, surface area, and volume of the Mayan Pyramid and interpret linear, quadratic and cubic functions.

The Mathematics of the **Inca** activities include:

- Data Analysis, Numbers and Number Sense for Grades K–3, where students collect, represent and analyze related data to illustrate base-10 place value concepts.
- Data Analysis, Numbers and Number Sense for Grades 4–8, where students create the Inca "quipu," knotted, color-coded cords, to explore the concept of ratio.
- Data Analysis, Numbers and Number Sense for Grades 9–12, where students create and explore the Inca "quipu," knotted, color-coded cords, to represent collected data, and use a graphing calculator to represent the data as scatter plots and calculate linear regressions.

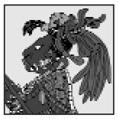
The Mathematics of the **Aztec** activities include:

- Number and Number Sense for Grades K–3, where students use the pictorial Aztec numbers to solve a variety of problems requiring arithmetic operations.
- Algebra for Grades 4–8, where students explore Aztec pyramids and the ratio of the height to the setback of the wall.
- Geometry and Data Analysis for Grades 9–12, where students create and explore the Aztec calendar to compute areas of common geometric figures.

The Mathematics of the **Navajo** activities include:

- Geometry and Measurement for Grades K–3, where students identify, sort, classify, and measure geometric shapes.
- Geometry and Measurement for Grades 4–8, where students create a Navajo rug investigating transformations, symmetry, and area of different geometric designs.
- Geometry for Grades 9–12, where students learn the effect of transformations on figures in the coordinate plane and space.

We hope you have as much fun using this booklet as we have had in writing it. We wish you all of the best in exploring the diverse mathematics one can encounter as our eyes and those of our students are opened to the wonderful uses of mathematics utilized by indigenous cultures around the world!



Math of the Maya

Numbers and Number Sense—Grades K–3

Objectives

Students gain rote and rational counting knowledge as they explore our Hindu-Arabic (base-10) counting system and learn about the Mayan vigesimal (base-20) number system. They learn to translate values from our base-10 counting system into their Mayan equivalent and from Mayan into base-10 to gain a deeper awareness of number systems and ways of counting.

NCTM Standards (Number and Operations)

Enable students to:

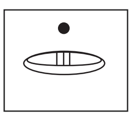
- understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Materials

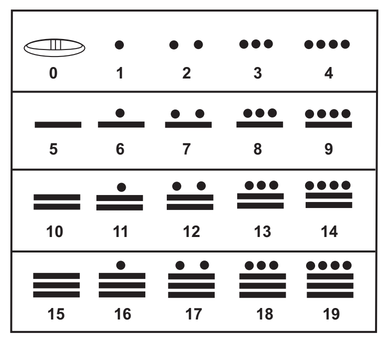
- Beans, toothpicks or popsicle sticks, shell macaroni
- 3 x 5-inch index cards

Introduce

Of all Native American mathematical systems, that of the Maya is the most complex. In the Mayan base-20 system, any whole number can be written with three symbols as indicated on the poster: the dot for 1, the bar for 5, and the shell for 0.



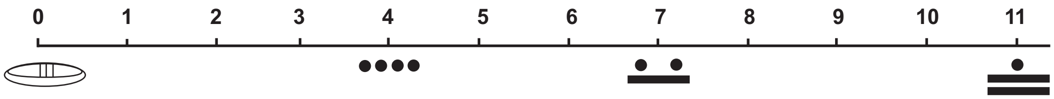
One of the most important mathematical inventions of the Maya was the concept of zero. The shell shape was used to represent 0 and also acted as a placeholder since large numbers were written vertically as powers of 20. This invention allowed the Maya to count past the digits of their fingers and toes. The number 20, for instance, was written as a dot above a shell to indicate one group of 20 and no units.



- Connect to the *Math of the Maya* poster by facilitating a discussion on the mathematical contributions of the Maya, as well as the use of the vigesimal (base-20) counting system.
- Model how to convert the Mayan numbers to base-10 decimal numbers using beans, toothpicks or popsicle sticks, and macaroni. Note that the bean represents one, the toothpick represents five, and the macaroni represents zero.
- Distribute a set of beans, toothpicks or popsicle sticks, macaroni, and five 3 x 5-inch index cards to each student. Ask students to create several Mayan numbers using the beans, toothpicks and macaroni.
- Once students are comfortable constructing Mayan numbers, ask the students to choose five Mayan numbers, from 1 to 26, and have them write each on a 3 x 5-inch index card, one per card. Have students work in pairs. Ask students to randomly pick a card and have the pair of students order their cards from greatest to least (to avoid negative numbers when subtracting) and then ask them to add, subtract, multiply, and divide, as appropriate, their Mayan numbers. Students should use all five cards, and perform all four operations with each set of cards, limiting multipliers and divisors to one-digit numbers.

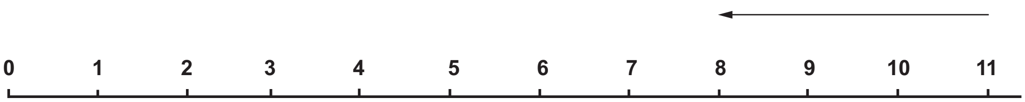
Explore and Create

Draw a number line across your board or create one on the floor using masking tape. Start at 0 and scale and label increments on the number line using values from our contemporary base-10 counting system. The length of the number line is determined by the grade level and interest of your students. Practice skip counting, forwards and backwards, using the drawn line as your model. Using a shuffled set of Mayan number cards, have students pull a card and place it or tape it on the number line on the correct position of its equivalent base-10 value. Can you place them all in their proper location?



Apply and Extend

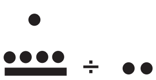
- Model addition, subtraction, multiplication, and division using the number line in the base-10 system. The example below shows one representation of the number sentence 11 – 3 = 8.



- Calculate and draw the results for the algorithms below using Mayan numbers.

  Example for adding two Mayan numbers:

- Compute the following using Mayan numbers for the solution.



- Have students suggest additional number sentences, illustrate them, and show the results using the Mayan numerical system.

Summarize and Assess

■ *Discussion Questions*

- How is counting in base-20 similar to counting in base-10? How is it different?
- How are operations in base-20 similar to operations in base-10? How are they different?
- How did this activity reinforce the meaning of each digit in base-10?
- Write a short explanation of the Mayan number system. How did learning about the math of the Maya help you better understand the base-10 system we use today?

Math of the Maya

Place Value and Number Operations—Grades 4–8

Objectives

Students explore the two different Mayan calendars and the cycle that occurs when both calendars are blended, known as the Calendar Round. Students focus on finding the least common multiple of two numbers by looking at their prime factorizations.

NCTM Standards (Number and Operations)

Enable students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another.

Materials

- Round paper plates or cardstock, at least 6 inches in diameter (2 for each student)
- Compass and protractor (if needed)
- Thumbtacks or pushpins
- Scissors
- One sheet of heavy cardboard (at least 9 x 12 inches) per student

Introduce

Start by connecting the lesson to the *Math of the Maya* poster by facilitating a discussion on the blend of the two Mayan cycles to create a calendar round, which consisted of a 52-year Mayan cycle. Emphasize that a calendar round is created by combining the astronomical year of 365 days, known as the "haab," and a ritual year of 260 days, known as the "tzolkin."

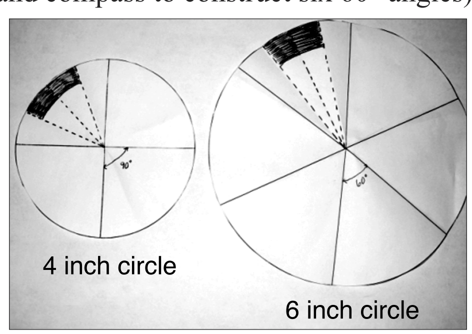
Explore and Create

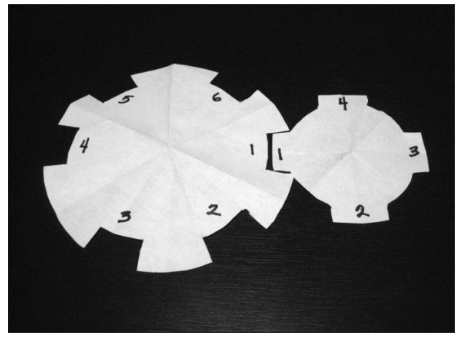
- Provide each student with two paper plates or cardstock. Have the students use a compass to construct a circle of radius 2 inches and a concentric circle of radius 1-1/2 inches on one paper plate and a circle of radius 3 inches and a concentric circle of radius 2-1/2 inches on the second paper plate. Have them cut around the large circle on each of the paper plates, resulting in two disks, each with a circle drawn on them. (Note: circles can be prepared in advance for students.)
- Have students divide the circles into the following sectors:

  4-inch circle: Divide into four congruent sectors by folding the circle in half and then again in half and draw lines on the folds to represent the four sectors (or draw two perpendicular lines through the center of the circle).

  6-inch circle: Divide into six congruent sectors by folding the circle in half and then again in thirds (as precisely as possible, or by using a protractor and compass to construct six 60° angles) and draw lines on the folds to represent the six sectors.

Have students divide each sector of their two circles into four equal wedges (this can be done by folding each sector in half and then again in half). For each sector, have students cut the outer section of the middle two wedges up to the circle drawn (see diagram). Have students mesh the two wheels together and turn them, one clockwise and the other counterclockwise to ensure a good fit. Smooth out any uneven edges if necessary.





- Label the wheels as follows (see figure):

  4-inch wheel: Label the teeth: 1, 2, 3 4, going counterclockwise

  6-inch wheel: Label the spaces: 1, 2, 3, 4, 5, 6 going clockwise

  - Facilitate a discussion on how each of the wheels created represents a different calendar. In this case, the 4-inch wheel represents a 4-day week, while the 6-inch wheel represents a 6-day week.
  - Have students blend the calendars by putting a pushpin through the center of the wheels and on the cardboard. Mesh the two calendars by turning the 4-day calendar clockwise and the 6-day calendar counterclockwise.
  - Pose the following questions to students: How many turns or revolutions of each of the calendars does it take before you are back to the starting position? How many days is that?
    - 2 turns of the 6-day calendar bring you back to the starting point (2 x 6 = 12).
    - 3 turns of the 4-day calendar bring you back to the starting point (3 x 4 = 12).

    After 12 days the two calendars return to their starting position.
  - At this point, facilitate a discussion on the least common multiple, in this case 12.
    - Prime factorization of 6 is 3 x 2.
    - Prime factorization of 4 is 2 x 2.

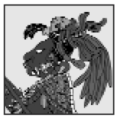
Apply and Extend

- What if we created two different Calendar Rounds, one with 10 teeth and one with 15 teeth? What is the number of revolutions that each would make before coming together at the starting position? How many days does it take for the two calendars to return to their starting positions? What is the least common multiple of 10 and 15? Students may realize that the number of days it takes to return to the starting position is the same as the least common multiple, and additional returns to the starting position result in other common multiples.
- What if we created the Calendar Rounds with 42 teeth and 70 teeth? 120 teeth and 135 teeth?
- We are now ready to blend the "tzolkin" and "haab" calendars to create the Calendar Round. What if we created the Calendar Rounds with 260 teeth (tzolkin calendar) and 365 teeth (haab calendar). What is the number of revolutions that each would make before coming together at the starting position? How many days does it take for the two calendars to return to their starting positions? What is the least common multiple of 260 and 375?
- Choose two numbers that are important in life and create your own Calendar Rounds. What did you discover about those numbers?

Summarize and Assess

■ *Discussion Questions*

- Have students write a short explanation of the Mayan calendar system stating similarities and differences between our 12 month, 365 day calendar and the ones used by the Mayan.
- How is the least common multiple related to the number of days it takes for two different Calendar Rounds to return to its starting position? Why does this work?
- Can every number be described with a set of prime factors? Is there an example of a number that cannot be described as a set of prime factors?



Math of the Maya

Place Value and Number Operations—Grades 9–12

Objectives

Students calculate perimeter, surface area and volume using a Mayan pyramid as a model. They develop polynomial functions for the pyramid's perimeter, surface area and volume, and interpret linear, quadratic and cubic functions using multiple representations.

NCTM Standards (Algebra, Geometry, and Measurement Standards)

Enable students to:

- understand patterns, relations, and functions, and generalize patterns using explicitly defined and recursively-defined functions (*Algebra Standards*);
- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships (*Geometry Standards*);
- understand measurable attributes of objects and the units, systems, and processes of measurement, and apply appropriate techniques, tools, and formulas to determine measurements (*Measurement Standards*).

Materials

- Model of a Mayan pyramid
- Diagram of top, side and front views of selected pyramid
- Cubes
- Rulers
- Graph paper and/or technology

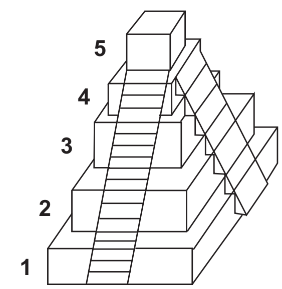


Figure A.

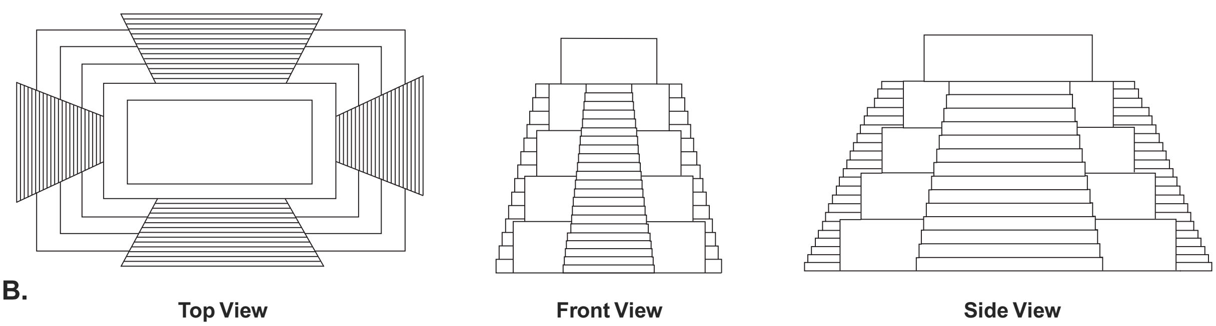


Figure B.

Introduce

- Extend *Math of the Maya* poster lessons for Grades K–8 by describing how Mayan cities across Meso-America were built using sophisticated architectural structures including pyramids. These cities were thought to have been built a thousand or more years ago.
- Show a 3D model of a Mayan pyramid or photos available from the Internet or have the students use cubes to build a model.
- Have students make observations and conjectures related to perimeter, surface area, and volume of the pyramid. Have students identify the measurements needed to calculate perimeter, surface area, and volume.
- Have students identify the formula for perimeter, surface area, and volume of a rectangular prism. (Figure A.)

Explore and Create

- Place students in groups of 3–4 and give enlarged copies of the front, side and top view of the Mayan pyramid. (Figure B.)
- Familiarize students with the activity using the pyramid with no steps.
    - Make a table of values for the data gathered on perimeter (using the top view), surface area and volume (using the top, side and front views) of the Mayan Pyramid. Let the independent variable represent the level of the pyramid. Let the dependent variable represent the perimeter, surface area or volume.
    - Measure the dimensions of each level of the pyramid with a ruler (units in mm). Record data in the appropriate table.
    - Calculate the perimeter of the top view of each level, the exposed surface area (if there were no steps) of each level and the volume of each level.
    - Graph the data for each calculation. Label all axes and use appropriate scales.
    - Derive a polynomial function representing each of the calculations made above.
    - Discuss each of the functions. How does the degree of the function relate to the dependent variable being calculated?

Apply and Extend

- Measure the dimensions of the stairs in the front and back of the pyramid using the top and front views:
    - Find the perimeter, surface area, and volume of each of the stairs of the pyramid measured.
    - Investigate if there is a proportional relationship in terms of cumulative number of stairs and the level number of a pyramid. If students discover this exists, have them calculate the perimeter, surface area, and volume of the base of a new pyramid constructed one level taller. If the new level was added to the top of the original pyramid, determine its perimeter (top view), surface area, and volume.
- Have the students use the Internet to explore some of the following types of questions about Mayan pyramids:
    - Were the number of steps of significance to the Mayan culture?
    - What were the actual dimensions of some of the well-known pyramids?
    - How long did it take to build these pyramids?
    - Have the students brainstorm further related questions, research them, then write up the information found.

Summarize and Assess

- Challenge students to compare the graphs and explain how are they similar and how are they different.
- Have a group of students use cubes to build a pyramid and determine the functions for the perimeter, surface area and volume of the pyramid.
- Have students describe, either verbally or in writing, the domain and range of each function.

Math of the Inca

Data Analysis, Numbers and Number Sense—Grades K–3

Objectives

Students create questions to be answered as they collect, represent, and analyze related data. The data representation illustrates base-10 place value concepts and techniques similar to those of the ancient Inca.

NCTM Standards (Data Analysis and Probability)

Enable students to:

- formulate questions that can be answered with data and collect, organize, and display relevant data to answer them;
- develop and evaluate inferences and predictions that are based on data.

NCTM Standards (Number and Operations)

Enable students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Materials

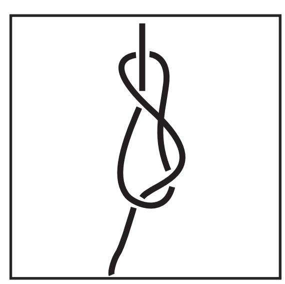
- Colored yarn (each one meter long)
- 3 x 5-inch index cards

Introduce

The Inca created an empire that reached its height of power in the 16th century, when it is thought to have controlled between five and sixteen million people. Organization of such an empire required leaders to be aware of many types of mathematical data, such as the number of people in a community, the amount of corn produced in a season, or how taxes were recorded and collected. The Inca invented a data recording device called the quipu, made of twisted colored cotton or wool cord on which combinations of knots were tied to represent numerical values. The word “quipu” means knot in Quechua, the native language of the Andes. The quipu was often constructed from multiple cords tied together to illustrate the accounting necessary to keep the empire organized and functional. The end at which the cords were connected represented the top or the highest numerical value and the loose end was the bottom or lowest value on the chord.

Values were recorded with knots using the positional base-10 system. A figure-8 knot was used to represent the numeral one. Numerals 2–9 were created using slipknots with the respective number of turns in each knot. Positional place values were created by how the knots were spaced; open spaces represented a change in place value or, if there were no knots in an open space, it represented zero. Spaces between knots or groups of knots represent place values of powers of 10. Groups of 10s, 100s, etc., were created with single knots depicting each count of 10 or 100.

- Connect to the *Math of the Inca* poster by facilitating a discussion of the need and purpose to develop a system of accounting and how such data was represented using the quipu.
- Model how to represent with knots a variety of values ranging from 1 to 19. Provide each student a length of yarn and teach him or her to create a figure-8 knot representing the value of 1. The Inca used slipknots with the appropriate number of turns to represent other values such as 2, 3, etc. For young children with less dexterity, teach them to represent these values as a sequence of individual knots. For instance four separate knots would represent the value of 4.



Tie a figure 8 to represent the value of 1.

- Discuss the importance of collecting and analyzing data and ask students what kinds of data are of interest to them. Suggestions may include number of members in each student's family or votes for a favorite food, pet, or color.

Explore and Create

The class and teacher collaboratively construct a class quipu cord representing the number of family members of the students. Students create individual quipu cords representing the number and gender of members in their families. They distinguish between the number of males and females in their families by using differently colored yarn on which knots are tied. When all students have completed this task, create the class quipu cord by having students tie their individual cords to a main cord. Each student cord is labeled with a masking tape tab printed with each student's name. The class quipu cord represents the number and gender of family members of all the students.

Apply and Extend

Students now are provided an opportunity to analyze and interpret the data of the quipu. The teacher can encourage students to formulate questions they had about the quipu's data. For instance, a student may ask how many total family members there are, how many are male or female, are there more females than males, and so on. The class will need to interpret the quipu data to answer such questions. This mathematics activity can be extended into a language arts activity in which students can draw pictures of their quipu cords and write stories about themselves and their families. This community quipu and the related written stories should be prominently displayed.

Summarize and Assess

■ *Discussion Questions*

- How does having a system of "keeping count" help a community keep track of important data and what can be done with this collected information?
- What data recording devices do students and their family use in their daily lives?

Math of the Inca

Data Analysis, Numbers and Number Sense—Grades 4–8

Objectives

Students create and explore the Inca “quipu,” a powerful device for accounting and transmitting information on knotted, color-coded cords. They use the quipu to explore concepts of ratio by collecting, comparing, and contrasting different sets of data.

NCTM Standards (Number and Operations)

Enable students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems.

NCTM Standards (Data Analysis and Probability)

Enable students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Materials

- Illustration of a quipu (refer to poster)
- Yarn (red, blue, yellow, black)
- Scissors
- Map of the Americas (optional)

Introduce

The Inca quipu provided a highly complex system of counting and record keeping by using various colors of strings with strategically placed knots attached to a base cord. The colors were used to represent different objects or items. For instance, different colored cords could be used to represent the number of people in one community as compared to people in another. Researchers speculate, but have never proven, that the knots may also have been used to represent sounds and consonants in the Quechua language, and that if these could be properly decoded, we could hear a “verbal” recording depicted in the numeric system.

Large values could be represented using the quipu system due to its organization of a positional base-10 system. Positional place values were created by spacing the knots; open spaces represented a change in place value while no knots in an open space represented zero. The number 213 is depicted in block B of the poster and shows knots of 2, 1, and 3 with spaces between them. Groups of 10s, 100s, etc. were created with single knots depicting each count of 10 or 100.

- Facilitate a discussion on the mathematics of the Inca by illustrating an example of a quipu, pointing out the various colors of string and the knots that indicate larger numbers in a decimal system.
- Refer back to the *Math of the Inca* poster to help model how to represent a variety of values ranging from single units to those of 10s, 100s, and 1000s. Distribute a piece of yarn to students and have them practice representing various multiple decimal numbers using the quipu system.
- Once students are comfortable making knots to represent different numbers, give each student three different colored pieces of yarn (yellow, blue, and red). Have the students find their pulse and, at your signal, count their number of heartbeats in one minute. Practicing how to count heartbeats for 15 seconds and having students report out their counts provides a quick assessment of their ability to count heartbeats.

- Have each student create a quipu cord to represent the number of heartbeats before and after moving or jumping vigorously for one minute. Students can use blue yarn to represent their resting heartbeats and red to indicate the heartbeats after movement. Students tie their blue and red yarn to the yellow to create a personal record. All students can tie their quipu cords to the black yarn to create a class count.

  Blue Yarn: Student's number of rested heartbeats before jumping or moving

  Red Yarn: Student's number of active heartbeats after jumping or moving

  Yellow Yarn: Student's total number of heartbeats

  Black Yarn: Class total number of heartbeats

Explore and Create

- Have groups of students compare their yellow quipus and discuss their method of comparing.
- Have students estimate the total number of heartbeats for the class from the data on the quipus.
- Define a ratio as an expression that compares quantities relative to each other. If necessary, facilitate a discussion on ratio by soliciting student responses. Show students the three ways that we can write a ratio (that is, 3:4, ¾, or 3 to 4). Ask students to discuss, with a partner, how their quipu data represents a ratio. What is their ratio of rested heartbeats to active heartbeats? Have them write this ratio in three ways. Does the ratio change if they compared active heartbeats to resting heartbeats? Does the written ratio also change?
- Have students tie their quipus to one black yarn. Looking at the class quipu. Have the class discuss the individual ratios of resting versus active heartbeats. Ask students how they think this might change the class ratio (resting versus active heartbeats) and why.

Apply and Extend

- Have students make or draw a quipu to show the number of students in their school/town/state. This not only extends the understanding of the quipu, but also emphasizes quantity and magnitude.
- Have students work with a partner and think about several additional sets of data they believe they could collect to make a class quipu. Challenge students to think about data that might require the use of five different colored yarns.

Summarize and Assess

■ *Discussion Questions*

- What other methods have people used to pass on their history besides writing and string records?
- Would you call a quipu a form of writing? Why or why not?
- Have students write a short explanation of the purpose and use of the Inca quipu and how learning about this accounting system helped them understand the need for data collection and recording today.
- How does the Inca system of recording quantities compare with our decimal system? How are the systems the same? How are they different?

Math of the Inca

Data Analysis, Numbers and Number Sense—Grades 9–12

Objectives

Students create and explore the Inca “quipu,” a device used to exhibit and communicate information on knotted, color-coded cords. The students use the quipu to represent collected data. Students then enter data into lists in graphing calculators, calculate the linear regression equation, choose the appropriate viewing window for their data, construct a scatter plot, enter the equation of the linear regression equation into the y = screen, and superimpose the line on the scatter plot.

NCTM Standards (Data Analysis and Probability)

Enable students to:

- formulate questions that can be addressed with data and collect, organize and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data.

NCTM Standards (Number and Operations)

Enable students to:

- understand numbers, ways of representing numbers, relationship among numbers and number systems;
- compute fluently and make reasonable estimates.

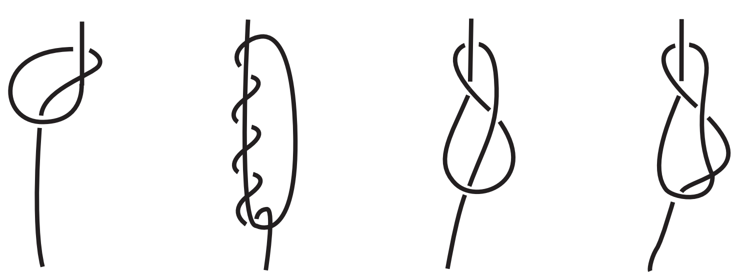
Materials

- Illustration of the quipu
- Different colored yarns
- Scissors
- Poster paper
- Graphing calculator
- Music listening devices
- Computer lab/classroom computers
- Graph paper

Introduce

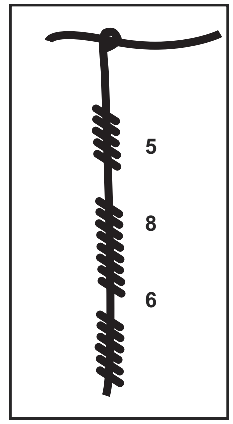
Quipus were recording devices used in the Incan Empire. A quipu consisted of colored, spun and plied thread or strings used to represent different objects or items, which were counted. For instance, different colored cords were used to represent the different types of agricultural produces produced in a given year.

The quipu was not a calculator; rather it was a storage device. The quipu consisted of strings, which were knotted to represent numbers. There were three basic knots: single, long and figure 8 (see diagram below).



Single Knot **Long Knot** **Figure-8 Knot** **Figure-8 Knot With an Extra Turn**

Knots in the string represented a number, using a positional base-10 representations. To represent the number 586 on a quipu (see diagram on the right), there would 6 touching knots near the free end of the string representing the units followed by a space, then 8 touching knots for the tens followed by another space, and finally 5 touching knots for the 100s.



586 on a quipu

For larger numbers, more knot groups were used, one for each power of 10, in the same way as the digits of our base-10 system occur in different positions to indicate the number of the corresponding power of 10 in that position.

- Use the *Math of the Inca* poster to facilitate a discussion on the use and purpose of quipu.
- Model how to create a quipu using knots with different values.
- Distribute a piece of yarn to students to represent different numerical values from small to really large numbers.

Explore and Create

The day before the lesson, ask students to list the number of songs per genre in their music files: iPods, iPhones, MP3 players, cell phones, CDs, etc.

A. Estimation of the data using the quipu:

- Place posters around the room titled with different genres of music.
- Give students five minutes to put the number of songs they listen to for each genre and to record a zero if they do not listen to a given genre of music.
- Depending on the number of small groups, assign 1–3 genres of music to analyze.
- Have the students create a quipu cord for each genre.
- Have the students use the quipu to make observations of the data and estimate the mean, median, and mode.

B. Extending data analysis:

- Have the students do the following: calculate the mean, median and mode. Construct a histogram and box-and-whisker plot. Next, calculate the linear regression equation, adjust the viewing window, construct a scatter plot, and superimpose the linear regression equation onto the scatter plot.
- Have students compare these results with their estimate in Part A.

Apply and Extend

In the computer lab and/or the classroom, the students design their PowerPoint presentations and present their project to the whole class.

Summarize and Assess

■ *Discussion Questions*

- When comparing estimated and calculated results what surprised you about the results?
- If a similar analysis were conducted in a different class or school, how might the data be the same or different than our class data? What might influence this?
- Compare the advantages and disadvantages of using quipus to represent data and with the data storage devices we have today.

Math of the Aztec

Numbers and Number Sense—Grades K–3

Objectives

Students use the pictorial Aztec numbers to solve a variety of addition and multiplication problems.

NCTM Standards (Number and Operations)

Enable students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Materials

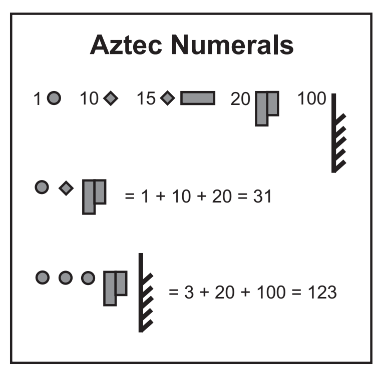
- Colored pencils or other markers
- 3 x 5-inch index cards
- Base-10 manipulatives

Introduce

The Aztec created a vast empire with a population in the millions, which rose to its greatest prominence between the 14th and 16th centuries. It is believed that the people known as the Aztec migrated from what is now northern Mexico to populate the Valley of Mexico as early as the 12th century. Legend has it that their Aztec god, Huitzilopochtli, commanded them on a journey south to seek a sign of an eagle eating a serpent. On an island in the middle of a lake in the Valley of Mexico, searchers witnessed an eagle perched on a cactus devouring a serpent. A major city named Tenochtitlan sprung up which now is known as Mexico City.

The Aztec, guided by a powerful leader Itzacoatl (1428–1440), battled other local populations for territorial control and eventually prevailed and expanded. The Aztec were industrious people and demonstrated amazing mathematical, scientific, economic, and engineering skills in the temples they constructed, the ways they converted rich land for farming from the lake, and in aqueducts constructed to provide fresh water. Commerce was developed on trade and production as skilled laborers, artisans, and craftspeople also made their physical contributions. Power and prestige grew through a strong emphasis on religion and government. The powerful Aztec were eventually conquered during the late 16th century by Spanish conquistador Cortez and his army strengthened by local enemies. Today, we marvel at the sophistication and power of this ancient culture, the amazing Aztec.

- Connect to the *Math of the Aztec* poster by discussing with students the mathematical contributions depicted which included the base-20 counting system with its pictorial numerals, the complex calendar system, and the unique way the Aztec calculated perimeter and area (a system that is still not fully understood).
- Model the Aztec pictorial numerals system and its reliance on counting groups of 20. Request students to draw depictions of the Aztec symbols or 1, 20, 100, … on separate 3 x 5 cards. Use the cards to model and describe how the Aztec depicted larger values as a series of drawings.
- Finally, using Aztec numeral cards to represent a variety of values demonstrates the base-10 equivalence of those values with base-10 manipulatives to show partial sums or products.



Explore and Create

Using the cards, the class and teacher collaboratively create a variety of values of Aztec numbers, appropriate for the grade level. These pictorial numbers on cards should then be represented with base-10 manipulatives to show partial sums or products (for example, $14 + 27 = (10 + 20 + 11) = (30 + 10) + 1 = 41$), or $12 \times 3 = (10 \times 3) + (2 \times 3) = 30 + 6 = 36$). The teacher and students collaboratively write the representative base-10 algorithm(s).

Apply and Extend

- Challenge students, working in pairs, to imagine the life of an Aztec youth near their age and think of an experience they might encounter with numbers. Have them draw the Aztec symbol for the specific number or numbers, draw the Aztec value and the picture of base-10 manipulatives, and compare these two symbol systems. Have students write a short description of their value and its use by imagining that they were an Aztec youth living centuries ago. The class pages should be collected and a class book of the mathematics of the Aztec compiled.
- Have students compare several of their Aztec numerical models with those of the Mayan numerals they constructed in a previous activity. How are they similar and different?
- Students wanting an extra challenge can suggest several Aztec values and use these values to experiment with adding, subtracting, multiplying, or dividing. They can reflect on the experience and write a short description of how calculating with the Aztec numerals is similar and different from those in base-10.

Summarize and Assess

■ *Discussion Questions*

- How is the mathematics of the Aztec similar and different from the Mayan and Incan systems presented on the posters?
- You have learned that the Aztecs used pictures to represent their numerals. Do the pictures help you better understand the values described? Why or why not?

Math of the Aztec

Algebra—Grades 4–8

Objectives

Students explore the slope of Aztec pyramids, the ratio of the height to the setback of the wall, and compare this ratio to the slope of modern day stairs. Students learn about the twin staircases of the Templo Mayor in Tenochtitlan and how the steepness of this pyramid supported religious beliefs.

NCTM Standards (Algebra)

Enable students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols.

Materials

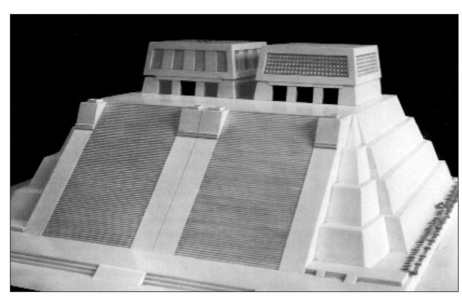
- Measuring tape or rulers
- Graph paper
- Pictures or Templo Mayor of Tenochtitlan and Egyptian pyramid Khufu (use Internet)
- Map of Mexico (optional)

Introduce

The Aztec, who dominated and ruled Southern and Central Mexico between the 14th and 16th century, established one of the most advanced civilizations in the Americas. It is believed that some of their cities, during their reign, were as large as any in Europe. By the late 1300s their capital city of Tenochtitlan, which was comprised of aqueducts, bridges, and chinapas (small islands formed by piled up mounds of mud), was completed. On these chinapas, the Aztecs grew corn, beans, chili peppers, squash, avocados, tomatoes, sweet potatoes, and tobacco. One of the most noticeable features of Tenochtitlan was the towering temples that were constructed to honor the Aztec gods.

The early Aztecs used a style very similar to the Maya when building their pyramids. One noticeable difference though is the use of a twin staircase built in order to accommodate two temples. A twin staircase can be found in the Templo Mayor of Tenochtitlan. In the Templo Mayor, each staircase was dedicated to a separate god. The left staircase was dedicated to Tlaloc (god of rain and fertility) and was colored in blue and white paint to symbolize water and moisture. The right staircase, on the other hand, was dedicated to Huitzilopochti (god of war and sun) and was painted in red and white. This pyramid was created in such a way that one could not see the temples unless one climbed to the top of the pyramid. Aztecs believed that gods lived in the sky, thus they typically built their pyramids very steep in order to be physically closer to the gods above.

- Connect to the *Math of the Aztec* poster by discussing the mathematical contributions of the Aztec people. Facilitate a discussion on the religious beliefs of the Aztecs and how pyramids were created as a way of accessing or being closer to the gods in the sky.
- Ask students to work in pairs to discuss what these Aztec pyramids might have looked like and what mathematical concepts might have been used when building them. Challenge students to think about the steepness of the pyramid and how the depth and height of the stairs might affect the steepness, where depth is the measure of the stair from front to back and height is its vertical measure.



Templo Mayor. Photo courtesy of Museo del Templo Mayor, INAH, Mexico. Credit: Matos Moctezuma, Eduardo, The , Great Temple of the Aztecs, Treasures of Tenochtitlan, Colin Renfrew (general editor), translated from the Spanish by Doris Heyden, Thames and Hudson, London, 1988, p. 19.

Explore and Create

- In pairs, have students measure the slope of several sets of stairs. They can use a measuring tape to measure the height and depth of the step and record their findings. Have students use graph paper to draw a representation of the two measurements. Older students can calculate the slope. If using graph paper, they should draw a side view of the stairs several times to help them "see" the steepness of the stairs they measured.
- Have students compare the stairs of their school to the Aztec pyramid stairs, which typically measure 9 inches high and 6 inches wide. Students who have used graph paper before can now show a side view of the pyramid's stairs. How do the two slopes compare?
- Have students write a brief paragraph about the difference in the slope of the two sets of stairways (school and Templo Mayor). Students should then share their responses with their partner.

Apply and Extend

- Have students make a conjecture about possible lengths and heights of stairs that would give steeper or less steep stairs than those of the Aztec pyramid, and then prove their conjecture.
- Ask students to compare the slope of the Khufu pyramid (in Egypt), which is a square based pyramid, to the Templo Mayor. The height of the Khufu pyramid is 482 ft. and each side of the base is 756 ft. It is believed that the Templo Mayor had a height of approximately 197 ft. and a base of 330 ft. by 260 ft.
- For the older grades, challenge students to find a linear equation that models the slope of the Khufu pyramid and Templo Mayor. Encourage students to graph both lines using different colors. How are the lines different? How are they similar?

Summarize and Assess

■ *Discussion Questions*

- Why do you think Aztec pyramids have stairs and Egyptian pyramids do not? Investigate their purposes and uses.
- How do you think the Aztecs created such precise and elaborate pyramids and temples? Besides the concept of slope, what other mathematical concepts do you think are intricately embedded in the way in which these pyramids were built?
- Investigate the differences between the Aztec pyramids, which typically consisted of twin stairways, and the Mayan Pyramids, which only consisted of a single stairway.

Math of the Aztec

Geometry, Data Analysis—Grades 9–12

Objectives

Students create and explore the Aztec calendar. Students construct an Aztec calendar using ®The Geometer's Sketchpad and estimate the areas of common geometric figures before they calculate them using technology. Students perform transformations within the circle to construct the calendar and compute areas of polygons.

NCTM Standards (Geometry)

Enable students to:

- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

NCTM Standards (Data Analysis and Probability)

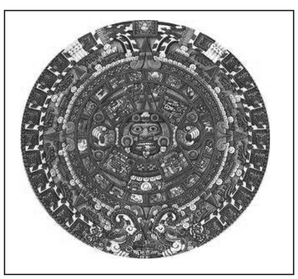
Enable students to:

- understand and apply basic concepts of probability.

Materials

- Illustration of the Aztec calendar
- 11 x 17-inch white paper
- Colored pencils
- Scientific and/or graphing calculator
- ®The Geometer's Sketchpad
- Compass
- Protractor

Introduce



The Aztec sun calendar is a circular stone with pictures representing how the Aztec measured days, months, and cosmic cycles. The Sun Stone or Calendar Stone is evidence of the Aztec's knowledge of astronomy and mathematics. The calendar contained the pictographs for their days, months and suns (cosmic cycles). The calendar was used by the Aztec to keep track of days, and the deities and rituals connected to the day's number. This information was vital because it was believed if it was forgotten the world would end. An examination of the Aztec calendar depicts concentric circles containing various symbols called glyphs. The circular constructions represented both the cyclical nature of time and the Aztec belief in the interrelatedness of differing rituals and beliefs.

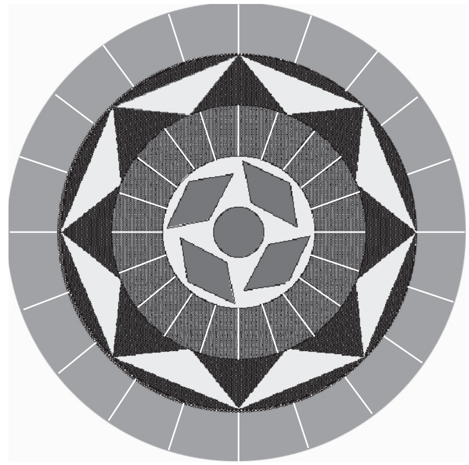
The stone is about 3.6 meters (12 feet) in diameter and weighs about 24 metric tons. It took 52 years to complete, from 1427–1479, presumably due to the use of only stone tools. This calendar is 103 years older than the Gregorian calendar, which is used worldwide today.

Originally the Calendar Stone was placed atop the main temple in Tenochtitlan, the capital of the Aztec empire. Today, Mexico City's cathedral stands on the site. The Aztec calendar faced south in a vertical position and was painted a vibrant red, blue, yellow and white. The Spaniards buried the stone when they conquered Tenochtitlan. The stone was lost for over 250 years until December of 1790 when it was found by accident during repair work on the cathedral. Today, it is located in the National Museum of Anthropology in Mexico City.

Use the *Math of the Aztec* poster to facilitate a discussion on the characteristics and meaning of the Aztec calendar.

Explore and Create

- Distribute white paper or create a diagram using ®The Geometer's Sketchpad.
- Draw five concentric circles with radii of 1, 3, 5, 7 and 9 inches. From the center out, the regions are numbered zones 1, 2, 3, 4, 5.
- Draw 4 evenly spaced congruent quadrilaterals in zone 2.
- Subdivide zone 3 into 20 congruent sections.
- Draw 8 evenly spaced congruent triangles in zone 4.
- Subdivide zone 5 into 20 congruent sections.



Apply and Extend

Use a separate piece of paper for the following:

- Find the area of each zone in the calendar created.
- In zones 2 and 4 find the area of each type of polygon.
- In zone 5, find the area of one of the sections.
- Consider your current age to the nearest year, and suppose the calendar you created represents your life events to date. The area of the whole calendar is your current age.
    - What percentage of time does each of the zones represent in your life?
    - What percentage of time does each type of polygon represent in your life?
    - Your life is divided into five zones. For each zone, describe a personal meaning or event(s) that reflects the percentage of your life. Color each zone and create a legend explaining the colors and their meaning in your life.
- The original Aztec calendar is about 12 feet in diameter. Estimate the radii of each of the concentric circles. Note that the distance between the circles is not uniform. Use proportions to estimate the radii.

Summarize and Assess

■ *Discussion Questions*

- Why does it make a difference which calendar we use?
- What are the advantages and disadvantages of using the Aztec calendar and the calendar we use today?
- Are there any attributes of our current calendar system that illustrate interrelatedness similar to the Aztec Calendar?
- If the radius of a whole circular calendar is twice the radius of its inner circular calendar, how does the area it covers compare to the area of the inner calendar?

Math of the Navajo

Geometry and Measurement—Grades K–3

Objectives

Students identify shapes to include triangles (isosceles, scalene, right, equilateral), rhombi, circles, squares, and rectangles. Students sort and classify their various shapes. Students develop and evaluate inferences and predictions that are based on data. Students measure length and width of rug samples using non-standard measurements.

NCTM Standards (Geometry)

Enable students to:

- use visualization, spatial reasoning, and geometric modeling to solve problems.

NCTM Standards (Measurement)

Enable students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement.

Materials

- Standard measuring tools (scales in both customary and metric standards)
- Colored yarn in one meter lengths
- 11.5 x 17-inch paper
- 3 x 5-inch index cards
- Colored construction paper

Introduce

The Navajo are a Native American Nation currently inhabiting tribal lands, which include vast expanses of Arizona, New Mexico, and Utah. The Navajo, who often refer to themselves as *Diné* (or the people), number more than 250,000 and comprise the largest Native American population. The Navajo have resided in the southwestern United States since approximately 1690. Historians trace their roots to Athabasca communities in northwestern Canada and Alaska where the Diné were typically hunters and gatherers. No one is certain exactly why or when the Diné made the apparent migration to their new home. Once in the Southwest, the Diné learned to farm and raise animals from their Pueblo neighbors. From them, they learned how to farm corn, beans, squash, and melons as well as raise sheep and a few horses.

The Diné also learned how to weave and incorporate Pueblo art styles in the items they produced. From the Spanish explorers in the area they gained a special type of sheep called the “Churro” that are highly valued for the high quality and long-fibered weaving wool. To the Diné, this sheep was a symbol of “Good Life” and the need in one’s life for order, harmony, balance, and beauty. A cornerstone of the Diné is the concept of *Hózhóa*, a term in Navajo which means, “to walk in beauty.” *Hózhóa* is displayed in the beautiful rug weavings, typically made by Diné women showing their intuitive understanding and application of numerous mathematical concepts. The rugs, originally woven for personal use, were later made and sold to tourists venturing into Navajo lands within the last century. Diné rug weavers have expanded their artistic creativity to incorporate a variety of styles that often depict a particular locale or period within their culture. The rugs, with their rich mathematical connections are highly valued today and can be studied by exploring various Websites. The Diné, a proud people with strong cultural traditions, continue to grow as they “walk in beauty” as Americans in the 21st century.

- Connect to the *Math of the Navajo* poster by facilitating a discussion of the types of shapes (include embedded shapes) on the poster and in rug images located on the Internet. Name the shapes viewed and describe their attributes. Discuss the perspective of Hózhóa and how this desire for balance and order in the patterns is incorporated in the rug designs.

- For younger students, cut examples of the shapes from colored paper. Collaborate to sort, classify, and count the number of various shapes (individual, embedded, similar, and congruent). Assess whether students recognize the same shapes regardless of orientation. Create a chart or graph to record the type and number(s) of each shape. Guide the students now to analyze the data. Ask students to suggest their own analyses. For instance, how many more of a certain shape are there than another? Continue the comparison and analysis.
- Define the concept of perimeter and demonstrate the perimeter of the poster or rug image. Invite the students to suggest items a Navajo weaver of the past may have used to measure the perimeter of their rug. Use their suggestions (such as, hands, feet, fingers, and so on) to model how to measure the perimeter of the poster, asking students to first guess their estimate before verifying. If two different people each measure the perimeter with their hands, will the measurements be exactly the same? Why or why not? Why does it matter?
- Investigate the perimeter and area of the poster using standard customary and metric units. Ask the students what they think defines the type of measurement a person uses.

Explore and Create

Provide students a large sheet of paper (11.5 x 17 inch) with which to design a model of their own rug to illustrate Navajo mathematical concepts. Encourage students to use various geometric shapes cut from colored construction paper to explore a rug design. (Younger students should be provided with a variety of shapes in various colors.) Remind the students of the Diné desire for balance and harmony, and ask students to create their rugs depicting their impressions of these aspects. Some Navajo rugs illustrate stories and provide a message to those who understand the symbols, shapes, or patterns used. Have students create their rug model so that it also communicates a message or story. On a separate sheet of paper, have the students draw or glue copies of the shapes they used. Have them write the name and a brief description of the shape’s characteristics.

Apply and Extend

Students can create a chart describing the types of geometric shapes they used in constructing their rug and the number of each shape used. On the back of their “rug” sheet, students write at least three statements describing an analysis of their data. The youngest students may, for instance, state, “I have more triangles than squares.” An older student could deduce that she has, “two times as many circles as rectangles and half as many triangles as squares.” Students share and discuss their insights and comment on what they notice about another’s rug and related analysis.

Summarize and Assess

■ *Discussion Questions*

- What was/is the role of mathematics in the life of a Navajo man, woman, or child? How might each have used mathematics in similar and different ways?
- Can Navajo rugs be identified by certain shapes and styles? What types of ideas or feelings do these Navajo patterns inspire for children?
- What mathematical designs and shapes can be traced to specific “cultures” students encounter in their daily lives?

Math of the Navajo

Geometry and Measurement—Grades 4–8

Objectives

Students create a Navajo rug by applying mathematical principles. Students investigate transformations, symmetry, and area of different geometric shapes.

NCTM Standards (Geometry)

Enable students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- apply transformations and use symmetry to analyze mathematical situations;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems.

NCTM Standards (Measurement)

Enable students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Materials

- Colored pens
- 18 x 24-inch construction paper
- Ruler
- Cardstock or graph paper, as appropriate
- Scissors
- ®The Geometer's Sketchpad (if appropriate)

Introduce

The Navajo, located mainly in Arizona, New Mexico, and Utah, are the largest tribe of Native Americans in the United States. The Navajo are famous throughout the world for their woven rugs and textiles.

Mathematical principles are intricately embedded in the geometric rug patterns. For example, repeated geometric shapes and lines are a result of translating, reflecting, rotating, and dilating shapes. Generally, the borders of the rugs are characteristic of a repeated pattern made by translations along the perimeter of the rug.

Women are usually the weavers in the Navajo culture. Navajo rugs are extremely detailed and usually display geometric symmetry. What is more amazing is that the intricate geometric shapes and patterns embedded in the rugs are not drawn on paper before weaving begins. Instead, Diné women mentally keep track of the pattern at a very detailed level.

- Refer students back to the *Math of the Navajo* poster and have them work in small groups to develop a list of mathematical concepts that are represented in the poster. They make mathematical observations about the design of the rug. As students report out, record the mathematical principles on the board or overhead so that students can refer back to them. Engaging the students in using descriptive mathematical language about the rug design prepares them for further work.

- Emphasize that Diné women mentally keep track of the rug pattern at a very detailed level when weaving rugs. Complement this by asking students to share any ideas on how it is possible to weave such a detailed and symmetric design without designing it on paper first. Use their suggestions to model how the coordinate plane can be used, in conjunction with transformations, to design geometrically detailed and symmetric rug designs.

Navajo Transitional Blanket, circa 1890, 61 by 42 inches.
Image courtesy of Mark Sublette Medicine Man Gallery, Tucson, AZ and Santa Fe, NM.

Explore and Create

- Ask students to create Navajo rugs using 18 x 24-inch construction paper. Students first create a geometric design and then use cardstock to trace and cut out the designs. Students then apply transformations to create designs for their rugs. Older students can use graph paper instead of cardstock so that they can apply transformations using coordinate points. Students can also use the ™Sketchpad to design their rugs.
- As part of the activity, students write detailed "instructions" on how to develop their designs. The goal is to encourage students to use mathematical language when describing their designs, as well as to conceptually understand the mathematical principles behind their Navajo designs.
- Finally, students present their Navajo rugs to the whole class. Encourage students to ask questions of each other about the mathematical concepts that were used to create the designs.

Apply and Extend

As an extension, ask students to work in groups of 3 or 4 and discuss how to find the:

- area of the entire rug;
- area of the geometric shape that was created using either the cardstock or graph paper.

Then, have students find the areas of their rugs and share with the whole class.

Summarize and Assess

■ *Discussion Questions*

- Navajo weavers almost never repeat a rug pattern a second time. Why do you think this is the case?
- What mathematical principles would be most useful to a Navajo rug weaver?
- Why do you think weavers generally do not draw their patterns on paper before weaving?
- Do you think the extensive use of symmetry found in so many Navajo rug designs makes it easier or more difficult to weave the rugs?

Math of the Navajo

Geometry—Grades 9–12

Objectives

- Students learn the effect of rigid motions (transformations) on figures in the coordinate plane and space, including translations, reflections, rotations, and dilations.

NCTM Standards (Geometry)

Enable students to:

- analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

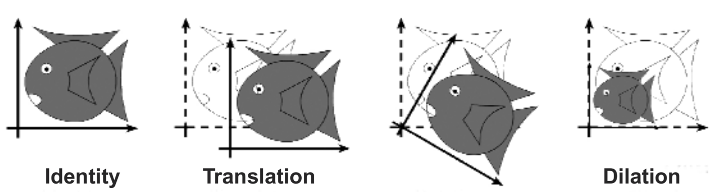
Materials

- Graph paper
- Rulers
- Colored pencils
- Computer lab/computers with access to the Internet
- Examples of Transformations sheet

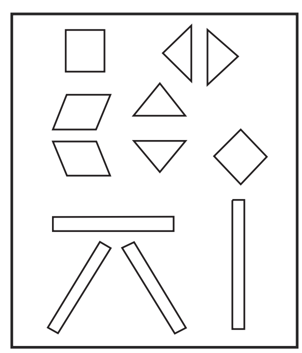
Introduce

The Navajo are the largest tribe of Native Americans in the United States. Located mainly in Arizona, New Mexico, and Utah, the Navajo are famous throughout the world for their woven rugs and textiles. The oldest Navajo textiles were blankets, done primarily in geometric patterns with the natural colors of sheep's wool: grey, cream, black and brown. Some vegetable dyes also were used to produce shades of red. Creating a blanket was a process of carding, spinning, dying, and weaving. Today, Navajo weavers primarily make rugs.

Transformations are abundant in geometric rug designs. The borders generally feature a repeated pattern made by translations.



Identity Translation Rotation Dilation

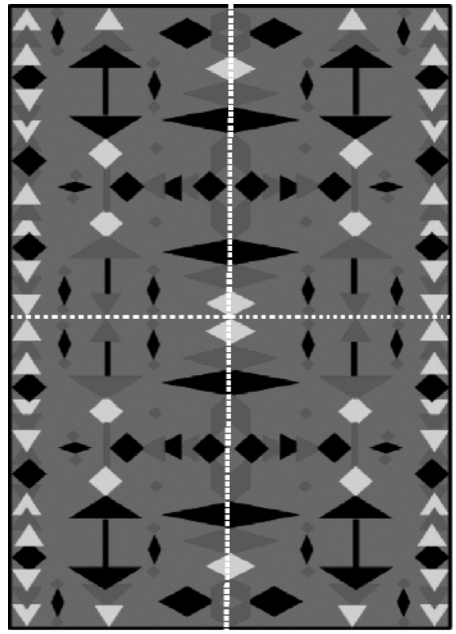


Examples of Transformations

- Use the *Math of the Navajo* poster to facilitate a discussion on the different types of transformations (translations, reflections, rotations, and dilations).
- Model how to create a Navajo rug by designing 1/4 of the rug in the first quadrant of a coordinate plane. Model transformations of shapes from the Examples of Transformations illustration to create the rest of the Navajo rug in the other three quadrants.
- The following Website can be used to model to the students how to create a Navajo rug: http://www.statemuseum.arizona.edu/activities/symmetry/dsgn_yr_own.shtml.

Explore and Create

- Distribute graph paper, rulers, colored pencils and a copy of the geometric shapes in the Examples of Transformations sheet.
- Some students may want to cut out the shape examples to transform and trace.
- Have students design a rug using the minimum design requirements below.
- Students can also use the ™Sketchpad or the Website link on page 28 to design their rugs. It is helpful to print the designed rugs on graph paper for the Apply and Extend activities.



Minimum Design Requirements

- Use at least three different shapes chosen from the examples.
- Use each type of transformation at least once.
- Compose a new figure by combining several shapes.

Apply and Extend

On a separate piece of paper, complete the following tasks:

- Choose one of the shapes (original figure) and identify its coordinates. Identify the coordinates of a translated image of the shape. Find the rule that generalizes the coordinates of the translated image.
- Choose another shape, different from the previous shape selected (original figure) and identify its coordinates. Identify the coordinates of a rotated image of the shape. Find the rule that generalizes the coordinates of the rotated image. Identify the point of rotation and an approximate angle of rotation.
- Repeat the process above for a reflected image. Also identify the line of reflection.
- Repeat the process for a dilated image. Determine an approximate percent of enlargement or reduction.

Summarize and Assess

■ *Discussion Questions*

- Why do you think earth tones (grey, brown, cream, black, etc.) are so frequently used in Navajo blankets and rugs?
- Describe the different lines of symmetry in your design. Do any sections of the design have their own symmetry?
- Describe at least three simple ways the design you created can be varied. That is, if you are starting over, how could you use templates differently but still produce a symmetric design?

References

Bishop, A. J. (1988). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

About *TODOS: Mathematics for ALL*

The mission of *TODOS: Mathematics for ALL* is to advocate for an equitable and high quality mathematics education for all students—in particular Hispanic/Latino students—by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

About *Mathematics of the Americas*

The poster set *Mathematics of the Americas* offers teachers an opportunity to make historical, cultural, and mathematical connections in the classroom. These connections transcend barriers and reinforce both the mathematics and the cultures being studied, including the Maya, Inca, Aztec, and Navajo.

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